PP4: Example Sheet 4

Quark model

1. Plot the baryon octet ($J^P = \frac{1}{2}^+$) as function of hypercharge Y versus third component of the isospin I_3 and give the quark model composition of each state. How is the charge Q of these baryons related to Y?

2. What are the main decay channels for the Λ^0 and the Σ^0 baryon? Why is the lifetime of the Λ^0 baryon 2.6×10^{-10} s while the lifetime of the Σ^0 is 7.4×10^{-20} s?

3. Using the Gell-Mann Okubo mass formula

$$\langle M \rangle = M_0 + YM_1 + M_2 \left[I(I+1) - Y^2/4 \right]$$

derive a mass relation between the different iso-multiplets of the baryon octet and compare this relation with the measured values.

4. Write down the wavefunction for a proton with spin up in full flavour and spin space. Derive the expression for the proton magnetic moment

$$\mu_p = \tfrac{1}{3}(4\mu_u - \mu_d)$$

5. The chromomagnetic spin-spin coupling gives the following mass formula for baryons:

$$m(q_1q_2q_3) = m_1 + m_2 + m_3 + b\left[\sum_{i>j} (\vec{\sigma_i} \cdot \vec{\sigma_j})/m_i m_j \right]$$

Using $m_u = m_d = 363$ MeV, $m_s = 538$ MeV, and $b/m_u^2 = 50$ MeV, calculate the mass of the following baryons: $N(939)$, Δ, Λ, Σ, Ξ, $\Sigma^\star$, $\Xi^\star$, Ω and compare with the observed values.