CPT Theorem

All interactions are invariant under CPT

- Necessary conditions:
  - Local interactions are Lorentz invariant
  - Pauli spin-statistics theorem
- Consequences:

  $\Longrightarrow$ Particles and anti-particles have identical masses and lifetimes

CP Violation and Baryogenesis

- Big Bang: matter and antimatter are symmetric

  $\Longrightarrow$ Annihilation of baryons with antibaryons

  Observe $n_{\rm b}/n_\gamma = 10^{-9}$
- 1966 Sakharov postulates 3 necessary conditions for existence of the universe

  Epoch with no thermal equilibrium

  Baryon number violation

  CP Violation
- Possible (partial) answer:

  Standard Model of weak interactions

CP Invariance

- Charged weak process $a + b \to c + d$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} j^\mu_{ca} j^\dagger_{\mu,bd}$$
$$\sim \mathrm{V_{ca}} \mathrm{V^*_{db}} \left(\bar{u}_c \gamma^\mu (1-\gamma^5) u_a\right) \left(\bar{u}_d \gamma_\mu (1-\gamma^5) u_b\right)$$

  Hermitian conjugate process $\bar{a} + \bar{b} \to \bar{c} + \bar{d}$

$$\mathcal{M}^\dagger = \mathcal{M}(c+d \to a+b) = \frac{G_F}{\sqrt{2}} (j^\mu_{ca})^\dagger j_{\mu,bd}$$
$$\sim \mathrm{V^*_{ca}} \mathrm{V_{db}} \left(\bar{u}_a \gamma^\mu (1-\gamma^5) u_c\right) \left(\bar{u}_b \gamma_\mu (1-\gamma^5) u_d\right)$$

- Total Hamiltonian: $\mathcal{M} + \mathcal{M}^\dagger$

  CP invariance if $\mathcal{M}_{CP} = \mathcal{M}^\dagger$

$$(j^\mu_{ca})_C = \mathrm{V_{ca}} (\bar{u}_c)_C \gamma^\mu (1-\gamma^5) (u_a)_C$$
$$= -\mathrm{V_{ca}} \bar{u}_a \gamma^\mu (1+\gamma^5) u_c$$
$$(j^\mu_{ca})_{CP} = -\mathrm{V_{ca}} \bar{u}_a \gamma^{\mu\dagger} (1-\gamma^5) u_c$$
$$\mathcal{M}_{CP} \sim \mathrm{V_{ca}} \mathrm{V^*_{db}} \left(\bar{u}_a \gamma^\mu (1-\gamma^5) u_c\right) \left(\bar{u}_b \gamma_\mu (1-\gamma^5) u_d\right)$$

- Find CP invariance if all CKM elements $\mathrm{V_{ik}}$ are real

  In general $\mathcal{M}_{CP} \neq \mathcal{M}^\dagger$

  $\Longrightarrow$ 3 generation Standard Model allows ~~CP~~ through phase δ in CKM matrix

Neutral Kaons

- Strangeness (mass) eigenstates with $J^{PC} = 0^{-+}$

$$|K^0\rangle = |\bar{s}d\rangle \qquad |\bar{K}^0\rangle = |-s\bar{d}\rangle$$

Phase convention for K^0 and $\bar{K}^0$ such that

$$CP\,|K^0\rangle = |\bar{K}^0\rangle \qquad CP\,|\bar{K}^0\rangle = |K^0\rangle$$

- Both states decay into $|\pi^+\pi^-\rangle$

$\Longrightarrow$ Mixing/Interference

CP eigenstates

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad CP\,|K_1^0\rangle = +|K_1^0\rangle$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad CP\,|K_2^0\rangle = -|K_2^0\rangle$$

- If weak decays conserve CP

$$K_1^0 \to \pi^+\pi^- \qquad K_2^0 \to \pi^+\pi^-\pi^0$$
$$\to \pi^0\pi^0 \qquad \to \pi^0\pi^0\pi^0$$
$$CP = 1 \qquad CP = -1$$

$K_2^0 \to 3\pi$ has less phase space ($m_K = 497.7$ MeV)

$$K_1^0 = K_S^0 \qquad \tau_{K_S^0} = 0.89 \times 10^{-10} s$$
$$K_2^0 = K_L^0 \qquad \tau_{K_L^0} = 51.7 \times 10^{-9} s$$

