CPT Theorem

All interactions are invariant under CPT

- Necessary conditions:
  - Local interactions are Lorentz invariant
  - Pauli spin-statistics theorem
- Consequences:

  $\Longrightarrow$ Particles and anti-particles have identical masses and lifetimes

CP Violation and Baryogenesis

- Big Bang: matter and antimatter are symmetric

  $\Longrightarrow$ Annihilation of baryons with antibaryons

  Observe $\quad n_{\mathrm{b}}/n_\gamma = 10^{-9}$
- 1966 Sakharov postulates 3 necessary conditions for existence of the universe

  Epoch with no thermal equilibrium

  Baryon number violation

  CP Violation
- Possible (partial) answer:

  Standard Model of weak interactions

CP Invariance

- Charged weak process $a + b \to c + d$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} j^{\mu}_{ca} j^{\dagger}_{\mu,bd}$$

$$\sim \mathrm{V_{ca}} \mathrm{V^*_{db}} \left(\bar{u}_c \gamma^{\mu} (1-\gamma^5) u_a\right) \left(\bar{u}_d \gamma_{\mu} (1-\gamma^5) u_b\right)$$

Hermitian conjugate process $\bar{a} + \bar{b} \to \bar{c} + \bar{d}$

$$\mathcal{M}^{\dagger} = \mathcal{M}(c + d \to a + b) = \frac{G_F}{\sqrt{2}} (j^{\mu}_{ca})^{\dagger} j_{\mu,bd}$$

$$\sim \mathrm{V^*_{ca}} \mathrm{V_{db}} \left(\bar{u}_a \gamma^{\mu} (1-\gamma^5) u_c\right) \left(\bar{u}_b \gamma_{\mu} (1-\gamma^5) u_d\right)$$

- Total Hamiltonian: $\mathcal{M} + \mathcal{M}^{\dagger}$

CP invariance if $\mathcal{M}_{CP} = \mathcal{M}^{\dagger}$

$$(j^{\mu}_{ca})_C = \mathrm{V_{ca}} (\bar{u}_c)_C \gamma^{\mu} (1-\gamma^5) (u_a)_C$$

$$= -\mathrm{V_{ca}} \bar{u}_a \gamma^{\mu} (1+\gamma^5) u_c$$

$$(j^{\mu}_{ca})_{CP} = -\mathrm{V_{ca}} \bar{u}_a \gamma^{\mu\dagger} (1-\gamma^5) u_c$$

$$\mathcal{M}_{CP} \sim \mathrm{V_{ca}} \mathrm{V^*_{db}} \left(\bar{u}_a \gamma^{\mu} (1-\gamma^5) u_c\right) \left(\bar{u}_b \gamma_{\mu} (1-\gamma^5) u_d\right)$$

- Find CP invariance if all CKM elements $\mathrm{V_{ik}}$ are real

In general $\mathcal{M}_{CP} \neq \mathcal{M}^{\dagger}$

$\Longrightarrow$ 3 generation Standard Model allows $\cancel{CP}$ through phase δ in CKM matrix

Neutral Kaons

- Strangeness (mass) eigenstates with $J^{PC} = 0^{-+}$

$$|K^0\rangle = |\bar{s}d\rangle \qquad |\bar{K}^0\rangle = |-s\bar{d}\rangle$$

Phase convention for K^0 and $\bar{K}^0$ such that

$$CP\,|K^0\rangle = |\bar{K}^0\rangle \qquad CP\,|\bar{K}^0\rangle = |K^0\rangle$$

- Both states decay into $|\pi^+\pi^-\rangle$
$\Longrightarrow$ Mixing/Interference
CP eigenstates

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad CP\,|K_1^0\rangle = +\,|K_1^0\rangle$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad CP\,|K_2^0\rangle = -\,|K_2^0\rangle$$

- If weak decays conserve CP

$$K_1^0 \to \pi^+\pi^- \qquad K_2^0 \to \pi^+\pi^-\pi^0$$

$$\to \pi^0\pi^0 \qquad \to \pi^0\pi^0\pi^0$$

$$CP = 1 \qquad CP = -1$$

$K_2^0 \to 3\pi$ has less phase space ($m_K = 497.7$ MeV)

$$K_1^0 = K_S^0 \qquad \tau_{K_S^0} = 0.89 \times 10^{-10} s$$

$$K_2^0 = K_L^0 \qquad \tau_{K_L^0} = 51.7 \times 10^{-9} s$$

$K^0 \bar{K}^0$ Oscillations

- Initial wavefunction of pure K^0 beam produced by strong interaction, eg. $\pi^- p \to \Lambda^0 K^0$

$$|\psi(t=0)\rangle = \left|K^0\right\rangle = \frac{1}{\sqrt{2}}\left(\left|K_L^0\right\rangle + |K_S\rangle\right)$$

