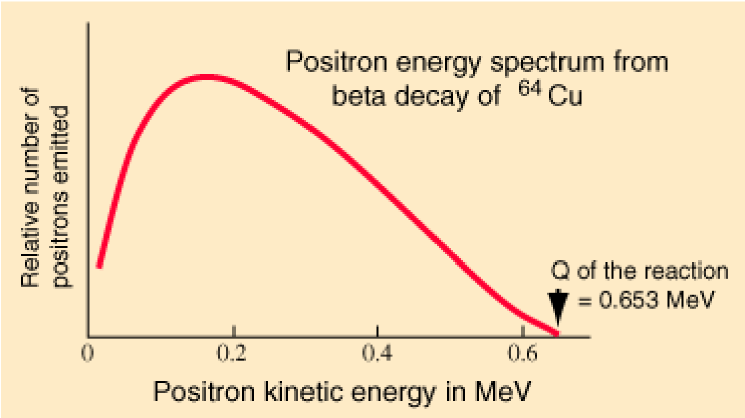
Weak Interactions

- Historically: nuclear β decay, continuous spectrum

$$^{A}_{Z}X \to\ ^{A}_{Z+1}Y^{(*)}e^{-}\bar{\nu}_e \qquad ^{A}_{Z}X \to\ ^{A}_{Z-1}Y^{(*)}e^{+}\nu_e$$



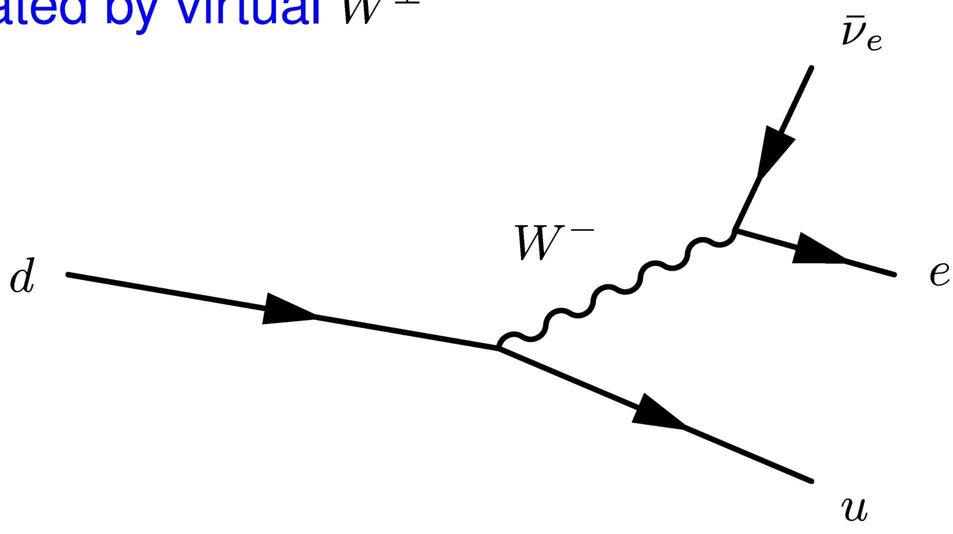
$\Longrightarrow$ Pauli postulates neutrino in 1930

- In 1930s explained by Fermi as bound n or p decay

$$n \to pe^{-}\bar{\nu}_e \qquad \text{bound } p \to ne^{+}\nu_e$$

$$\tau_n = 886.7 \pm 1.9 \text{ s} \qquad \tau_p > 10^{32} \text{ years}$$

- Today: charged weak current $d \to ue^{-}\bar{\nu}_e, u \to de^{+}\nu_e$ mediated by virtual $W^{\pm}$

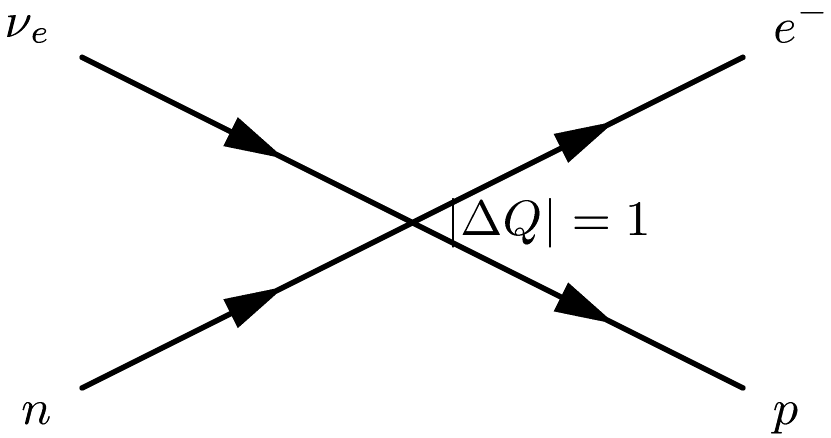


β Decay à la Fermi

- 1932 Fermi, 1st theory of weak interactions
  Charged weak current analogous to $e^- p$ scattering

$$\mathcal{M}(e^- p \to e^- p) = (e \bar{u}_p \gamma^\mu u_p) \frac{-1}{q^2} (-e \bar{u}_e \gamma_\mu u_e) = j_e^\mu \frac{-e^2}{q^2} j_{\mu p}$$

- Point interaction for β decay
  or crossed reaction $n \nu_e \to p e^-$



$$\mathcal{M}(n \nu_e \to p e^-) = G_F \, (\bar{u}_p \gamma^\mu u_n) \, (\bar{u}_e \gamma_\mu u_{\nu_e})$$

- Fermi coupling constant $G_F = 1.166 \times 10^{-5}$ GeV^{-2}
  limit of Feynman propagator for small $q^2 \ll M_W^2$

$$\lim_{q^2 \to 0} \frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_W^2)}{q^2 - M_W^2} = \frac{i g_{\mu\nu}}{M_W^2}$$

  Setting $G_F = g^2 / M_W^2$ and $g = e$
  predicts weak mass scale of $\sim$ 100 GeV