$K^0\bar{K}^0$ Oscillations

- Initial wavefunction of pure K^0 beam produced by strong interaction, eg. $\pi^- p \to \Lambda^0 K^0$

$$|\psi(t=0)\rangle = |K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_L^0\rangle + |K_S\rangle\right)$$

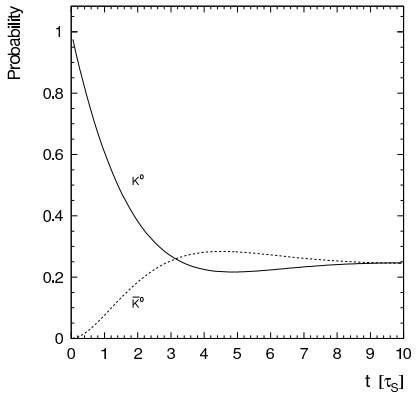
- K_S^0 and K_L^0 time evolution and weak decay with different lifetimes and masses $\Delta m = m_L - m_K$

$$|K_S^0(t)\rangle = |K_S^0\rangle\, e^{-im_S t - \frac{\Gamma_S}{2}t} \qquad |K_L^0(t)\rangle = |K_L^0\rangle\, e^{-im_L t - \frac{\Gamma_L}{2}t}$$

- Oscillations between K^0 and $\bar{K}^0$ eigenstates

$$|\psi_{K^0}(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\Delta m t\right)$$

$$|\psi_{\bar{K}^0}(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\Delta m t\right)$$

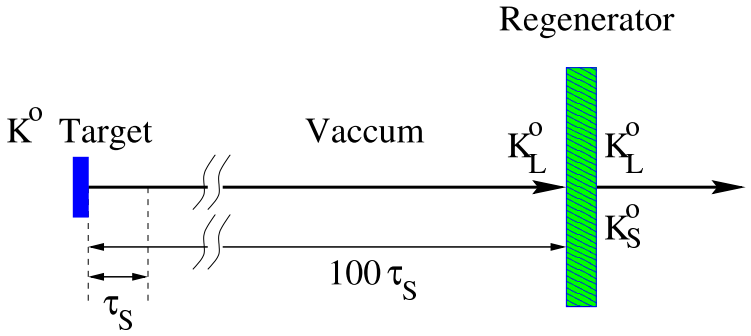


Measure

$$\Delta m = 3.5 \times 10^{-6} eV$$
$$\frac{\Delta m}{m_{K^0}} = 7 \times 10^{-15}$$
$$\Delta m\, \tau_S = 0.474$$

K^0_S Regeneration

- Produce K^0_L beam
  after 100 τ_S K^0_S have all decayed
  Beam traverses material



- K^0 and $\bar{K}^0$ absorbed differently in matter
  only $\bar{K}^0$ can produce hyperons

$$|K^0\rangle \longrightarrow f\,|K^0\rangle \qquad |\bar{K}^0\rangle \longrightarrow \bar{f}\,|\bar{K}^0\rangle$$

$$\bar{f} < f < 1$$

- After material K^0_S state is regenerated

$$|K^0_L\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(f\,|K^0\rangle - \bar{f}\,|\bar{K}^0\rangle\right)$$
$$= \tfrac{1}{2}(f+\bar{f})\,|K^0_L\rangle + \tfrac{1}{2}(f-\bar{f})\,|K^0_S\rangle$$

CP Violation in K^0_L Decays

- 1964 Christenson, Cronin, Fitch and Turlay observe
  45±9 $K^0_L \to \pi^+\pi^-$ events in 22700 K^0_L decays

$\Longrightarrow$ **CP Violation in Weak Interactions**

Small effect PDG average 1998

$$\mathcal{B}(K^0_L \to \pi^+\pi^-) = (2.067 \pm 0.035) \times 10^{-3}$$

- K^0_S and K^0_L are slightly different from
  CP eigenstates K^0_1 and K^0_2
  Redefine