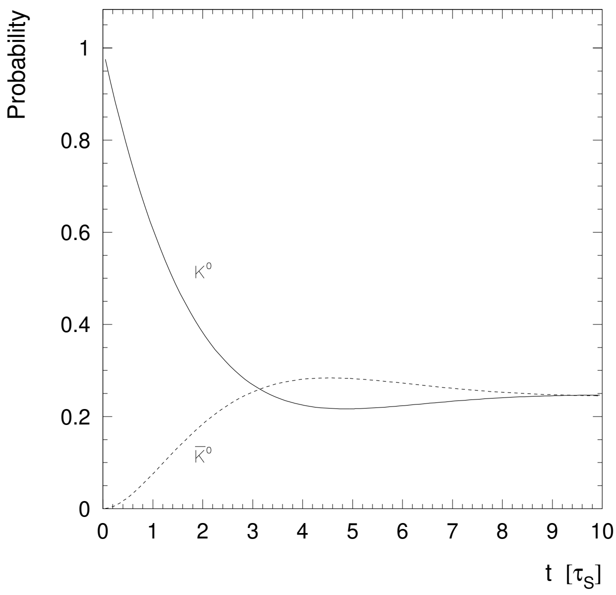
- K_S^0 and K_L^0 time evolution and weak decay with different lifetimes and masses $\Delta m = m_L - m_K$

$$\left|K_S^0(t)\right\rangle = \left|K_S^0\right\rangle e^{-im_S t - \frac{\Gamma_S}{2}t} \quad \left|K_L^0(t)\right\rangle = \left|K_L^0\right\rangle e^{-im_L t - \frac{\Gamma_L}{2}t}$$

- Oscillations between K^0 and $\bar{K}^0$ eigenstates

$$|\psi_{K^0}(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\Delta m t\right)$$

$$|\psi_{\bar{K}^0}(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos\Delta m t\right)$$



Measure

$$\Delta m = 3.5 \times 10^{-6} eV$$

$$\frac{\Delta m}{m_{K^0}} = 7 \times 10^{-15}$$

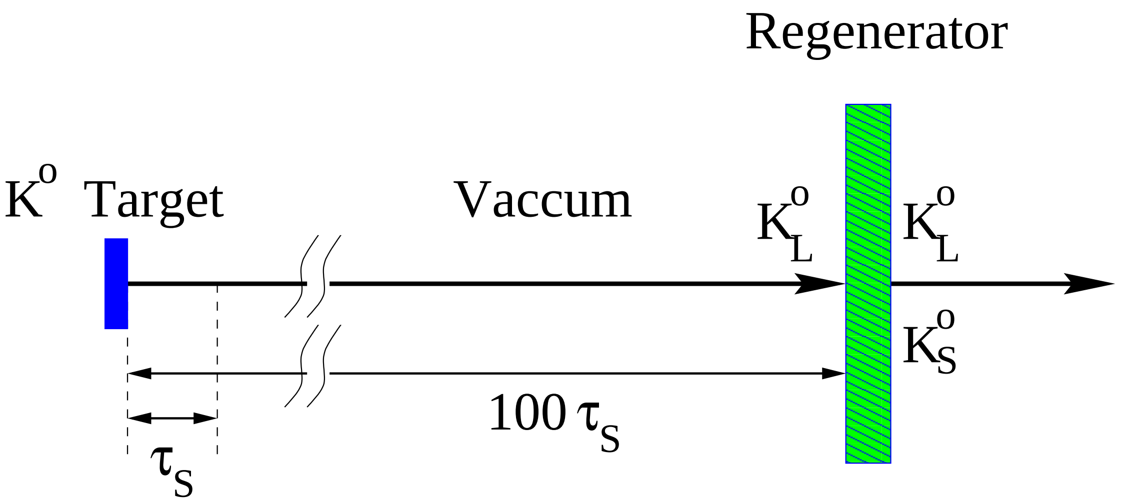
$$\Delta m \ \tau_S = 0.474$$

K_S^0 Regeneration

- Produce K_L^0 beam

  after 100 τ_S K_S^0 have all decayed

  Beam traverses material



- K^0 and $\bar{K}^0$ absorbed differently in matter
  only $\bar{K}^0$ can produce hyperons