- Short range $R = \hbar c / (m_W c^2) \ll 0.1$ fm

Parity Violation

- 1950s: $\theta - \tau$ puzzle: same mass, spin, charge

$$\theta^+ \to \pi^+\pi^0 \qquad \text{Parity } P_{\theta+} = P_\pi P_\pi (-1)^{L=0} = +1$$

$$\tau^+ \to \begin{cases} \pi^+\pi^-\pi^+ \\ \pi^+\pi^0\pi^0 \end{cases} \qquad P_{\tau+} = P_\pi P_\pi P_\pi (-1)^{L=0} = -1$$

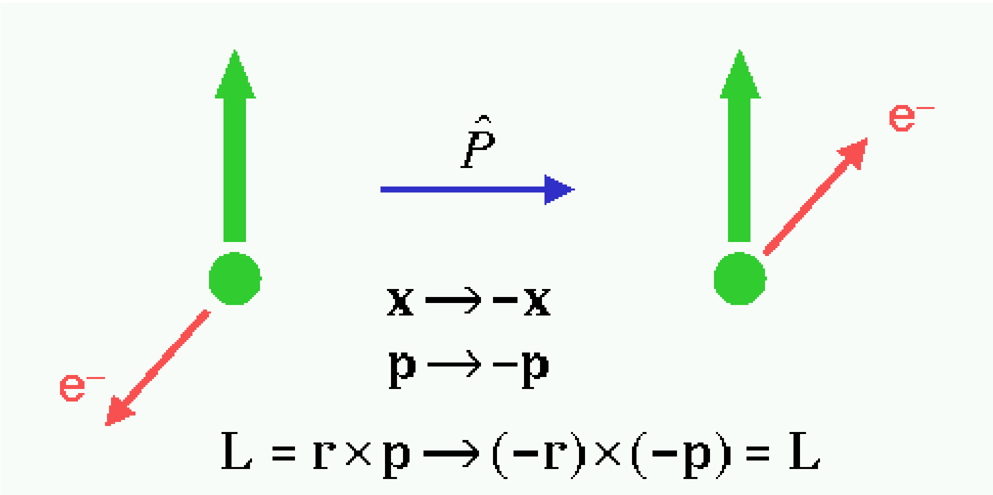
- 1956: Lee and Yang propose Parity Violation in weak decays

  $\Longrightarrow$ θ^+ and τ^+ are same particle: K^+

- 1957: Wu et al. study ${}^{60}Co \to {}^{60}Ni^* + e^-\bar{\nu}_e$

  polarised ${}^{60}Co$ spins aligned to B-field, T = 0.01 K

  emitted e^- prefers direction opposite spin



Expect equal rates if parity is conserved

$\Longrightarrow$ **Parity is violated in weak decays**

Parity

- Parity operator for spinors: $\psi' = P\psi \equiv \gamma^0\psi$

$$P(\bar{\psi}) \equiv (\bar{\psi})\gamma^0 = \psi^\dagger(\gamma^0)^\dagger\gamma^0 = \bar{\psi}\gamma^0$$

$$P(\bar{\psi}\gamma^0\psi) \equiv \bar{\psi}\gamma^0\gamma^0\gamma^0\psi = +\bar{\psi}\gamma^0\psi$$

$$P(\bar{\psi}\gamma^k\psi) \equiv \bar{\psi}\gamma^0\gamma^k\gamma^0\psi = -\bar{\psi}\gamma^k\psi$$

  Spatial components of a vector current $V^\mu = \bar{\psi}\gamma^\mu\psi$ change sign under P

$$P(V^0, \vec{V}) = (V^0, -\vec{V})$$

- Example QED: current-current interaction

$$\mathcal{M}(e^-\mu^- \to e^-\mu^-) \sim g_{\mu\nu}\left(\bar{u}_3\gamma^\mu u_1\right)\left(\bar{u}_4\gamma^\nu u_2\right)$$
$$= g_{\mu\nu}V_e^\mu V_\mu^\nu$$

  QED conserves parity: $\mathcal{M}$ unchanged under P

$$\mathcal{M} \sim V_e^0 V_\mu^0 - \vec{V}_e \cdot \vec{V}_\mu$$

$$P(\mathcal{M}) \sim V_e^0 V_\mu^0 - (-\vec{V}_e)\cdot(-\vec{V}_\mu) = V_e^0 V_\mu^0 - \vec{V}_e \cdot \vec{V}_\mu$$

- Exists also axial-vector current $\bar{\psi}\gamma^5\gamma^\mu\psi$

$$P(\bar{\psi}\gamma^5\gamma^0\psi) \equiv \bar{\psi}\gamma^0\gamma^5\gamma^0\gamma^0\psi = -\bar{\psi}\gamma^5\gamma^0\psi$$

$$P(\bar{\psi}\gamma^5\gamma^k\psi) \equiv \bar{\psi}\gamma^0\gamma^5\gamma^k\gamma^0\psi = +\bar{\psi}\gamma^5\gamma^k\psi$$

Charge Conjugation

- Dirac equation for e^- and e^+ in an el. mag. field

$$[\gamma^\mu(i\delta_\mu + eA_\mu) - m]\,\psi = 0$$
$$[\gamma^\mu(i\delta_\mu - eA_\mu) - m]\,\psi_C = 0$$

- Charge conjugation operator for spinors

$$\psi_C = C\gamma^0\psi^* = C\bar{\psi}^T$$

$$C = i\gamma^2\gamma^0 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

- Free particle (at rest) $\psi_C = i\gamma^2\gamma^0\gamma^0\psi^*$

$$\psi_C^{(1)} = i\gamma^2\left[\exp(-ip\cdot x)u^{(1)}(p)\right]^*$$
$$= \exp(ip\cdot x)u^{(4)}(-p) = \exp(ip\cdot x)v^{(1)}(p)$$

$$\psi_C^{(2)} = -\psi^{(3)} \qquad \psi_C^{(3)} = -\psi^{(2)} \qquad \psi_C^{(4)} = \psi^{(1)}$$

