11 Quantum Chromodynamics (QCD)

In deep inelastic scattering experiments we have learned that, inside the protons, there are electrically charged and neutral constituents which were given the name partons. The charged partons are identified with the quarks and the neutral partons are identified with gluons. In this chapter, we will study the dynamics of the strong interactions of quarks and gluons. The theory that describes how gluons mediate the force which binds the quarks in nucleons. is called **quantum chromodynamics (QCD)**. We will start the discussion by comparing QCD as an analogous theory to QED. We then look at the properties of QCD and discuss its behaviour, such as the concept of confinement and some of its consequences, such as fragmentation. We will end up with an example where the combination of QED and QCD is put into action, namely electron-positron annihilation into hadrons.

11.1 SU(3) Colour

The fundamental constituents in hadrons are **quarks** which have a **colour** charge. The interaction between quarks is mediated by **gluons**. Just as QED describes the interaction of electrically charged particles with the exchange of virtual photons, quantum *chromo*dynamics describes the strong interactions with quarks emitting and absorbing gluons. In QCD colour takes the same role as the electric charge in QED. Quarks carry one of three colour charges, say red r, green g, and blue b whereas antiquarks carry anti-colour, $\bar{r}$, $\bar{g}$, and $\bar{b}$. In Table 14 we give a summary of this analogy between QCD and QED.

	QED	**QCD**
Symmetry	U(1)	SU(3)
Conserved charge	Electric charge	3 colour charges (r, g, b)
Coupling constant	$e = \sqrt{4\pi\alpha}$	$g_S = \sqrt{4\pi\alpha_S}$
Gauge bosons	1 Photon	8 gluons

Table 14: Analogy between QED and QCD.

Mathematically, the SU(3) colour symmetry is identical to the SU(3) flavour symmetry discussed in detail in Chapter 8. We simply have to replace the three quarks flavour by the three colour charges

$$u \to r \qquad\qquad d \to g \qquad\qquad s \to b$$

Thus we introduce the *colour vector* c which has three eigenstates r, g, and b which can be written as follows:

$$r = \begin{pmatrix}1\\0\\0\end{pmatrix} \text{“red”}, \qquad g = \begin{pmatrix}0\\1\\0\end{pmatrix} \text{“green”}, \qquad b = \begin{pmatrix}0\\0\\1\end{pmatrix} \text{“blue”}\,. \tag{200}$$

Since gluons are massless, the three colour states form an exact SU(3) colour symmetry. The strong interactions are invariant under rotations

$$U = e^{-i\vec{\theta}\cdot\vec{T}}\,.$$

The 8 generators T^a of the group SU(3) are defined by Equation 116 which we rewrite here for convenience:

$$\text{Generators} \qquad \left[\lambda^a, \lambda^b\right] = 2if^{abc}\lambda^c \qquad\qquad T^a = \tfrac{1}{2}\lambda^a$$

These generators may be taken as the λ-matrices, see Equation 117.

A fundamental vertex for the strong interactions is shown in Figure 56. Quarks interact strongly by exchanging virtual gluons. We notice immediately that for a quark to be able to change its colour at a vertex the gluons must carry colour themselves. This is in contrast to the electromagnetic interactions where the photon which mediates the interaction has zero charge itself. This feature has dramatic consequences for QCD, some of which we will encounter below.

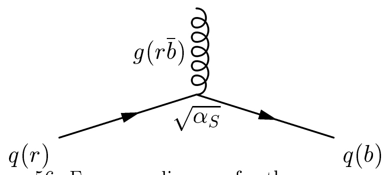

Figure 56: Feynman diagram for the process $q \to qg$.

