PP4: Example Sheet 7

Weak Interactions

1. Which particle mediates muon decay $\mu^- \to e^-\bar{\nu}_e\nu_\mu$? Write down its Feynman propagator in the limit of small momentum transfers q^2. The total muon decay rate is

$$\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3} \quad .$$

   How is the Fermi coupling constant G_F related to the propagator? Which two experimental measurements are used to determine G_F?

2. When neglecting the electron mass the double differential decay rate for $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ in the rest frame of a polarised the muon is given by

$$\frac{\mathrm{d}\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu)}{\mathrm{d}x\,\mathrm{d}\cos\theta} = \frac{G_F^2 m_\mu^5}{192\pi^3} x^2\,(3-2x)\left(1+\frac{1-2x}{3-2x}\cos\theta\right)$$

   where $x = E_e/(m_\mu/2)$ is the reduced energy of the emitted electron and θ is the angle between the electron direction and the spin $\vec{s}_\mu$ of the muon. Draw a diagram showing the particle helicities in the muon rest frame in the case where the electron has its maximum kinematically allowed energy. Explain in the limit $m_e = 0$ why the rate for an electron being emitted along (opposite) the direction of the spin $\vec{s}_\mu$ of the muon is zero (maximum).

3. The B_d^0 meson with a mass $m_B = 5.279$ GeV is the lightest neutral B meson. What force is responsible for its decay? In the spectator model the decay of the B_d^0 is dominated by the decay of the $\bar{b}$ quark and the d quark acts only as a "spectactor" in the decay. What are the main decay modes of the B_d^0 in this simplified model. Draw a Feynman diagrams for these decays. Which of these decays will have considerable phase space suppression?

   Neglecting phase space effects estimate the semileptonic branching ratio $\mathcal{B}(B_d^0 \to Xe^+\nu_e$, where X stands for hadrons.

   Explain how the charge of the lepton in this decay mode allows you to infer whether it was a B_d^0 or a $\bar{B}_d^0$ that decayed.

   Using muon decay as a prototype V-A interaction write down the decay rate for $B_d^0 \to Xe^+\nu_e$. Given the mass and lifetime of the muon, $m_\mu = 105.7$ MeV and $\tau_\mu = 2.2\mu s$, the mass of the b quark, $m_b = 4.5$ GeV, the branching ratio $\mathcal{B}(B_d^0 \to Xe^+\nu_e) = 10.5\%$ and $|V_{cb|} = 0.04$ estimate the lifetime of the B_d^0 meson.

4. Write down the Wolfenstein parameterisation of the CKM matrix. Using the measured values of $\mid \mathrm{V_{us}} \mid= 0.2196 \pm 0.0023$, $\mid \mathrm{V_{cb}} \mid= (41 \pm 2) \times 10^{-3}$, and $\mid \mathrm{V_{ub}} \mid= (4.0 \pm 0.8) \times 10^{-3}$, find the values for λ, A, and ϱ.

Use the above values to predict the *relative* decay rates for the decays

(a) $B \to \pi \ell \nu_\ell$ and $B \to D \ell \nu$

(b) $\bar{B}^0 \to D^{*-}\pi^+$ and $\bar{B}^0 \to \varrho^- \pi^+$.

5. What criteria must be satisfied for particle–antiparticle mixing to be possible. List the particles which satisfy the criteria and try to draw appropriate Feynman diagrams.

6. By considering the decays of the K^0 and $\overline{\mathrm{K}}^0$ derive the following expression for the semi-leptonic *charge asymmetry*:

$$\delta_\ell = \frac{\Gamma(\mathrm{K}_\mathrm{L}^0 \longrightarrow \pi^- \ell^+ \nu_\ell) - \Gamma(\mathrm{K}_\mathrm{L}^0 \longrightarrow \pi^+ \ell^- \bar{\nu}_\ell)}{\Gamma(\mathrm{K}_\mathrm{L}^0 \longrightarrow \pi^- \ell^+ \nu_\ell) + \Gamma(\mathrm{K}_\mathrm{L}^0 \longrightarrow \pi^+ \ell^- \bar{\nu}_\ell)} = \frac{2Re(\epsilon)}{1+|\epsilon|^2} \approx 2Re(\epsilon)$$

A non-zero value of δ is a signal for CP-violation in the $\mathrm{K}^0 - \overline{\mathrm{K}}^0$ mixing.

7. Consider the time evolution of an initially pure $\overline{\mathrm{K}}^0$ beam: what are the K^0 and $\overline{\mathrm{K}}^0$ content at a later time such that $\tau_S \ll t < \tau_L$? How does this compare to the case of an initially pure K^0 beam?