- C conserved for electromagnetic interactions

$$j_C^\mu = +e\bar{\psi}\gamma^\mu\psi = -j^\mu$$
$$\mathcal{M}_C \sim j_C^\mu A_{\mu,C} = (-j^\mu)(-A_\mu) = j^\mu A_\mu$$

Bilinear Covariants

- Dirac spinor does not transform as 4-vector
  electron current $ej_e^\mu(p,x)$ is vector
  probability density ϱ

$$ej_e^\mu(p,x) = -e\bar{\psi}_e\gamma^\mu\psi_e \qquad \varrho = j^0 = \bar{\psi}\gamma^0\psi = \psi^\dagger\psi$$

  Scalar quantity

$$\bar{\psi}\psi = \psi^\dagger\gamma^0\psi = |\psi_1^*\psi_1|^2 + |\psi_2^*\psi_2|^2 - |\psi_3^*\psi_3|^2 - |\psi_4^*\psi_4|^2$$

  is also relativistic invariant

- Most general currents: bilinear covariants

$$\begin{array}{cl} \bar{\psi}\psi & \text{Scalar } S \\ \bar{\psi}\gamma^5\psi & \text{Pseudoscalar } P \\ \bar{\psi}\gamma^\mu\psi & \text{Vector } V \\ \bar{\psi}\gamma^5\gamma^\mu\psi & \text{Axial} - \text{vector } A \\ \bar{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\psi & \text{Tensor } T \end{array}$$

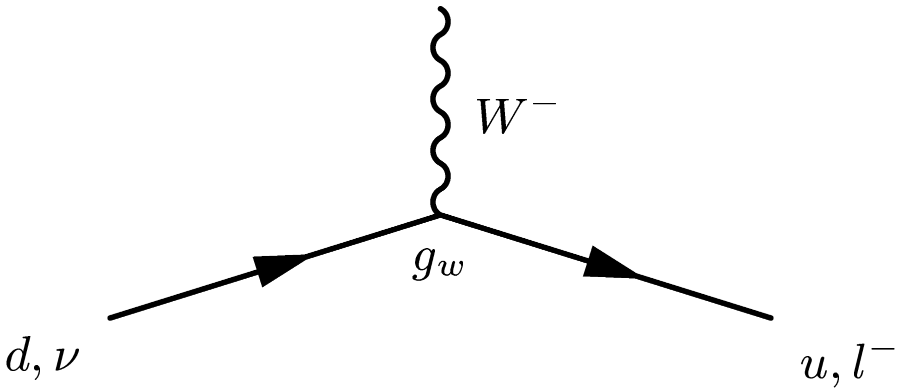
- Matrix γ^5

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(\gamma^5)^2 = 1 \qquad \{\gamma^5, \gamma^\mu\} = 0$$

V-A Interaction

- After 1957: Experimentally find V-A interaction for charged weak current $\bar{\psi}(\gamma^\mu - \gamma^5\gamma^\mu)\psi$



- Feynman rule for $W^\pm$ vertex
$-i\frac{g_w}{\sqrt{2}}\gamma^\mu\frac{1}{2}(1-\gamma^5)$
- β decay matrix element

$$\mathcal{M}(d \to ue^-\bar{\nu}_e) = \frac{G_F}{\sqrt{2}}\left(\bar{u}_u\gamma^\mu(1-\gamma^5)u_d\right)\left(\bar{u}_e\gamma_\mu(1-\gamma^5)v_{\nu_e}\right)$$

- Comparing with propagator $\frac{ig_{\mu\nu}}{M_W^2}$

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2} \qquad g_w = 0.66$$

$$\alpha_{\text{weak}} = \frac{g_w^2}{4\pi} = \frac{1}{29} > \alpha_{\text{em}} = \frac{1}{137}$$

Weak interaction weak due to large M_W,
not small α_{weak}

Handedness

- Chiral spinor operators project left (right) handed part of particle

$$\tfrac{1}{2}(1-\gamma^5)u = u_L \qquad \tfrac{1}{2}(1+\gamma^5)u = u_R$$
$$\tfrac{1}{2}(1-\gamma^5)v = v_R \qquad \tfrac{1}{2}(1+\gamma^5)v = v_L$$

- Only left-handed particles or right-handed antiparticles participate in charged weak current interactions, $W^{\pm}$ only couples to u_L and v_R