We are now able to construct the gluon colour wavefunction. A gluon must carry one unit of each, colour c and anti-colour $\bar{c}$. Mathematically, this is analogous to the combination of a quark q with an antiquark $\bar{q}$ into mesons, and hence results in an *octet* plus a *singlet* of colour-anti-colour states. By now we should be familiar with denoting this decomposition as $3\otimes\bar{3} = 8\oplus 1$, a notation which we have borrowed from the language of group theory. The list of all the octet and singlet colour-anti-colour states is

$$\begin{array}{ll} \text{Octet} & r\bar{b} \quad r\bar{g} \quad b\bar{g} \quad b\bar{r} \quad g\bar{r} \quad g\bar{b} \\ & \frac{1}{\sqrt{2}}\left(r\bar{r} - g\bar{g}\right) \quad \frac{1}{\sqrt{6}}\left(r\bar{r} + g\bar{g} - 2b\bar{b}\right) \\ \text{Singlet} & \frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right) \end{array} \tag{201}$$

Which of the combinations - octet or singlet states - Nature has chosen has to be decided by experiment. If a singlet gluon were to exist it should also occur as a free particle since colour singlet states are observable. In addition, a colour singlet gluon would give rise to long-range strong forces. We conclude that Nature has chosen the octet option and that there must be 8 physical gluons. As a consequence, we realise that both quarks and gluons carry net colour and hence, must be permanently confined within hadrons.

11.2 Feynman Rules for QCD

We now discuss the Feynman rules for QCD. Again we will not derive these, but the different terms can be obtained from the Feynman rules for QED by making the corresponding substitutions listed in Table 14 by taking into account the colour vectors. It is evident that all possible Feynman diagrams must be accounted for. In Figure 56 above, we have discussed the fundamental quark-gluon vertex. But the fact that gluons carry colour themselves has a consequence, namely that, as opposed to photons which are electrically neutral, gluons do interact with each other. This leads to two additional fundamental diagrams. Firstly, a gluon can radiate or absorb another gluon which results in *three-gluon vertex.* Secondly two gluons can "touch" each other which results in an effective *four-gluon vertex.* The corresponding Feynman diagrams for these two new processes are shown in Figure 57.

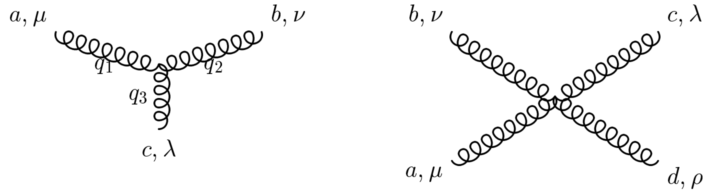

Figure 57: Feynman diagrams for a three-gluon and a four-gluon vertex.

The Feynman rules for QCD can be written down as follows:

- Notation: as in QED but add colour for quark lines.
- Add a colour vector to the spinor $u^{(s)}(p)c^a$ for each external quark line. The spinors are the same as for QED.
- The gluon propagator is $\frac{-ig_{\mu\nu}\delta^{ab}}{q^2}$.
- Each quark-gluon vertex has a factor $\frac{-ig_S}{2}\lambda^a\gamma^\nu$. The additional factor of 2 arises due to the historical definition of g_s.
- Each three-gluon vertex has a factor

$$-g_S f^{abc}\left[g_{\mu\nu}(q_1-q_2)_\lambda + g_{\nu\lambda}(q_2-q_3)_\mu + g_{\lambda\mu}(q_3-q_1)_\nu\right]$$

- Each four-gluon vertex gives a factor

$$\begin{aligned}&-ig_S^2[f^{abe}f^{cde}(g_{\mu\lambda}g_{\nu\rho}-g_{\mu\rho}g_{\nu\lambda})\\ &+f^{ade}f^{bce}(g_{\mu\nu}g_{\lambda\rho}-g_{\mu\lambda}g_{\nu\rho})+f^{ace}f^{dbe}(g_{\mu\rho}g_{\nu\lambda}-g_{\mu\nu}g_{\lambda\rho})]\end{aligned}$$

11.3 Quark-Quark Interaction

Let us look at the interaction between two quarks or a quark and an antiquark. In the simplest diagrams, sometimes called *tree-level* or **lowest order diagrams**, only one gluon is exchanged. We show the Feynman diagram for the scattering of a quark-antiquark pair at lowest order in Figure 58. We assume that the two quarks are of different flavour so that this is the only diagram

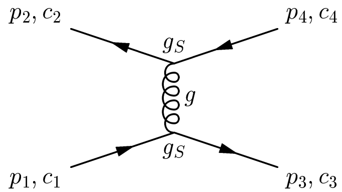

Figure 58: Feynman diagram for the process $q\bar{q} \to q\bar{q}$.

