Introduction to the Standard Model

Lecture 15

The Yukawa sector continued

$$\mathcal{L}_{\text{Yukawa(quark)}} = \left(\overline{d'}_R, \overline{s'}_R, \overline{b'}_R\right) C_q \begin{pmatrix} \vec{\phi}^{\,\dagger} \cdot \begin{pmatrix} u \\ d' \end{pmatrix}_L \\ \vec{\phi}^{\,\dagger} \cdot \begin{pmatrix} c \\ s' \end{pmatrix}_L \\ \vec{\phi}^{\,\dagger} \cdot \begin{pmatrix} t \\ b' \end{pmatrix}_L \end{pmatrix} + \left(\overline{u}_R, \overline{c}_R, \overline{t}_R\right) C'_q \begin{pmatrix} \vec{\phi}^{\,\mathrm{T}} \cdot \varepsilon \cdot \begin{pmatrix} u \\ d' \end{pmatrix}_L \\ \vec{\phi}^{\,\mathrm{T}} \cdot \varepsilon \cdot \begin{pmatrix} c \\ s' \end{pmatrix}_L \\ \vec{\phi}^{\,\mathrm{T}} \cdot \varepsilon \cdot \begin{pmatrix} t \\ b' \end{pmatrix}_L \end{pmatrix}$$
$$+ \text{ Herm. conj.}$$

We may choose the basis using the transformations

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix}_R \to U_2 \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R, \quad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_R \to U_3 \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_R, \quad \begin{pmatrix} \begin{pmatrix} u \\ d' \end{pmatrix}_L \\ \begin{pmatrix} c \\ s' \end{pmatrix}_L \\ \begin{pmatrix} t \\ b' \end{pmatrix}_L \end{pmatrix} \to V_2 \begin{pmatrix} \begin{pmatrix} u \\ d' \end{pmatrix}_L \\ \begin{pmatrix} c \\ s' \end{pmatrix}_L \\ \begin{pmatrix} t \\ b' \end{pmatrix}_L \end{pmatrix}$$

The quark sector

Using U_2, V_2 one finds analogously to the lepton case

$$C'_q \to U_2^\dagger C'_q V_2 = \begin{pmatrix} \lambda_u & 0 & 0 \\ 0 & \lambda_c & 0 \\ 0 & 0 & \lambda_t \end{pmatrix}$$

$$C_q \to U_3^\dagger C_q \underbrace{V_3 V_3^\dagger}_{\mathbb{1}} V_2 = \begin{pmatrix} \lambda_d & 0 & 0 \\ 0 & \lambda_s & 0 \\ 0 & 0 & \lambda_b \end{pmatrix} \underbrace{V_3^\dagger V_2}_{\equiv V^\dagger}$$

Redefine $C_q \to V C_q$. Without a loss of generality, C_q can be brought to the form

$$C_q \to V \begin{pmatrix} \lambda_d & 0 & 0 \\ 0 & \lambda_s & 0 \\ 0 & 0 & \lambda_b \end{pmatrix} V^\dagger$$

where V is the Cabibo-Kobyashi-Maskawa matrix (1973). We now have a unitary transform that works on the u, c, t, and d, s, b quarks.

There is still some freedom that allows us to restrict $V \in U(3)$ further. We are free to apply a phase transformation on $\psi_{L/R}$ since the wavefunctions are defined up to a global phase.

$$V = \begin{pmatrix} e^{-i\varphi_1} & 0 & 0 \\ 0 & e^{-i\varphi_2} & 0 \\ 0 & 0 & e^{-i\varphi_3} \end{pmatrix} V \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix}$$

Consider first the case of two generators:

$$V_{2\text{ gens.}} = \begin{pmatrix} e^{-i(\varphi_1-\chi_1)}V_{11} & e^{-i(\varphi_1-\chi_2)}V_{12} \\ e^{-i(\varphi_2-\chi_1)}V_{21} & e^{-i(\varphi_2-\chi_2)}V_{22} \end{pmatrix}$$

The three phase differences can be chosen such that: $V_{11} \geq 0$, $V_{12} \geq 0$, $V_{21} \leq 0$.
The 4th phase is fixed as $\varphi_2 - \chi_2 = (\varphi_2 - \chi_1) + (\varphi_1 - \chi_2) - (\varphi_1 - \chi_1)$. This gives

$$V = \begin{pmatrix} V_{11} & V_{12} \\ -|V_{21}| & e^{i\rho}V_{22} \end{pmatrix}, \qquad V^\dagger = \begin{pmatrix} V_{11} & -|V_{12}| \\ V_{21} & e^{-i\rho}V_{22} \end{pmatrix}$$

as $V \in U(2) \Rightarrow VV^\dagger = \mathbb{1} \Rightarrow$

$$\begin{aligned} V_{11}^2 + V_{21}^2 &= 1 \\ -V_{11}|V_{21}| + V_{12}V_{22}e^{-i\rho} &= 0 \\ |V_{21}|^2 + V_{22}^2 &= 1 \end{aligned}$$

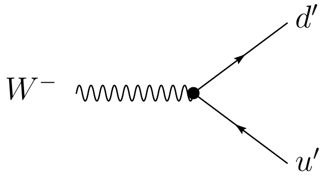
To fulfill these three conditions we need $\rho = 0$, an imaginary phase is not allowed. Hence

$$\left.\begin{aligned} V_{11} = V_{22} &= \cos\theta_C \\ V_{11} = |V_{21}| &= \sin\theta_C \end{aligned}\right\} \theta_C \in [0, \frac{\pi}{2}] \text{ is called the \textit{Cabibo angle}}$$

The Cabibo matrix for two generations,

$$V = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

acts on the d' and s' quarks and introduces $d' - s'$ mixing.