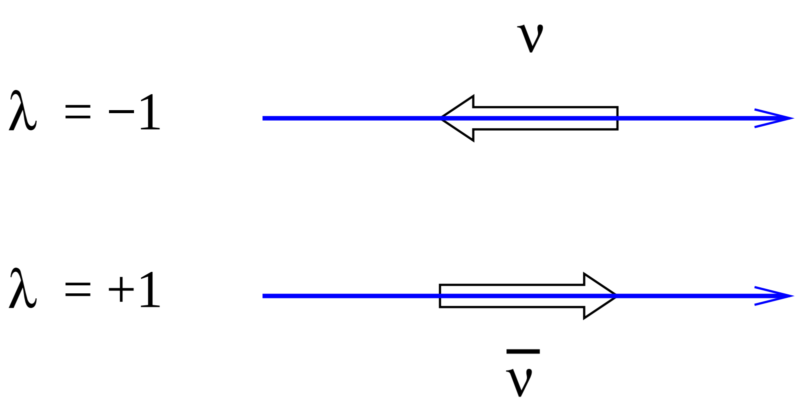
- For fermions with $E \gg m$

  Left-handed particle has negative helicity

  Right-handed antiparticle has positive helicity

  $\Longrightarrow$ Neutrinos are left-handed

  Antineutrinos are right-handed



CP Symmetry

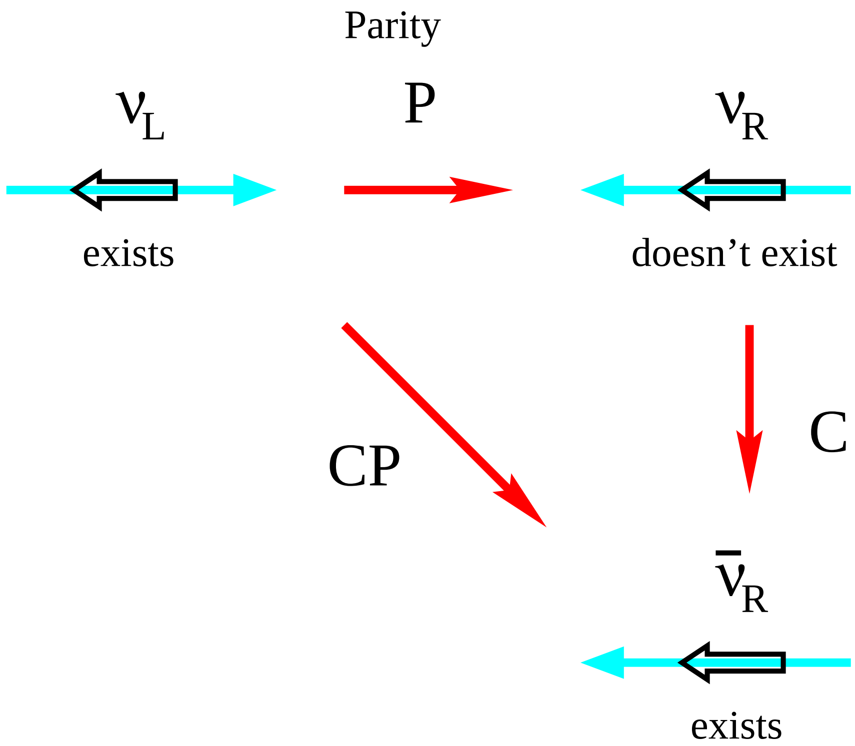
- Weak interactions violate parity P

  eg muon decay: θ = angle between e^- direction and spin $\vec{s}_\mu$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\cos\theta} = \frac{1}{\tau_\mu} 2(1 - cos\theta)$$

- Weak interactions violate charge conjugation C,

  $\bar{\nu}_L = C\nu_L$ does not exist



- CP believed to be conserved in weak interactions,

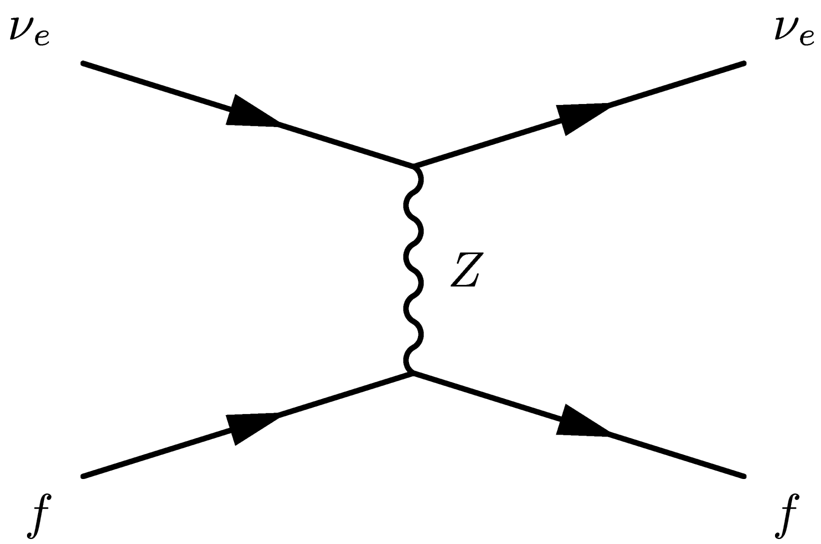
  ν_L and $\bar{\nu}_R$ exist, but not ν_R and $\bar{\nu}_L$

- 1964 Christenson, Cronin, Fitch and Turlay observe

  CP violating weak decay $K^0_L \to \pi^+\pi^-$

Neutral Weak Currents

- $\exists$ also neutral weak current interactions
  $\nu_e q(l) \rightarrow \nu_e q(l)$



- small q^2: Neutral weak currents masked by el.mag. currents

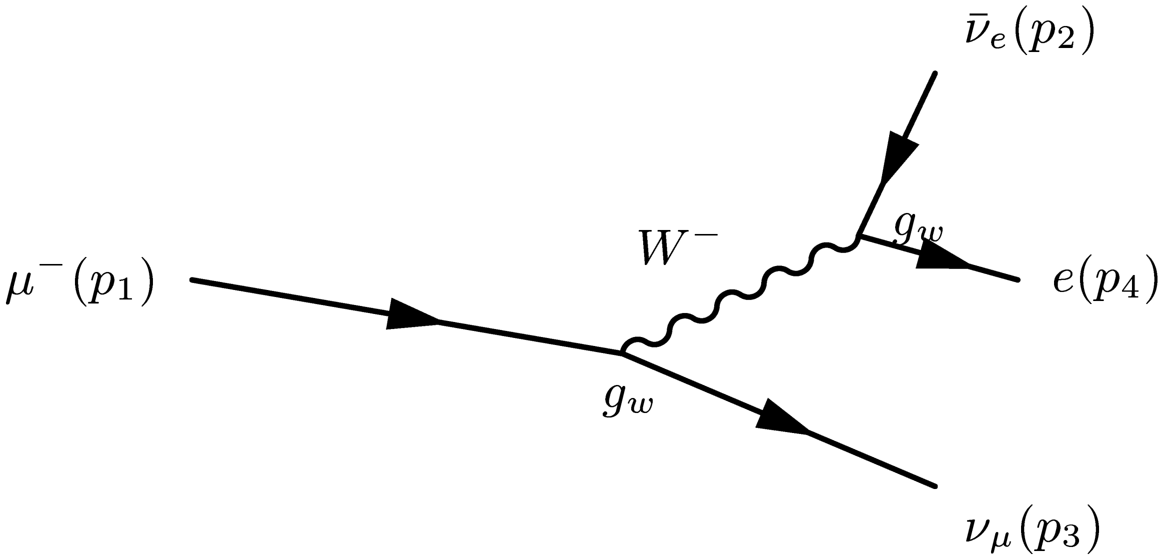
- $W^{\pm}$ and Z bosons discovered at CERN (1983), UA1 experiment