for this process. The matrix element or amplitude is then given by

$$\begin{aligned}-i\mathcal{M} &= \left[\bar{u}_3c_3^\dagger\right]\left[\frac{-ig_S}{2}\lambda^a\gamma^\mu\right][u_1c_1]\left[\frac{-ig_{\mu\nu}\delta^{ab}}{q^2}\right]\left[\bar{v}_2c_2^\dagger\right]\left[\frac{-ig_S}{2}\lambda^b\gamma^\nu\right][v_4c_4]\\ \mathcal{M} &= \frac{-g_S^2}{4q^2}\left[\bar{u}_3\gamma^\mu u_1\right]\left[\bar{v}_2\gamma_\mu v_4\right](c_3^\dagger\lambda^a c_1)(c_2^\dagger\lambda^a c_4)\end{aligned} \tag{202}$$

where a summation over the index a is implied. The result looks very close to the matrix element for electron-positron scattering, except that e is replaced by g_s and that, in addition, we have a **colour factor** f

$$f = \frac{1}{4}(c_3^\dagger\lambda^a c_1)(c_2^\dagger\lambda^a c_4)\ . \tag{203}$$

This comparison with QED shows that, by a simple analogy to the electrostatic Coulomb potential between two opposite charges, we are allowed to write down the strong force potential $V_{q\bar{q}}$ describing the $q\bar{q}$ interaction as

$$V_{q\bar{q}}(r) = -f\frac{\alpha_S}{r}\ . \tag{204}$$

The colour factors f can be calculated in different ways. A straightforward way is multiplying the colour vectors and λ matrices in Equation 203. As an example, we calculate f for the interaction of a red quark and a red antiquark exchanging a gluon and turning into a blue quark and a blue antiquark $r\bar{r} \to b\bar{b}$:

$$\begin{aligned}f &= \frac{1}{4}(0,0,1)\lambda^a\begin{pmatrix}1\\0\\0\end{pmatrix}(1,0,0)\lambda^a\begin{pmatrix}0\\0\\1\end{pmatrix} = \frac{1}{4}\lambda^a_{31}\lambda^a_{13}\\ &= \frac{1}{4}(\lambda^4_{31}\lambda^4_{13}+\lambda^5_{31}\lambda^5_{13}) = \frac{1}{4}(1\cdot 1 + i\cdot(-i)) = \frac{1}{2}\end{aligned} \tag{205}$$

The method used above is simple but rather tedious. Another way is to assign a *colour coefficient* c_i at each vertex. This "pedestrian" method describes the exchange of a single gluon between two colour charges a and b as a product of the two colour coefficients

$$f = -\tfrac{1}{2} c_a c_b \tag{206}$$

The colour coefficient is defined as the coupling of the quark colour charge to the gluon wavefunction with the convention that antiquark coefficients obtain an additional minus sign. We now recalculate the colour factor for $r\bar{r} \to b\bar{b}$ scattering which corresponds to Figure 58 with the settings $c1 = r$, $c2 = \bar{r}$, $c3 = b$, and $c4 = \bar{b}$. The exchanged gluon is $r\bar{b}$ and hence the colour factors are $c_a = 1$ and $c_b = -1$, respectively, which yields $f = \frac{1}{2}$. As expected this results is identical to the calculation of Equation 205. Below is a summary of the different $q\bar{q}$ colour exchanges.

$$\begin{array}{cccc}
q\bar{q}\ \text{state} & \text{gluon} & c_a c_b & f \\
r\bar{r} \to b\bar{b} & r\bar{b} & 1 \times (-1) = -1 & \frac{1}{2} \\
r\bar{b} \to r\bar{b} & \frac{(r\bar{r}+g\bar{g}-2b\bar{b})}{\sqrt{6}} & \frac{1}{\sqrt{6}} \times (-\frac{-2}{\sqrt{6}}) = \frac{1}{3} & -\frac{1}{6} \\
r\bar{r} \to r\bar{r} & \frac{(r\bar{r}+g\bar{g}-2b\bar{b})}{\sqrt{6}} & \frac{1}{\sqrt{6}} \times \frac{-1}{\sqrt{6}} = -\frac{1}{6} & \\
 & \frac{1}{\sqrt{2}}\,(r\bar{r} - g\bar{g}) & \frac{1}{\sqrt{2}} \times \frac{-1}{\sqrt{2}} = -\frac{1}{2} & \\
 & \text{combined} & -\frac{1}{6} - \frac{1}{2} = -\frac{2}{3} & \frac{1}{3} \\
b\bar{b} \to b\bar{b} & \frac{(r\bar{r}+g\bar{g}-2b\bar{b})}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \times -\frac{-2}{\sqrt{6}} = -\frac{2}{3} & \frac{1}{3}
\end{array} \tag{207}$$

