Introduction to the Standard Model

Lecture 13

The Standard Model

Up to this point we have been gathering all the materials needed from which to construct the Standard Model. The Standard Model with the gauge theory with the gauge group:

\[ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \]

It defines the fundamental interaction of leptons and quarks. It is an application of spontaneous symmetry breakdown.

We divide our approach to the model into two sectors:

I.) \( SU(2)_L \otimes U(1)_Y \)

- referred to as
  - Glashow-Salam-Weinberg Theory (c. 1970)
  - or Electro-Weak Theory
  - or Quantum Flavour Dynamics

- describes electromagnetic and weak interactions

\[ SU(2)_L \otimes U(1)_Y \xrightarrow{\text{SSBreakdown}} U(1)_{EM} \text{ remains} \]

- it is a chiral theory:
  \( \psi_L, \phi_L \) transforms as an \( SU(2)_L \) doublet
  \( \psi_R, \phi_R \) transforms as an \( SU(2)_L \) singlet

- does not distinguish between quarks, \textit{colour blind}
  this implies universal couplings to \( q^r,q^b,q^g \)

II.) \( SU(3)_C \)

- referred to as Quantum Chromodynamics

- describes strong interactions, nuclear binding forces

- the theory is exact in that the gauge bosons are strictly massless (unbroken)

- this is a vector-like theory: universal coupling to \( L \) and \( R \)-handed states

- \textit{flavour blind}; universal coupling to flavour eigenstates: \( u,d,s,c,b,t \)
Glashow-Salam-Weinberg Theory

- P. W. Higgs, *Phys. Lett.* (1964)

We can write the theory as the sum of Lagrangian densities:

$$
\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{Yukawa}}[+ \mathcal{L}_{\text{ghost}}]
$$

Note that the Yukawa sector, $\mathcal{L}_{\text{Yukawa}}$, allows for boson and fermion interactions; the “ghost” sector is needed to maintain covariance whilst quantising a non-Abelian gauge theory.

**The gauge sector: $\mathcal{L}_{\text{gauge}}$**

Weak interactions are short range hence we need a massive exchange particle

$$
\text{massless} \sim \frac{1}{r} \rightarrow \text{massive} \sim e^{-\frac{m_r}{r}}
$$

Parity violation in weak interaction is observed (no $L/R$ symmetry)

Electromagnetic interactions forms a good gauge theory as it has a massless boson with an unbroken generator; charge is conserved leaving the boson (photon) massless.

Many Lagrangians were constructed to explain the data; Glashow, Salam, and Weinberg wrote the right one.

**gauge group:** $SU(2)_L \otimes U(1)_Y$

**generators:** $T^1, T^2, T^3$

**weak isospin**

**Y**

**gauge coupling:**

$g \quad g'$

The Lagrangian may be written as

$$
\mathcal{L}_{\text{gauge}} = -\frac{1}{4} W^{j\mu\nu} W^{j\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}
$$

where

$$
W^{j\mu\nu} = \partial_{\mu} W^{j\nu} - \partial_{\nu} W^{j\mu} - g\varepsilon^{jlm} W_l^{\mu} W_m^{\nu} \quad j \in [1, 2, 3]
$$

$$
B_{\mu\nu} = \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} \quad \text{the } U(1)_Y \text{ field}
$$
This theory describes four massless vector bosons but we need only one.

Recall: explicit mass terms are not allowed as they would destroy gauge invariance; we need a Higgs sector.

The Higgs sector: $\mathcal{L}_{\text{Higgs}}$

We need the generation of gauge boson masses by spontaneous symmetry breakdown: 1 massless gauge boson (photon) and 3 massive gauge bosons (weak interactions)

Minimal choice: introduce a scalar field as an $SU(2)$ doublet

$$\vec{\phi} = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \pi_1 + i\pi_2 \\ \sigma + i\pi_3 \end{array} \right) = \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right)$$

where $T(\vec{\phi}) = \frac{1}{2}$ transforms under the fundamental representation of $SU(2)_L$.

Assign hypercharge: $Y(\vec{\phi}) = \frac{1}{2}$.

To make the theory invariant under local transformations, we need

$$D_\mu = \partial_\mu + ig\frac{1}{2} \vec{\sigma} \cdot \vec{W}_\mu + ig'YB_\mu$$

The Lagrangian is then

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \vec{\phi})^\dagger (D^\mu \vec{\phi}) - V(\vec{\phi}^\dagger \vec{\phi})$$

where

$$V(\vec{\phi}^\dagger \vec{\phi}) = \mu^2 \vec{\phi}^\dagger \vec{\phi} + \lambda (\vec{\phi}^\dagger \vec{\phi})^2$$

(It can be shown that globally symmetric $SU(2)$ scalar theory is isomorphic to the $SO(4)$ sigma-model. See tutorial for more details.)