- Z^0 couples also to right-handed fermions
  Feynman rule for Z^0 vertex

$$-i\frac{g_w'}{2}\gamma^\mu(c_V - c_A\gamma^5)$$

  will discuss g_w', c_V, c_A later

Muon Decay



- Invariant amplitude for $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\bar{u}_{\nu_\mu} \gamma^\lambda (1-\gamma^5) u_\mu\right) \left(\bar{u}_e \gamma_\lambda (1-\gamma^5) v_{\nu_e}\right)$$

- Spin: average initial states, sum final states, s_f
  Using trace theorems (without derivation)

$$\begin{aligned}
\langle | \mathcal{M} |^2 \rangle &= \frac{1}{2} \Sigma_{s_f} |\mathcal{M}|^2 \\
&= \frac{G_F^2}{2 \times 2} Tr \left(\gamma^\mu (1-\gamma^5)(\not{p}_1 + m_\mu)\gamma^\nu (1-\gamma^5)\not{p}_3\right) \\
&\quad \times \; Tr \left(\gamma_\mu (1-\gamma^5)\not{p}_2 \gamma_\nu (1-\gamma^5)(\not{p}_4 + m_e)\right) \\
&= 64\, G_F^2\, (p_1 \cdot p_2)(p_3 \cdot p_4)
\end{aligned}$$

- In muon rest frame

$$p_1 = (m_\mu, \vec{0}) \qquad p_1 \cdot p_2 = m_\mu E_2$$

$$(p_3 + p_4)^2 = m_e^2 + 2 p_3 \cdot p_4 \qquad p_3 \cdot p_4 = \tfrac{1}{2}(m_\mu^2 - m_e^2) - m_\mu E_2$$

Muon Decay II

- Setting $m_e = 0$

$$\langle | \mathcal{M} |^2 \rangle = 32 G_F^2 m_\mu^2 E_2 (m_\mu - 2E_2)$$

- Differential decay rate, see chapter 4, Equ 48

$$\begin{aligned} \mathrm{d}\Gamma &= \frac{|\mathcal{M}|^2}{F} \mathrm{d}Q \\ &= \frac{|\mathcal{M}|^2}{2E_1} (2\pi)^4 \delta^{(4)}(p_a - \sum_{i=1}^{n} p_f) \prod_{f=1}^{n} \frac{\mathrm{d}^3 p_f}{(2\pi)^3 2E_f} \end{aligned}$$

  Integrate over neutrino 4-momenta p_2, p_3

- Muon decay rate for V-A interaction, electron energy $E = E_4$,
  $x = \frac{E}{(m_\mu/2)}$ θ = angle between e^- direction and spin $\vec{s}_\mu$

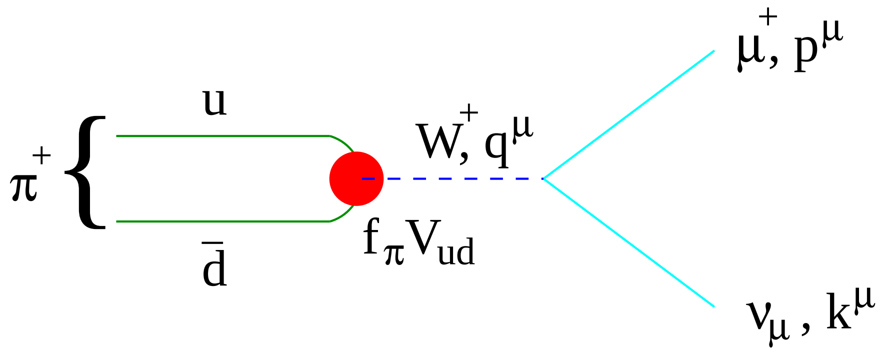
$$\begin{aligned} \frac{\mathrm{d}\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}{\mathrm{d}x \, \mathrm{d}\cos\theta} &= \frac{G_F^2 m_\mu^5}{192\pi^3} x^2 (3 - 2x) \left(1 + \frac{1-2x}{3-2x} \cos\theta \right) \\ \frac{\mathrm{d}\Gamma}{\mathrm{d}E} &= \frac{G_F^2 m_\mu^2}{12\pi^3} E^2 (3 - 4E/m_\mu) \\ \Gamma &= \frac{G_F^2 m_\mu^5}{192\pi^3} \end{aligned}$$

- Use muon lifetime $\tau_\mu = \hbar/\Gamma$ to measure G_F

- Weak interaction has universal coupling $-i \frac{g_w}{\sqrt{2}} \gamma^\mu \frac{1}{2}(1 - \gamma^5)$ for all lepton currents, $e^- \to \nu_e W^-$, $\mu^- \to \nu_\mu W^-$, $\tau^- \to \nu_\tau W^-$, and for all quark currents of weak eigenstates, discussed below

Pion Decay

- $\pi^+ \to \mu^+\nu_\mu$ and $\pi^- \to \mu^-\bar{\nu}_\mu$



- Matrix element, invariant amplitude

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} (X)^\mu \left(\bar{u}_\mu \gamma_\mu (1-\gamma^5) v_{\nu_\mu}\right)$$

- X^μ describes $\pi \to W$ blob $\quad \langle 0|\, A^\mu \,|\pi\rangle$

$$X^\mu = f(q^2) q^\mu V_{ud}$$

Pion decay constant $f_\pi = f(q^2)$ but $q^2 = m_\pi^2$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} f_\pi V_{ud} m_\mu \left(\bar{u}_\mu (1-\gamma^5) v_{\nu_\mu}\right)$$