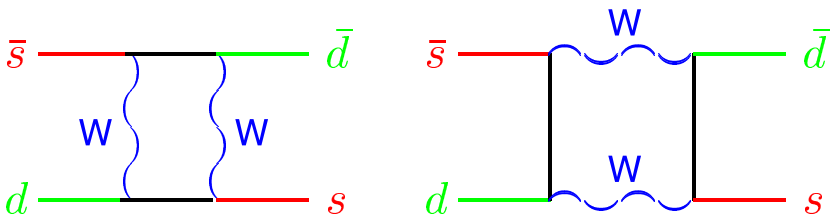
$$|K^0_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K^0_1\rangle + \varepsilon\,|K^0_2\rangle\right)$$
$$|K^0_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\varepsilon\,|K^0_1\rangle + |K^0_2\rangle\right)$$

- Interference in $\pi^+\pi^-$ signal

$$|\psi_{K^0}(t)|^2 = |\psi_{K^0}(0)|^2 \Big(e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2|\eta_{+-}| e^{-\frac{\Gamma_L+\Gamma_S}{2}t} \cos(\Delta m t + \phi_{+-}) \Big)$$

~~CP~~ via $K^0\bar{K}^0$ Mixing

- $K^0\bar{K}^0$ oscillations in the Standard Model



$$|K_S> = \frac{1}{\sqrt{2(1+\varepsilon^2)}}\left[(1+\varepsilon)|K^0> + (1-\varepsilon)|\bar{K}^0>\right]$$

$$|K_L> = \frac{1}{\sqrt{2(1+\varepsilon^2)}}\left[(1+\varepsilon)|K^0> - (1-\varepsilon)|\bar{K}^0>\right]$$

- ~~CP~~ in decays $K_L \to \pi^+\pi^-, \pi^0\pi^0$ and

$$\delta = \frac{\Gamma(K_L \to \pi^-\ell^+\nu) - \Gamma(K_L \to \pi^+\ell^-\nu)}{\Gamma(K_L \to \pi^-\ell^+\nu) + \Gamma(K_L \to \pi^+\ell^-\nu)} = 2\,\mathcal{R}e\,\varepsilon > 0$$

$$\Longrightarrow |\varepsilon| = (2.258 \pm 0.0018) \times 10^{-3}$$

- CKM Matrix (including $\mathcal{O}(\lambda^5)$ terms)

$$|\varepsilon| = C_\epsilon B_K A^2\lambda^6\eta \left(-\eta_1 S(x_c) + \eta_2 A^2\lambda^4(1-\rho)S(x_t) + \eta_3 S(x_c, x_t)\right)$$

$$\Longrightarrow \eta \neq 0 \text{ and } \text{~~CP~~}$$

Direct CP Violation

- ~~CP~~ arises either through

$$\varepsilon \neq 0 \qquad \text{or} \qquad K_2^0 \to \pi^+\pi^-(\varepsilon' \neq 0)$$

Need more than one ~~CP~~ process to distinguish

- Compare $\pi^+\pi^-$ and $\pi^0\pi^0$ final states

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|\,\mathcal{H}_W\,|K_L^0\rangle}{\langle\pi^+\pi^-|\,\mathcal{H}_W\,|K_S^0\rangle} \qquad \eta_{00} = \frac{\langle\pi^0\pi^0|\,\mathcal{H}_W\,|K_L^0\rangle}{\langle\pi^0\pi^0|\,\mathcal{H}_W\,|K_S^0\rangle}$$

- Different allowed isospin combinations for $\pi^+\pi^-$ and $\pi^0\pi^0$. Full calculation gives

$$\eta_{+-} = \varepsilon + \varepsilon' \qquad \eta_{00} = \varepsilon - 2\varepsilon'$$

$$R = \frac{\eta_{00}}{\eta_{+-}} \qquad Re(\frac{\varepsilon'}{\varepsilon}) = \frac{1}{6}(1-R)$$

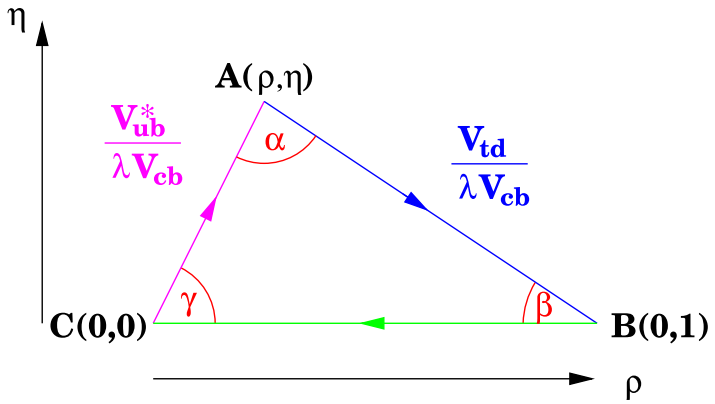
- Experimentally $|\varepsilon'| \ll |\varepsilon|$
  Is ε' non-zero?
- Measurements of $Re(\frac{\varepsilon'}{\varepsilon})$