$$\left|K^0\right\rangle \longrightarrow f\left|K^0\right\rangle \qquad \left|\bar{K}^0\right\rangle \longrightarrow \bar{f}\left|\bar{K}^0\right\rangle$$

$$\bar{f} < f < 1$$

- After material K_S^0 state is regenerated

$$\left|K_L^0\right\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(f\left|K^0\right\rangle - \bar{f}\left|\bar{K}^0\right\rangle\right)$$
$$= \tfrac{1}{2}(f+\bar{f})\left|K_L^0\right\rangle + \tfrac{1}{2}(f-\bar{f})\left|K_S^0\right\rangle$$

CP Violation in K_L^0 Decays

- 1964 Christenson, Cronin, Fitch and Turlay observe 45±9 $K_L^0 \to \pi^+\pi^-$ events in 22700 K_L^0 decays

$\Longrightarrow$ **CP Violation in Weak Interactions**

Small effect PDG average 1998

$$\mathcal{B}(K_L^0 \to \pi^+\pi^-) = (2.067 \pm 0.035) \times 10^{-3}$$

- K_S^0 and K_L^0 are slightly different from CP eigenstates K_1^0 and K_2^0
  Redefine

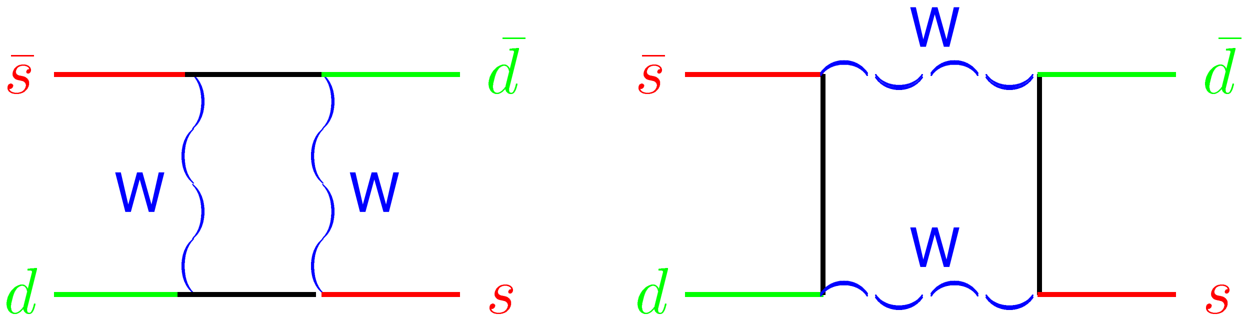
$$\left|K_S^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\left|K_1^0\right\rangle + \varepsilon\left|K_2^0\right\rangle\right)$$
$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\varepsilon\left|K_1^0\right\rangle + \left|K_2^0\right\rangle\right)$$

- Interference in $\pi^+\pi^-$ signal

$$|\psi_{K^0}(t)|^2 = |\psi_{K^0}(0)|^2 \left(e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2|\eta_{+-}|e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos(\Delta m t + \phi_{+-})\right)$$

$\cancel{CP}$ via $K^0\bar{K}^0$ Mixing

- $K^0\bar{K}^0$ oscillations in the Standard Model



$$
\begin{aligned}
|K_S> &= \frac{1}{\sqrt{2(1+\varepsilon^2)}}\left[(1+\varepsilon)|K^0> +(1-\varepsilon)|\bar{K}^0>\right] \\
|K_L> &= \frac{1}{\sqrt{2(1+\varepsilon^2)}}\left[(1+\varepsilon)|K^0> -(1-\varepsilon)|\bar{K}^0>\right]
\end{aligned}
$$