As expected the colour factor is invariant under rotations of colours and we find

$$f(r\bar{r} \to r\bar{r}) = f(b\bar{b} \to b\bar{b}) = f(g\bar{g} \to g\bar{g}) \; . \tag{208}$$

11.4 Binding of Mesons and Baryons

From the static model of quarks and gluons we already know that only *colour singlet* states are observed in Nature. In $q\bar{q}$ mesons a quark with colour charge is combined with a anti-coloured antiquark into the singlet state

$$|\psi\rangle = \frac{1}{\sqrt{3}} \left| r\bar{r} + g\bar{g} + b\bar{b} \right\rangle \tag{209}$$

The matrix element of a meson can be cast in the form $\langle\psi|\, G \,|\psi\rangle$ where the operator G describes the strong interaction between the $q\bar{q}$ pairs. Expanding the matrix element in eigenstates of colours we find 3 diagonal terms where the $q\bar{q}$ pair does not change colour and 6 cross terms where colour charge is exchanged

$$\begin{array}{lll}
\text{diagonal} & \text{e.g.} & \langle r\bar{r}|\, G \,|r\bar{r}\rangle \\
\text{cross-terms} & \text{e.g.} & \langle r\bar{r}|\, G \,\left|b\bar{b}\right\rangle \quad .
\end{array}$$

To calculate the colour factor for the matrix element $\langle\psi|\, G \,|\psi\rangle$ we have to sum the corresponding products of colour coefficients $c_a c_b$ for the 3 diagonal and the 6 cross terms and obtain

$$f = -\tfrac{1}{2}\tfrac{1}{3}\left[3\left(-\frac{2}{3}\right) + 6(-1)\right] = \frac{4}{3} \tag{210}$$

We find that f is positive which when comparing with Equation 204 results in the following strong interaction potential

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{\alpha_S}{r} \; . \tag{211}$$

Hence the QCD potential $V_{q\bar{q}}(r)$ is negative. This results in an attractive force between quarks and antiquarks, at least at short range, which binds the $q\bar{q}$ pair into a colour singlet meson.

If the $q\bar{q}$ pair is in a colour octet state, for example $r\bar{b}$, the same calculation gives the following result:

$$V_{q\bar{q}}(r) = \frac{1}{6}\frac{\alpha_S}{r} \; .$$

The strong potential for colour octet states is repulsive at short distances. Consequently, no colour octet mesons have been observed.

Similarly we can calculate the colour factors for the quark-quark interaction. When combining 2 colour charges we find the decomposition $3 \otimes 3 = 6 \oplus \bar{3}$. The sextet is symmetric and only the antisymmetric triplet can be combined with the colour charge of the third quark to form a totally antisymmetric colour singlet state within a baryon.

$$(rgb - rbg + gbr - grb + brg - bgr)/\sqrt{6} \; . \tag{212}$$

The colour factor for three quarks in a singlet state is $f = \frac{2}{3}$ and the corresponding strong interaction potential between the three quarks in a baryon is

$$V_{q\bar{q}}(r) = -\frac{2}{3}\frac{\alpha_S}{r}$$

which is attractive as is expected for a colour single state.

Quantum chromodynamics is a perturbative theory and works only in the regime of $\alpha_S < 1$. At distances larger than about 0.2 fm, the strong coupling constant becomes large and the QCD perturbation series diverges. Thus the long range part of the strong interaction potential cannot be predicted in QCD. The breakdown of the perturbation series is believed to be due to the three-gluon and four-gluon self interactions which produce additional vertices which do not have an analogue in QED. These gluon self interactions squeeze the field lines of the strong force between a $q\bar{q}$ pair into a *flux tube*, as is sketched in Figure 59. The field lines in this flux tube contain a positive energy density which corresponds to an additional term in the strong interaction potential

$$V_{q\bar{q}}(r) = \lambda r \; .$$

Figure 59: Sketch of field lines for electromagnetic and strong interactions.