	1993		1999/2000
E731	$(7.4 \pm 5.9) \times 10^{-4}$	KTEV	$(28.0 \pm 4.1) \times 10^{-4}$
NA31	$(23.0 \pm 6.5) \times 10^{-4}$	NA48	$(14.0 \pm 4.3) \times 10^{-4}$

Edinburgh is part of NA31 and NA48

Unitarity Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$V_{ub}^* - \lambda V_{cb} + V_{td} = 0$$



$\Longrightarrow$ 3 sides $\sim \frac{\mathcal{O}(\lambda^3)}{\mathcal{O}(\lambda\lambda^2)} = \mathcal{O}(1)$

Relations between angles and CKM matrix elements

$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$

$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$

~~CP~~ if area of triangle $\neq 0$ $\quad (\eta \neq 0)$

$\Longrightarrow$ Expect Large ~~CP~~ in Decays of B mesons

~~CP~~ in B Mesons

Formalism

Mass eigenstates

$$|B_L> = p|B^0> + q|\bar{B}^0>$$
$$|B_H> = p|B^0> - q|\bar{B}^0>$$

Schrödinger equation

$$i\frac{d}{dt}\begin{pmatrix} p \\ \pm q \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} p \\ \pm q \end{pmatrix}$$

$|q/p| = 1,$ since $\Gamma_{12} << M_{12}$

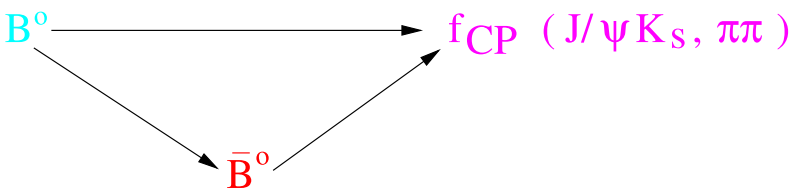
Decays into CP eigenstates $|f_{CP}>$

$$A = <f_{CP}|\mathcal{H}|B^0>$$
$$\bar{A} = <f_{CP}|\mathcal{H}|\bar{B}^0>$$

CP asymmetry $a_{f_{CP}}(t)$

$$a_{f_{CP}}(t) = \frac{\Gamma(B^0(t) \to f_{CP}) - \Gamma(\bar{B}^0(t) \to f_{CP})}{\Gamma(B^0(t) \to f_{CP}) + \Gamma(\bar{B}^0(t) \to f_{CP})}$$

~~CP~~ in $B\bar{B}$ Mixing



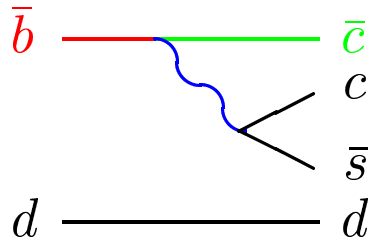
$$(q/p)_{B_d^0} \sim (V_{tb}V_{td}^*)^2$$

$$\Longrightarrow |A/\bar{A}| = 1 \quad \text{and} \quad |r_{f_{CP}}| = 1, \quad r_{f_{CP}} = \frac{q}{p}\frac{\bar{A}}{A}$$

$$a_{f_{CP}}(t) = -\mathrm{Im}(r_{f_{CP}})\sin(\Delta M t)$$

Example:

$B_d^0 \to J/\psi K_S^0$



$$r_{f_{CP}} = \underbrace{\left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)}_{B^0\bar{B}^0 \text{ Mixing}} \underbrace{\left(\frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}\right)}_{b \to c\bar{c}s} \underbrace{\left(\frac{V_{cs}^* V_{cd}}{V_{cd} V_{cs}^*}\right)}_{K^0\bar{K}^0 \text{ Mixing}}$$

$$\Longrightarrow \mathrm{Im}(r_{f_{CP}}) = -\sin(2\beta)$$