PP4: Example Sheet 6

QED, DIS, QCD

1. The Dirac equation for a free particle in momentum space can be written as

$$(\gamma^\mu p_\mu - m)u(p) = 0$$

Give the solutions for $u(p)$ and the corresponding energy eigenvalues for a particle at rest. Explain the negative energy solutions. Why are there four solutions?

2. Draw the Feynman diagram for electron muon scattering $e^-\mu^- \to e^-\mu^-$ and write down the matrix element for this process.

How many of the 16 possible spin configurations contribute to the the matrix element of $e^-\mu^- \to e^-\mu^-$ in the non-relativistic limit? Using the following relations

$$\bar{u}^{(s)}\gamma^\mu u^{(s)} = 2m \quad \text{if} \quad \mu = 0 \qquad\qquad \bar{u}^{(s)}\gamma^\mu u^{(s')} = 0 \quad \text{if} \quad s \neq s'$$

derive the spin-averaged matrix element squared for unpolarised e^- and μ^-

$$\langle| \mathcal{M} |^2\rangle = 16e^4 m_e^2 m_\mu^2 \left(\frac{1}{t^2}\right)$$

where $t = (p_1 - p_3)^2$, m_e (m_μ) is the electron (muon) mass, and p_1 and p_3 are the 4-momenta of the incoming and outgoing electron, respectively.

3. The cross section in the laboratory frame for scattering of electrons off target particles with mass M at rest in the limit where the recoil of M can be neglected is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{lab}} = \frac{1}{64\pi^2 M^2}\langle| \mathcal{M} |^2\rangle$$

Derive the Rutherford cross section for the scattering process $e^-p \to e^-p$ for point-like protons in the non-relativistic limit

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{lab}} = \frac{m_e^2\alpha^2}{4p^4 \sin^4 \frac{\theta}{2}}$$

where $p = |\ \vec{p_1}\ | = |\ \vec{p_3}\ |$ and θ is the scattering angle between the incoming and outgoing e^-.

4. Determine the total hadronic cross sections and the value of

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

for at CoM energies $\sqrt{s}$ of 2 GeV, 5 GeV, and 30 GeV. How many events are produced per year at each of these energies with a collider that has a mean luminosity of 10^{31} cm^{-2}s^{-1}?

A look ahead to weak interactions: Why is this approach not valid around $\sqrt{s} = 3.1$ GeV, $\sqrt{s} = 9.4 - -10.6$ GeV and at $\sqrt{s} = 91.2$ GeV?

5. Write down the Feynman diagram for the scattering of a charged lepton off a nucleon at rest. For an elastic collision the cross section is given by

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{lab}} = \frac{\alpha^2}{4E_1^2 \sin^4 \frac{\theta}{2}} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{1+\tau} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right)$$

where E_1 and E_3 are energy of the initial and final lepton, respectively, and θ is the scattering angle. What is the relationship between q^2 and the energy transfer $\nu = E_1 - E_3$? What is the physical interpretation of q^2 and $G_E(q^2)$ and $G_M(q^2)$? How and why does $G_{(E,M)}(q^2)$ vary as a function of q^2? The data are well described by the following form for $G(q^2)$

$$G(q^2) = \left(\frac{G(0)}{1 + \mid \vec{q} \mid^2 / 0.71} \right)^2$$

with q^2 in GeV^2. Show that an exponential charge distribution in the proton

$$\varrho(r) = \varrho_0 \exp(-r/r_0)$$

leads to this form for $G^2(q^2)$ and calculate r_0. Hint: Put back the $\hbar$'s amd c's and use $\hbar c = 197$ MeV $\cdot$ fm.