W couples to the doublet $\begin{pmatrix} u \\ d' \end{pmatrix}_L = \Bigg(\begin{matrix} u \\ d'\cos\theta_C + s'\sin \end{matrix}$

as a linear combination of mass eigenstates.
This allows K^+ to decay via a vector boson → K^+, K^-, $\Lambda\ldots$ can decay via charged currents.

In the 3-generation case we start with

$$V = \begin{pmatrix} V_{ud} & V_{ud} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

which can be written as a matrix defined by 3 angles and 1 phase if we follow the same reasoning as in the 2-generation case. We employ a reduced notation to express this more compactly: $\cos\theta_i \to c_i$, $\sin\theta_i \to s_i$. The mixing matrix is then

$$\begin{pmatrix} c_1 & s_1 c_2 & s_1 s_2 \\ -s_1 c_2 & \left(c_1 c_2 c_3 - s_2 s_3 e^{i\delta}\right) & \left(c_1 c_2 s_3 + s_2 c_3 e^{i\delta}\right) \\ -s_1 s_2 & \left(c_1 s_2 c_3 + c_2 s_3 e^{i\delta}\right) & \left(c_1 s_2 s_3 - c_2 c_3 e^{i\delta}\right) \end{pmatrix}$$

where $\theta_i \in [0, \frac{\pi}{2}]$, $\delta \in [0, 2\pi]$.

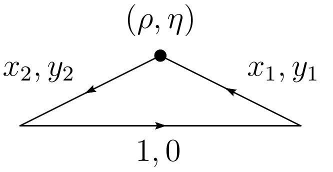
Remarks: 1.) Of all parameters in the Standard Model, $e^{i\delta}$ is the only explicitly complex one. Such terms are not invariant under CP transformations (→ CP violation). This has two important applications:
i.) mixing in kaon and anti-kaon → first experimental indication of CP violation in nature.
ii.) for baryon asymmetry → need CP violation.

2.) V_{ub},V_{td} are poorly restricted experimentally. As $VV^\dagger = \mathbb{1}$ there are g relations, for example

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0 \quad \Leftrightarrow \quad 1 + \underbrace{\frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}}}_{\in\mathbb{C}} + \underbrace{\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}}_{\in\mathbb{C}} = 0$$

$$\Leftrightarrow \quad \left(1, 0\right) + \left(x_2, y_2\right) + \left(x_1, y_1\right) = 0$$

We can identify these as vectors in the complex plane forming a *unitary triangle*:



Collecting things together, the Yukawa sector of the Standard model leads to mass terms for fermions, $m_f = \lambda\frac{v}{\sqrt{2}}$, and Higgs-fermion interactions:

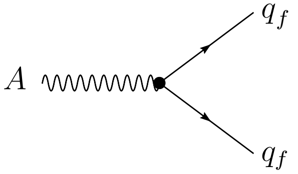
$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} = -\Bigg\{ & \left(\overline{e}, \overline{\mu}, \overline{\tau}\right)\begin{pmatrix} M_e & 0 & 0 \\ 0 & M_\mu & 0 \\ 0 & 0 & M_\tau \end{pmatrix}\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + \\ & + \left(\overline{u}, \overline{c}, \overline{t}\right)\begin{pmatrix} M_u & 0 & 0 \\ 0 & M_c & 0 \\ 0 & 0 & M_t \end{pmatrix}\begin{pmatrix} u \\ c \\ t \end{pmatrix} + \left(\overline{d}, \overline{s}, \overline{b}\right)\begin{pmatrix} M_d & 0 & 0 \\ 0 & M_s & 0 \\ 0 & 0 & M_b \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix} \Bigg\}\left(1 + \frac{H}{v}\right)\end{aligned}$$

Recall that dsb is a linear combination of electroweak eigenstates:

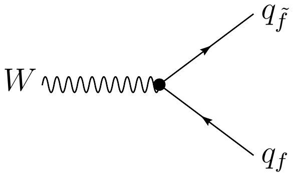
$$\underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}_{\text{mass eigenstates}} = V \underbrace{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}}_{\text{e.w. eigenstates}}$$

We see that we have 9 masses + 3 angles + 1 phase = 13 parameters. The flavour sector has many parameters to be fixed by experimental observation.

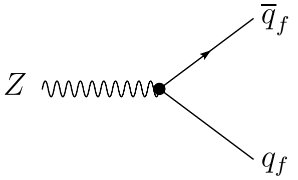
Feynman Rules of fermion, gauge and Yukawa sectors



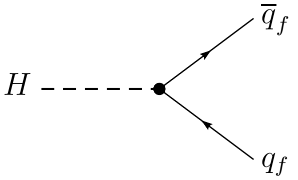
$$\sim ieQ_f\gamma^\mu$$



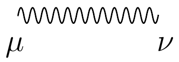
$$\sim \tfrac{-ig}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)\left(V^\dagger\right)_{f\tilde{f}}$$



$$\sim \tfrac{-ig}{2\cos\theta_W}\gamma^\mu(V_f - A_f\gamma_5)$$



$$\sim -i\tfrac{m_f}{v} = \tfrac{-igm_f}{2M_W}$$



$$\sim i\left(\frac{-g^{\mu\nu}+\dfrac{p^\mu p^\nu}{M_Y^2}}{p^2-M_Y^2+i\delta}\right)$$

The massive gauge boson propagator is given in the unitary gauge. In this gauge no Goldstone bosons are present.