- Using Dirac equations for muon and neutrino

$$\bar{u}_\mu (\not{p} - m_\mu) = 0 \qquad\qquad \not{k}\, v_\nu = 0$$

- Sum over outgoing spins using trace theorems

$$Tr(\not{a}\,\not{b}\,) = 4a\cdot b \qquad\qquad Tr(\gamma^5) = Tr(\gamma^5 \not{a}\,\not{b}\,) = 0$$

$$\langle |\,\mathcal{M}\,|^2\rangle = \frac{G_F^2}{2} f_\pi^2 |\,V_{ud}\,|^2 m_\mu^2 Tr\left((\not{p} + m_\mu)(1-\gamma^5)\not{k}\,(1+\gamma^5)\right)$$
$$= 4G_F^2 f_\pi^2 |\,V_{ud}\,|^2 m_\mu^2 (p\cdot k)$$

Pion Decay II

- In pion rest frame

$$q^\mu = p^\mu + k^\mu \qquad\qquad q^2 = p^2 + k^2 + 2p\cdot k$$

$$\Longrightarrow p\cdot k = \tfrac{1}{2}(m_\pi^2 - m_\mu^2)$$

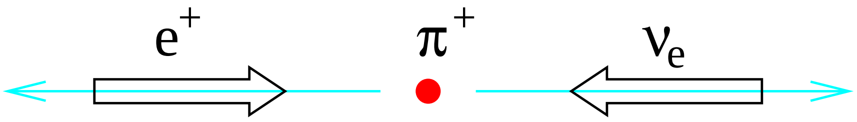
$$\Gamma = \frac{|\vec{p}\,|}{8\pi m_\pi^2}\langle |\mathcal{M}|^2\rangle \qquad |\vec{p}\,| = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

- We obtain for the total decay rate

$$\Gamma = \frac{1}{\tau_{\pi^+}} = \frac{G_F^2}{8\pi} f_\pi^2 |V_{ud}|^2 m_\mu^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

- Measure $f_\pi = 130.7 \pm 0.1 \pm 0.36$ MeV
  from lifetime τ_{π^+} = 26 ns

- Helicity suppression for $\pi^+ \to e^+\nu_e$



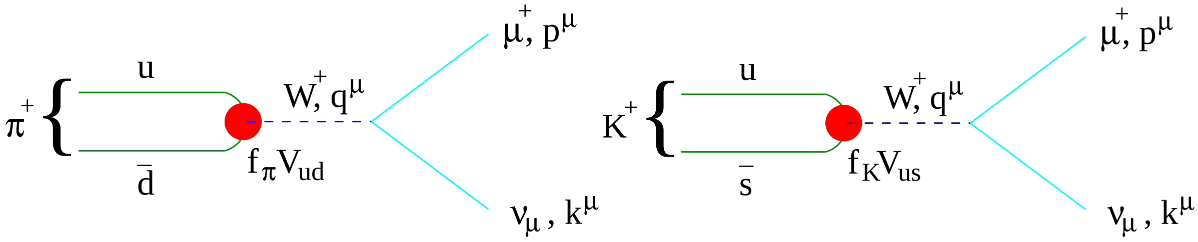
$$\frac{\Gamma(\pi^+ \to e^+\nu_e)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)$$

$$= 1.230 \cdot 10^{-4}$$

due to V-A interaction

Cabibbo Angle

- Decay rates for strange hadrons ($\Delta S = 1$) only about 5 % of expected values from phase space



- G_F measured in muon decay $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ slightly larger than in nuclear β decay $d \to u e^- \bar{\nu}_e$
- Cabibbo hyothesis:
  $W^\pm$ couples to u and d' quark

$$d' = d \cos\theta_C + s \sin\theta_C \qquad \theta_C = \text{Cabibbo angle}$$

$$s' = -d \sin\theta_C + s \cos\theta_C$$

Weak eigenstates different from mass eigenstates
Weak vertices for quarks have rotation angles

$$\begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix} = 1 \quad \begin{pmatrix} u \\ d \end{pmatrix} = \cos\theta_C = V_{ud} \quad \begin{pmatrix} u \\ s \end{pmatrix} = \sin\theta_C = V_{us}$$

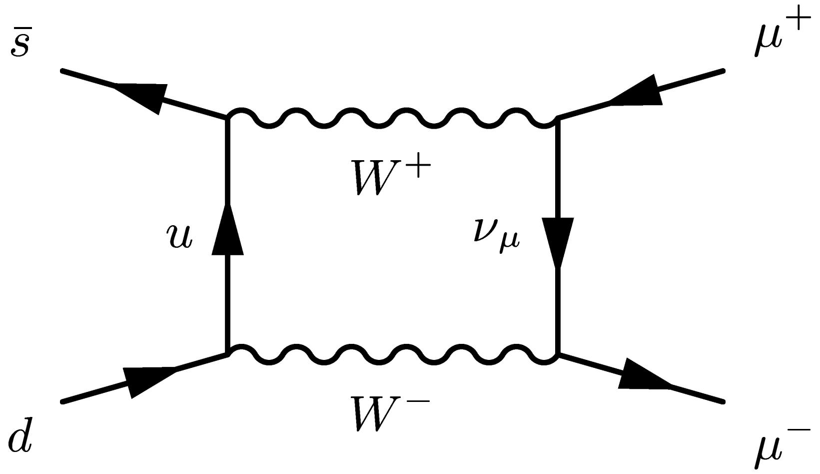
- All data consistent with $\sin^2\theta_C = 0.05$

$$\pi^+ \to \mu^+ \nu_\mu \sim \cos^2\theta_C = |\, V_{ud}\,|^2 \qquad \mu^- \to e^- \bar{\nu}_e \nu_\mu \sim 1$$

$$K^+ \to \mu^+ \nu_\mu \sim \sin^2\theta_C = |\, V_{us}\,|^2 \qquad d \to u e^- \bar{\nu}_e \sim \cos^2\theta_C$$