- $\cancel{CP}$ in decays $K_L \to \pi^+\pi^-, \pi^0\pi^0$ and

$$\delta = \frac{\Gamma(K_L \to \pi^-\ell^+\nu) - \Gamma(K_L \to \pi^+\ell^-\nu)}{\Gamma(K_L \to \pi^-\ell^+\nu) + \Gamma(K_L \to \pi^+\ell^-\nu)} = 2\,\mathcal{R}e\,\varepsilon > 0$$

$$\Longrightarrow |\varepsilon| = (2.258 \pm 0.0018) \times 10^{-3}$$

- CKM Matrix (including $\mathcal{O}(\lambda^5)$ terms)

$$
\begin{aligned}
|\varepsilon| &= C_\epsilon B_K A^2 \lambda^6 \eta \\
&\quad (-\eta_1 S(x_c) + \eta_2 A^2 \lambda^4 (1-\rho) S(x_t) + \eta_3 S(x_c, x_t))
\end{aligned}
$$

$\Longrightarrow \eta \neq 0$ and $\cancel{CP}$

Direct CP Violation

- $\cancel{CP}$ arises either through

$$\varepsilon \neq 0 \qquad \text{or} \qquad K_2^0 \to \pi^+\pi^- (\varepsilon' \neq 0)$$

  Need more than one $\cancel{CP}$ process to distinguish

- Compare $\pi^+\pi^-$ and $\pi^0\pi^0$ final states

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | \mathcal{H}_W | K_L^0 \rangle}{\langle \pi^+\pi^- | \mathcal{H}_W | K_S^0 \rangle} \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | \mathcal{H}_W | K_L^0 \rangle}{\langle \pi^0\pi^0 | \mathcal{H}_W | K_S^0 \rangle}$$

- Different allowed isospin combinations for $\pi^+\pi^-$ and $\pi^0\pi^0$. Full calculation gives

$$\eta_{+-} = \varepsilon + \varepsilon' \qquad \eta_{00} = \varepsilon - 2\varepsilon'$$

$$R = \frac{\eta_{00}}{\eta_{+-}} \qquad Re(\frac{\varepsilon'}{\varepsilon}) = \frac{1}{6}(1 - R)$$

- Experimentally $|\varepsilon'| \ll |\varepsilon|$

  Is ε' non-zero?

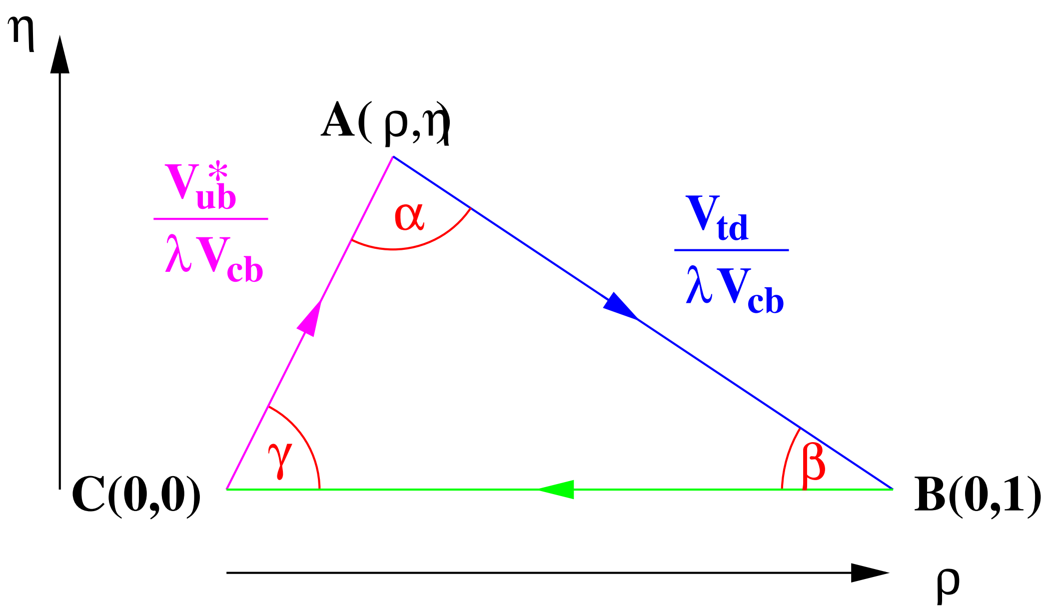
- Measurements of $Re(\frac{\varepsilon'}{\varepsilon})$

	1993		1999/2000
E731	$(7.4 \pm 5.9) \times 10^{-4}$	KTEV	$(28.0 \pm 4.1) \times 10^{-4}$
NA31	$(23.0 \pm 6.5) \times 10^{-4}$	NA48	$(14.0 \pm 4.3) \times 10^{-4}$