This linear potential results in a constant restoring force. The precise dependence of $V_{q\bar{q}}(r)$ at large r is not that important. Some authors prefer the potential of a harmonic oscillator $V \sim r^2$ as in a mechanical spring, others a logarithmic potential $V \sim \log(r)$. Using the linear term, the overall QCD potential for $q\bar{q}$ pairs is

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{\alpha_S}{r} + \lambda r \qquad (213)$$

The functional form of this potential is shown in Figure 60. This potential gives a good description

Figure 60: Strong interaction potential $V_{q\bar{q}}(r)$ of a $q\bar{q}$ pair as a function of the separation r.

for the spectra of charmonia, $c\bar{c}$, and bottomonia, $b\bar{b}$ quark states.

As a consequence of the positive term in the strong interaction potential $V_{q\bar{q}}(r)$, a $q\bar{q}$ pair cannot be separated since infinite energy were required. The quarks can never escape. Instead additional $q\bar{q}$ pairs are produced as the distance between quarks and hence their kinetic energy or tension increases[31]. This feature of the strong interaction is called **confinement**. All objects which carry colour charge, i.e. quarks and gluons, are permanently trapped within hadrons. When quarks and gluons are probed within hadrons, as for example in deep inelastic scattering experiments, quarks

[31] The numerical value of λ is about the same as the gravitational force of a mass of 16 tons.

produce additional quark-antiquark pairs. This process is called **fragmentation**. All quarks produced must combine into colour singlets, i.e. hadrons – **hadronisation**.

We call the regime where the strong interactions cannot be calculated, **non-perturbative**. A full calculation of QCD is attempted by putting quarks and gluons on discrete lattices, and calculating the interactions between these object by Monte Carlo methods. Very powerful computers are needed for these *lattice gauge theory* calculations[32].

11.5 Running Coupling Constants

Here we qualitatively look at the effect of higher order diagrams. In QED, a bare electric charge e is surrounded by a halo of virtual e^+e^- pairs. At low momentum transfers $q^2 \approx 0$, one probes not only the bare electron but also the halo which effectively produces a *screening* of the bare electric charge. When calculating the cross section of $e^-e^- \to e^-e^-$ scattering we actually have to sum an infinite series of diagrams with an ever increasing number of electromagnetic vertices. Unfortunately, even in QED, where the coupling constant $\alpha \ll 0$, some terms in these calculations diverge. However, these infinite terms can be reabsorbed into the coupling constants by a procedure called **renormalisation**. As a consequence the coupling constant is no longer constant but increases when q^2 of the process is increased.

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{q^2}{\mu^2}\right)} \qquad (214)$$

This effect is measurable. For e^+e^- collisions produced by the LEP collider at $\sqrt{s} = m_Z = 91.2$ GeV, the electromagnetic coupling constant is $\alpha(q^2 = m_Z^2) = 1/128$, an increase of 7 percents with respect to the value of $\alpha(q^2 \approx 0) = 1/137$ which we know from the Millikan experiment.

For QCD we use the same renormalisation method. The bare colour charge of the quarks is screened by virtual $q\bar{q}$ pairs. However, there are the additional gluon vertices. It turns out that they produce an effect with an opposite sign with respect to the screening. A red colour charge is preferentially surrounded by other red charges. This *anti-screening* has to be included and leads to the following result for the strong coupling constant

$$\alpha_S(q^2) = \frac{\alpha_S(\mu^2)}{1 + \beta\alpha_S(\mu^2)\log\left(\frac{q^2}{\mu^2}\right)} \qquad (215)$$

$$\beta = (11N_{\mathrm{colour}} - 2N_{\mathrm{flavour}})/(12\pi) \qquad (216)$$

The function β contains both, the contributions from screening and anti-screening. For our universe with $N_{\mathrm{colour}} = 3$ and $N_{\mathrm{flavour}} \leq 6$ the anti-screening effect dominates and the strong coupling constant decreases for large q^2 or correspondingly at short distances. This feature of QCD is called **asymptotic freedom**. Quarks and gluons which are probed with high q^2 deep within a hadron

[32] The particle physics theory group at the University of Edinburgh is currently building a very fast Teraflop machine to improve the precision of their lattice calculations.

are essentially free objects as has been established by deep elastic scattering experiments. We realise that the structure of QCD is much richer than QED.

