PP4: Example Sheet 2

Relativistic Kinematics

1. A pion decays at rest into a muon plus a neutrino ($\pi^+ \to \mu^+\nu_\mu$). What is the speed of the muon? How far will the muon travel in vacuum before disintegrating?

   Cosmic ray muons are produced high in the atmosphere, say at 10 km, and have an energy of about 2 GeV. What is the speed $\beta = v/c$ of such a muon? How far will the muon travel before disintegrating? The pion has mass $m_\pi = 139.6$ MeV, the muon has mass $m_\mu = 105.7$ MeV and lifetime $\tau_\mu = 2.197\ \mu$s.

2. In the rest frame of a particle with mass M, decaying into 2 particles with mass m_1 and m_2, derive the relation for the momenta $\vec{p_1}$ and $\vec{p_2}$ of the final state particles.

$$| \vec{p_1} | = | \vec{p_2} | = \frac{\left[\left(M^2 - (m_1 + m_2)^2\right)\left(M^2 - (m_1 - m_2)^2\right)\right]^{\frac{1}{2}}}{2M}$$

   How do these relations simplify for the two cases, $m_1 = 0$, and $m_1 = m_2 = 0$, respectively?

3. What are the Centre-of-Momentum (CoM) energies of the following machines: (See section 22 of the PDG for a summary of HEP machines.)

   LEP1: e^+e^- collider, both beams 45.6 GeV

   LHC: *pp* collideer, both beams 7 TeV

   HERA: ep collider, $E_\mathrm{e} = 30$ GeV and $E_\mathrm{p} = 820$ GeV (m_p=938.272 MeV/c^2)
   If HERA were a fixed target machine what energy would the electron require to give an equivalent CoM energy?

4. The cross section for the process $\pi^- p \to \Sigma^0 K^0$ is about 0.350 mbarn. The liquid hydrogen target is 180 cm long and has a density of 0.07 g cm^{-3}. The beam hits the target in bunches of 10^3 pions with 6 bunches per minute. How many of these reactions are produced per minute? Note that 1 barn = 10^{-24} cm^2.

5. The LEP accelerator has a circumference of 26.66 km, it contains counter circulating electron and positron beams, each of 8 bunches. The bunches have transverse sizes $\sigma_\mathrm{horiz} = 200\ \mu$m and $\sigma_\mathrm{vert} = 8\ \mu$m and contain 2×10^{11} particles each. What is the luminosity in units of cm^{-2} s^{-1} or pb^{-1} s^{-1}?

   Use the relation that if particle bunches in an accelerator have Gaussian profiles then their effective transverse area is:

$$A = 4\pi\sigma_\mathrm{horiz}\sigma_\mathrm{vert}$$