  Edinburgh is part of NA31 and NA48

Unitarity Triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$V_{ub}^* - \lambda V_{cb} + V_{td} = 0$$



$\Longrightarrow$ 3 sides $\sim \frac{\mathcal{O}(\lambda^3)}{\mathcal{O}(\lambda\lambda^2)} = \mathcal{O}(1)$

Relations between angles and CKM matrix elements

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$$

$$\gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$\cancel{CP}$ if area of triangle $\neq 0$ $(\eta \neq 0)$

$\Longrightarrow$ Expect Large $\cancel{CP}$ in Decays of B mesons

Formalism

Mass eigenstates

$$
\begin{aligned}
|B_L> &= p|B^0> + q|\bar{B}^0> \\
|B_H> &= p|B^0> - q|\bar{B}^0>
\end{aligned}
$$

Schrödinger equation

$$
i\frac{d}{dt}\begin{pmatrix} p \\ \pm q \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} p \\ \pm q \end{pmatrix}
$$

$|q/p| = 1,$ since $\Gamma_{12} << M_{12}$

Decays into CP eigenstates $|f_{CP}>$

$$
\begin{aligned}
A &= < f_{CP}|\mathcal{H}|B^0> \\
\bar{A} &= < f_{CP}|\mathcal{H}|\bar{B}^0>
\end{aligned}
$$

CP asymmetry $a_{f_{CP}}(t)$

$$
a_{f_{CP}}(t) = \frac{\Gamma(B^0(t) \to f_{CP}) - \Gamma(\bar{B}^0(t) \to f_{CP})}{\Gamma(B^0(t) \to f_{CP}) + \Gamma(\bar{B}^0(t) \to f_{CP})}
$$

~~CP~~ in $\mathrm{B\bar{B}}$ Mixing

$$\begin{tikzcd} \mathrm{B^o} \arrow[rr] \arrow[dr] & & \mathrm{f_{CP}}\ (\mathrm{J}/\psi \mathrm{K_S},\ \pi\pi) \\ & \mathrm{\bar{B}^o} \arrow[ur] & \end{tikzcd}$$

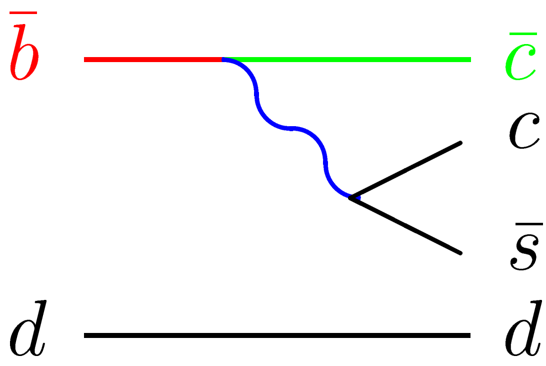
$$(q/p)_{B_d^0} \quad \sim \quad (\mathrm{V_{tb}V_{td}^*})^2$$

$\Longrightarrow |A/\bar{A}| = 1$ and $|r_{f_{CP}}| = 1$, $r_{f_{CP}} = \frac{q}{p}\frac{\bar{A}}{A}$

$$\boxed{a_{f_{CP}}(t) = -\mathrm{Im}(r_{f_{CP}})\sin(\Delta M t)}$$

Example:

$B_d^0 \rightarrow J/\psi K_S^0$



$$r_{f_{CP}} \quad = \quad \underbrace{\left(\frac{\mathrm{V_{tb}^*V_{td}}}{\mathrm{V_{tb}V_{td}^*}}\right)}_{\mathrm{B^0\bar{B}^0\ Mixing}} \quad \underbrace{\left(\frac{\mathrm{V_{cs}^*V_{cb}}}{\mathrm{V_{cs}V_{cb}^*}}\right)}_{\mathrm{b \rightarrow c\bar{c}s}} \quad \underbrace{\left(\frac{\mathrm{V_{cs}^*V_{cd}}}{\mathrm{V_{cd}V_{cs}^*}}\right)}_{\mathrm{K^0\bar{K}^0\ Mixing}}$$

$$\boxed{\Longrightarrow \mathrm{Im}(\mathrm{r_{f_{CP}}}) = -\sin(2\beta)}$$