11.6 Electron-Positron Annihilations

We now look at the process of electron-positron annihilation into quarks, $e^+e^- \to q\bar{q}$. As a consequence of the confinement of quarks and gluons within hadrons, the quarks fragment and hadronise. During this process momentum and energy must be conserved. Especially at high momenta the average hadron momentum along the original quark direction is large when compared to its transverse momentum which is limited to $\hbar c/r_0 \approx 500$ MeV where r_0 is a typical size of a hadron. Thus the fragmentation follows closely the direction of the leading quarks and produces a collimated **jet** of many hadrons in the detector. At the LEP collider operating at $\sqrt{s} = 91.2$ GeV, typically 20 charged particles, mostly pions, are produced. This is illustrated in Figure 61 where we show a hadronic event recorded by the ALEPH detector at LEP.

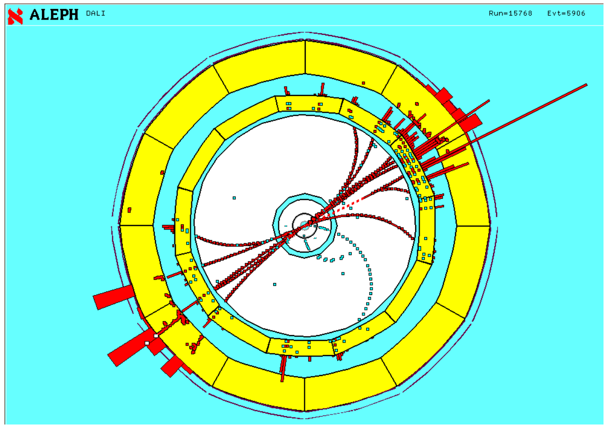

Figure 61: A typical 2-jet hadronic event recorded by the ALEPH detector.

When a quark radiates a *hard gluon* which carries a large fraction of the quark energy, we have the process $e^+e^- \to q\bar{q}g$ at quark level and we expect to see three jets in the detector. This is how in 1979, the gluons have first been observed at the PETRA e^+e^- collider operating at a centre-of-mass energy of 30 to 40 GeV. In Figure 62 we show a three-jet event measured by the L3 detector at LEP. The production rate of three-jet versus two-jet events is clearly related to the strong coupling constant α_S, see below.

The cross section for $e^+e^- \to q\bar{q}$ can be calculated in QED. The matrix element is analogous to the one for the process $e^+e^- \to \mu^+\mu^-$ as is illustrated in Figure 63. We can trivially exchange the

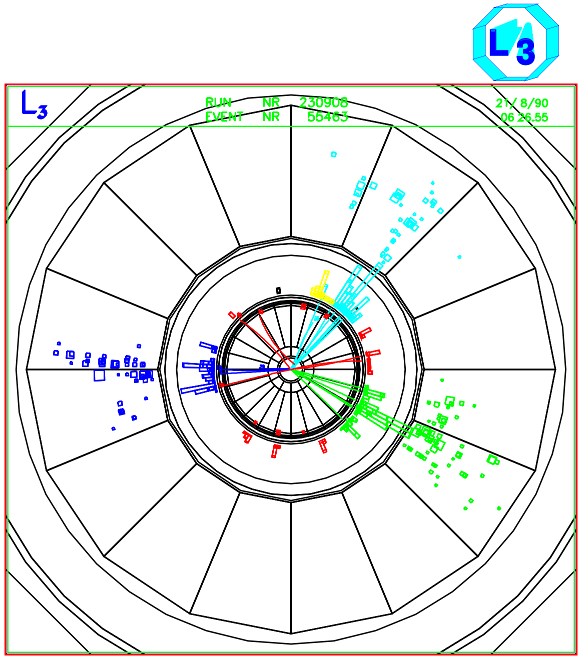

Figure 62: A typical 3-jet hadronic event recorded by the L3 detector.