Note that $\mu^2 < 0$ and $\lambda > 0$ leads to SSBreakdown.

$$\langle \vec{\phi} \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v \end{array} \right) \quad \text{with} \quad v = \sqrt{-\frac{\mu^2}{\lambda}}$$

Consider
\[
(T^3 + Y) \langle \vec{\phi} \rangle = \left[ \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \langle \vec{\phi} \rangle \Rightarrow e^{i\lambda Q} \langle \vec{\phi} \rangle = \langle \vec{\phi} \rangle
\]

\[
T^1 \langle \vec{\phi} \rangle = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix} \neq \langle \vec{\phi} \rangle
\]

Hence, \( T^1 \) is a broken generator. The same is true for \( T^2 \) and \( T^3 - Y \) meaning that we have three Goldstone bosons which we may gauge away. The \( SU(2)_L \) doublet can be written as (tutorial)

\[
\vec{\phi} = \exp(-i \frac{1}{v} T^3 \theta^j) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H + v \end{pmatrix} = U^\dagger \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H + v \end{pmatrix}
\]

The term \( U^\dagger = \exp(-i \frac{1}{v} T^3 \theta^j) \) looks like a \( SU(2)_L \) local group element. The Goldstone bosons, \( \theta^j \), play here the role of the space-time dependent parameters.

Applying the gauge transformation \( U \) leads to the so-called unitary gauge:

\[
\vec{\phi} \rightarrow \vec{\phi}' = U \vec{\phi} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H + v \end{pmatrix}
\]

We obtain

\[
\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \left[ D_\mu \begin{pmatrix} 0 \\ H + v \end{pmatrix} \right]^\dagger D^\mu \begin{pmatrix} 0 \\ H + v \end{pmatrix} + V \left( \frac{1}{2} (H + v)^2 \right)
\]

\[
D^\mu \begin{pmatrix} 0 \\ H + v \end{pmatrix} = \left( \partial_\mu + \frac{i g}{2} \vec{\sigma} \cdot \vec{W}_\mu + \frac{i g'}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} 0 \\ H + v \end{pmatrix}
= \left( \partial_\mu H \right) + \frac{ig}{2} (g' B_\mu - g W^3_\mu) \begin{pmatrix} v + H \\ 0 \end{pmatrix} + \frac{ig'}{2} (g' B_\mu - g W^3_\mu)
\]

\[
\left( D^\mu \begin{pmatrix} 0 \\ H + v \end{pmatrix} \right)^\dagger = (0, \partial_\mu H) - i \frac{g}{2} (W^1_\mu + iW^2_\mu) (0, v + H) - i \frac{g'}{2} (\frac{1}{\sqrt{2}} W^+)(0, v + H)
\]

\[
\mathcal{L}_{\text{Higgs}} = \frac{1}{2} \partial_\mu H \partial^\mu H - \frac{\mu^2}{2} (v + H)^2 - \frac{\lambda}{4} (v + H)^4 + \frac{g^2}{2} (v + H)^2 W^\mu_\mu W^\mu + \frac{1}{8} (g' B^\mu - g W^{3\mu}) (g' B_\mu - g W^3_\mu) (v + H)^2
\]

This Lagrangian defines interaction and mass terms
Note:
\[ \vec{\sigma} \cdot \vec{W}^\mu = \sigma^1 W^1_\mu + \sigma^2 W^2_\mu + \sigma^3 W^3_\mu \]
\[ = \begin{pmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & W^3_\mu \end{pmatrix} = \begin{pmatrix} W^3_\mu & W^+_\mu \\ W^-_\mu & W^3_\mu \end{pmatrix} \]

The charged vector boson masses can be read off directly from
\[ \frac{g^2 v^2}{8} 2W^+_{\mu}W^{-\mu} = \frac{g^2 v^2}{4} ((W^1_{\mu})^2 + (W^1_{\mu})^2) \]
leading to
\[ M_{W^\pm} = M_{W^{1,2}} = \left( \frac{g v}{2} \right) \]

For the quadratic term in the $W^3_{\mu}, B_{\mu}$ bosons we find
\[ \mathcal{L}_{\text{quad}} = \frac{v^2}{8} (W^3_{\mu}, B_{\mu}) \begin{pmatrix} g^2 & -gg' \\ -g'g & g'^2 \end{pmatrix} \begin{pmatrix} W^3_{\mu} \\ B_{\mu} \end{pmatrix} \]
The mass matrix has two eigenvalues, one of which is 0 implying that the determinant is 0 which means that there is the massless gauge boson. After the field redefinition,
\[ \begin{pmatrix} W^3_{\mu} \\ B_{\mu} \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} , \quad \sin \theta_W = \frac{g'}{(g^2 + g'^2)^{1/2}} \]
we get
\[ \mathcal{L}_{\text{quad}} = \frac{v^2}{8} (g^2 + g'^2) \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z_{\mu} \\ A_{\mu} \end{pmatrix} \]
We see that $A^\mu$ is massless and that $Z^\mu$ is a massive vector boson with mass $M_Z^2 = \frac{g^2 + g'^2 v^2}{4}$, and the other massive state in Higgs sector is the Higgs boson with mass $M_H^2 = -2\mu^2 = 2\lambda v^2$. 

5