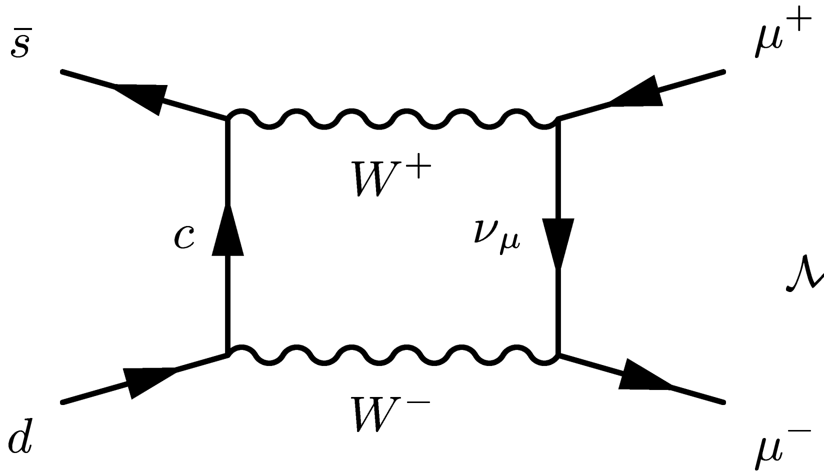
GIM Mechanism

- Limit for $K^0 \to \mu^+\mu^-$ much lower than expected rate



$$\mathcal{M} \sim G_F^2 \cos\theta_c \sin\theta_c$$

- 1970: Glashow, Iliopoulos, and Maiani (GIM) postulate existence of charm quark



$$\mathcal{M} \sim -G_F^2 \cos\theta_c \sin\theta_c$$

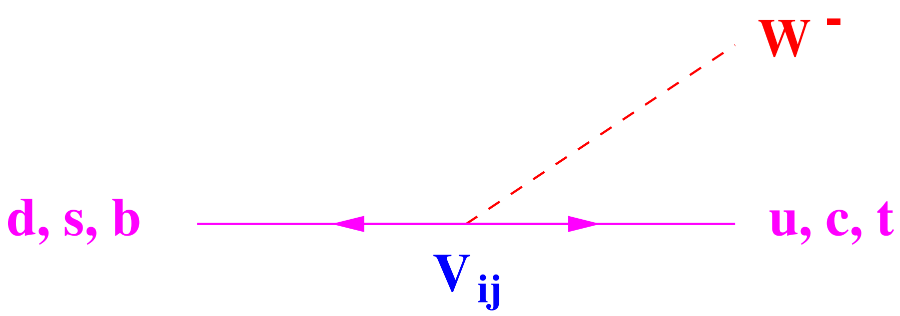
- 2 diagrams have opposite sign and approx. cancel up to $m_c \neq m_u$ effects

- Charged current, $W^\pm$, couples $u \Leftrightarrow d'$ and $c \Leftrightarrow s'$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Weak Mixing Angles

- Charged weak current for 3 generations
  GIM mechanism generalised for 6 quark flavours
  rotates mass into weak flavour eigenstates



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \mathrm{V}_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$j^{\mu} = \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix} \gamma^{\mu}(1-\gamma^{5})\mathrm{V}_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- Cabbibo-Kobayashi-Maskawa Matrix
  (1973, before discovery of charm quark)

$$\mathrm{V}_{\mathrm{CKM}} = \begin{pmatrix} \mathrm{V}_{\mathrm{ud}} & \mathrm{V}_{\mathrm{us}} & \mathrm{V}_{\mathrm{ub}} \\ \mathrm{V}_{\mathrm{cd}} & \mathrm{V}_{\mathrm{cs}} & \mathrm{V}_{\mathrm{cb}} \\ \mathrm{V}_{\mathrm{td}} & \mathrm{V}_{\mathrm{ts}} & \mathrm{V}_{\mathrm{tb}} \end{pmatrix}$$

CKM Matrix

- Properties of quark flavour changing CKM Matrix

  Unitarity: $\mathrm{V}^\dagger \mathrm{V} = 1$

  Independent parameters: $(n = 3$ generations$)$

  $$2n^2 - n^2 - (2n-1) = (n-1)^2$$

  $\Longrightarrow$ 3 rotation angles, 1 complex phase

  $\Longrightarrow$ CP violation possible

- $\mathrm{V_{CKM}}$ parameterisation (PDG): angles θ_{12}, θ_{23}, θ_{13}, $s_{12} = \sin\theta_{12}$, $c_{12} = \cos\theta_{12}$, and phase δ

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_23c_13 \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_23c_13 \end{pmatrix}$$