$\mu^+\mu^-$ pair with a $q\bar{q}$ pair since both are point-like spin $\frac{1}{2}$ particles and their spinors are identical. We only need to replace the charge of the muon e with the fractional charge of the quarks e_q and to introduce an additional factor $N_c = 3$ since quarks come with three different possible colour charges. We find the result

$$\sigma(e^+e^- \to q\bar{q}) = 3\left(\frac{e_q}{e}\right)^2 \sigma(e^+e^- \to \mu^+\mu^-) \tag{217}$$

What we observe in an experiment are jets of hadrons.

$$e^+e- \to q\bar{q}, \qquad\qquad q\bar{q} \to \text{ hadrons.} \tag{218}$$

Experimentally, it is advantageous to measure ratios of cross sections since many uncertainties, like luminosity, cancel. We define

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\sum_q \left(\frac{e_q}{e}\right)^2$$

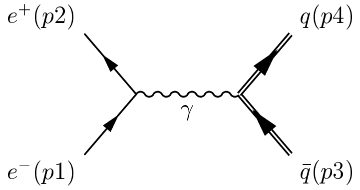

Figure 63: Diagram for $e^+e- \to q\bar{q}$.

We expect a step function for R as a function of the centre-of-mass energy, $\sqrt{s}$. At values of $\sqrt{s} < 3$ GeV below the $c\bar{c}$ threshold, only u, d, and s quarks can contribute. At values above the Υ resonances, u, d, s, c, and b quarks contribute and we predict that

$$\begin{aligned} R &= 3\left[(2/3)^2 + (-1/3)^2 + (-1/3)^2\right] = 2 && u\ d\ s \\ &= 2 + 3(2/3)^2 = 10/3 && u\ d\ s\ c \\ &= 10/3 + 3(-1/3)^2 = 11/3 && u\ d\ s\ c\ b \end{aligned} \tag{219}$$

In Figure 64 we show measurements of R in the region up to 40 GeV. The data are clearly consistent with these predictions for $N_c = 3$ and rule out $N_c = 1$. This is another experimental fact which shows that there are three colours in a quark. Including higher order processes, such

Figure 64: The ratio R as a function of the centre-of-mass energy up to 40 GeV.

as $e^+e^- \to q\bar{q}g$ gives the following result

$$R = 3\sum_q \left(\frac{e_q}{e}\right)^2 \left[1 + \frac{\alpha_S(q^2)}{\pi}\right] \tag{220}$$

This can be used to measure the strong coupling constant and a value of $\alpha_S \approx 0.14$ has been obtained at $\sqrt{s} = 30$ GeV.

In Figure 65 we show the cross section for $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to$ hadrons up to a centre-of-mass energy of 120 GeV. At energies up to about 50 GeV, the cross section for $e^+e^- \to \mu^+\mu^-$

follows the functional form derived in QED, see Equation 179.

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} = \frac{87\ \text{nb}}{s(\text{GeV}^2)} \tag{221}$$

and falls off proportional to s. At higher energies we find that the cross section increases again and we observe the Z resonance at $\sqrt{s} = 91.2$ GeV. This must be due to additional diagrams, which are caused by the weak interaction which we will study in the next chapter.

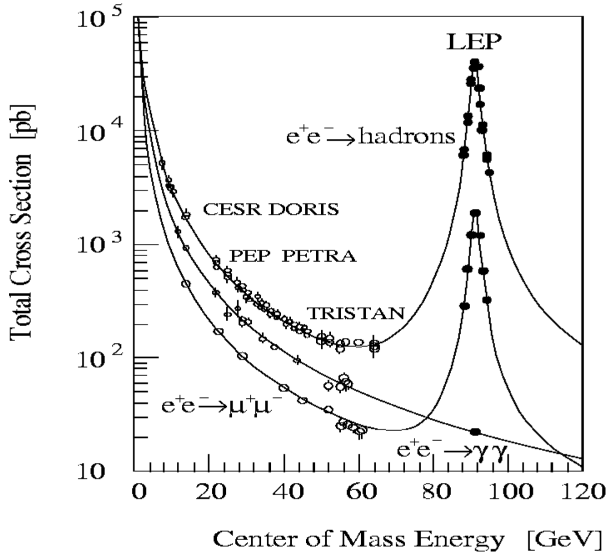

Figure 65: Cross sections for $e^+e^- \to$ hadrons $e^+e^- \to \mu^+\mu^-$ as a function of the centre-of-mass energy.