- Wolfenstein Parameterisation

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

  4 real parameters: λ, A, ρ and η

- No weak mixing for leptons if $m_\nu = 0$

  $\Longrightarrow$ corresponding mixing matrix $\mathrm{V_{MNS}} = 1$

$\mathrm{V_{ij}}$ Measurements

- CKM elements $\mathrm{V_{ij}}$ must be measured experimentally
- $\mathrm{V_{ud}} \approx \cos\theta_C$: Nuclear β decay
  $\Longrightarrow \quad |\,\mathrm{V_{ud}}\,| = 0.9740 \pm 0.0010$
- $\mathrm{V_{us}} \approx \sin\theta_C$: $K^+ \to \pi^0 e^+ \nu_e$
  $\Longrightarrow \quad |\,\mathrm{V_{us}}\,| = 0.2196 \pm 0.0023$
- $\mathrm{V_{cs}} \approx \cos\theta_C$: $D^+ \to \bar{K}^0 e^+ \nu_e$
  $\Longrightarrow \quad |\,\mathrm{V_{cs}}\,| = 1.04 \pm 0.16$
  Improve with indirect $W^- \to \bar{c}s$ decays
  $\Longrightarrow \quad |\,\mathrm{V_{cs}}\,| = 0.97 \pm 0.11$
- $\mathrm{V_{cd}} \approx \sin\theta_C$: $\nu_\mu d \to \mu^- c$
  $\Longrightarrow \quad |\,\mathrm{V_{cd}}\,| = 0.224 \pm 0.016$
- $\mathrm{V_{cb}} \approx \mathcal{O}(\lambda^2)$: $B \to D^{(*)} \mu^+ \nu_\mu$ and $B \to D^{(*)} e^+ \nu_e$
  $\Longrightarrow \quad |\,\mathrm{V_{cb}}\,| = (41 \pm 2) \times 10^{-3}$
- $\mathrm{V_{ub}} \approx \mathcal{O}(\lambda^3)$: $B \to \pi(\rho) \mu^+ \nu_\mu$ and $B \to \pi(\rho) e^+ \nu_e$
  $\Longrightarrow \quad |\,\mathrm{V_{ub}}\,| = (4.0 \pm 0.8) \times 10^{-3}$
- $\mathrm{V_{tb}} \approx 1$: $t \to b \ell^+ \nu_\ell$
  $\Longrightarrow \quad |\,\mathrm{V_{tb}}\,| = 0.99 \pm 0.29$
- $\mathrm{V_{td}} \approx \mathcal{O}(\lambda^3)$: $B_d^0 \bar{B}_d^0$ mixing
  $\Longrightarrow \quad |\,\mathrm{V_{tb}^* V_{td}}\,| = 0.0083 \pm 0.0016$
- $\mathrm{V_{ts}} \approx \mathcal{O}(\lambda^2)$: $B_s^0 \bar{B}_s^0$ mixing
  $\Longrightarrow \quad |\,\mathrm{V_{td}}\,| \,/\, |\,\mathrm{V_{ts}}\,| < 0.24$

Charged Weak Currents

- **Purely leptonic interactions** $(\ell = e, \mu)$

Muon decay	$\mu^- \to e^- \bar{\nu}_e \nu_\mu$
Tau decay	$\tau^- \to e^- \bar{\nu}_e \nu_\tau, \quad \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$
ν scattering	$\nu_\ell e^- \to \ell^- \nu_e$

- **Semi-leptonic interactions** (leptons and hadrons)

Neutron decay	$n \to p e^- \bar{\nu}_e$
Pion decay	$\pi^+ \to \mu^+ \nu_\mu$
Kaon decay	$K^+ \to \pi^0 e^+ \nu_e$
D meson decay	$D^0 \to K^- \ell^+ \nu_\ell$
B meson decay	$B \to X \ell^+ \nu_\ell$
Lambda β decay	$\Lambda \to p \ell^- \nu_\ell$

- **Non-leptonic interactions** (only hadrons)
  observable if change of additive quantum number S, C, B

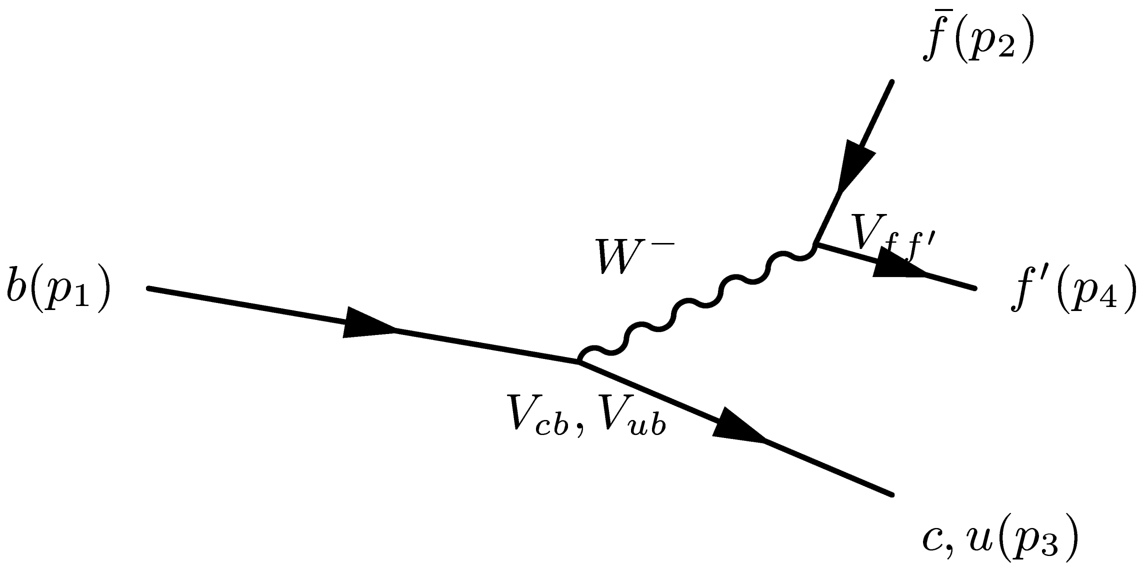
Lambda decay	$\Lambda \to p\pi^-, \; n\pi^0$
Pion decay	None, lighest hadron
Kaon decay	$K^+ \to \pi^+\pi^0, \; \pi^+\pi^-\pi^+$
	$K^0 \to \pi^+\pi^-, \pi^0\pi^0, \pi^+\pi^-\pi^0, \pi^0\pi^0\pi^0$
D meson decay	$D^0 \to K^-\pi^+, K^-\pi^+\pi^0$
B meson decay	$B^+ \to \bar{D}^0\pi^+, \bar{D}^{*0}\pi^+, \pi^+\pi^0$
Lambda$_c$ decay	$\Lambda_c^+ \to pK^-\pi^+$

Spectator Model

- For weak decays heavy mesons with b or c quark,

$$D^+ = c\bar{d} \qquad D^0 = c\bar{u} \qquad D_s^+ = c\bar{s}$$

$$B^+ = u\bar{b} \qquad B^0 = d\bar{b} \qquad B_s^0 = s\bar{b}$$



- Decay of heavy meson equal to decay of heavy quark, light quark or antiquark u, d, s is only spectator

- Consequences

  Decay rates should be equal for

  D^+, D^0, D_s^+ and B^+, B^0, B_s^+

  For D mesons, lifetimes $\tau_{D+} \approx 2.5\tau_D^0 \approx \tau_{D_s^+}$

  For B mesons, lifetimes are equal to 10%