Introduction to the Standard Model

Lecture 17

Quantum Chromodynamics (QCD)

Non-abelian gauge theories	Yang, Mills (1954)
Renormalizability	Fadeev, Popov (1969)
Quark model and non-abelian g.t.	Fritzsch, Gell-Mann, Leutwyler (1972/73)
Asymptotic freedom	Gross, Politzer, Wilczek (1973)

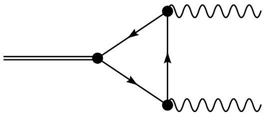
Hints that we must go beyond the quark model:

a.) The quark model has state: Δ^{++}, Ω^- $\rightarrow$ bound states of 3 identical fermions, $\Delta^{++} \sim u^\uparrow u^\uparrow u^\uparrow$, which is not compatible with the exclusion principle.
Motivates a distinguishing, extra label organised in an anti-symmetric way called *colour*:

$$\Delta^{++} \sim \sum_{i,j,k \in \{b,r,g\}} u_i^\uparrow u_j^\uparrow u_k^\uparrow \epsilon_{ijk}$$

b.) $\Gamma(\pi_0 \rightarrow \gamma\gamma)$

$\approx$



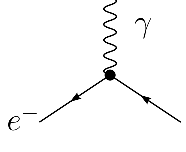
$\approx \left(\frac{N_C}{3}\right)^2$

If $N_C = 1$ (only 1 colour), then the theoretical predictions for decay cross-sections is a factor of 10 too small (see tutorial).

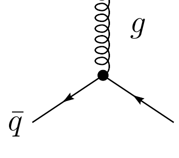
c.) Quark dynamics must explain why no free quarks (confinement):

- → all hadrons are colour singlets
- → non point-like substructure of hadrons observed in collider experiments
- → parton model where partons are quarks + gluons (Feynman, Bjorken 1972)

QED was very well understood; thus it was proposed that quark dynamics should follow from a non-Abelian gauge theory.



$\Leftrightarrow$



QCD was formulated as an $SU(3)_C$ gauge theory with quarks (fermions) in the fundamental representation.

$$\mathcal{L} = \mathcal{L}_{\text{gluon}} + \mathcal{L}_{\text{quarks}} + \mathcal{L}_{\text{gauge-fixing}} + \mathcal{L}_{\text{ghost}}$$

The ghost sector, $\mathcal{L}_{\text{ghost}}$, is needed for the quantisation of non-abelian gauge theories.

$$\begin{aligned}
\mathcal{L}_{\text{gauge}} &= -\frac{1}{4}\sum_{a=1}^{N^2-1} F^a_{\mu\nu}F^{a\mu\nu} \qquad \text{where } F^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a - g f_{abc} A^\mu_b A^\nu_c \\
&= -\frac{1}{2} tr\big(F_{\mu\nu}F^{\mu\nu}\big) \qquad \text{where } F^{\mu\nu} = F_a{}^{\mu\nu} T_a \\
&= -\frac{1}{4N} tr_{\text{Ad}}\big(\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\big) \qquad \text{where } \mathcal{F}_{\mu\nu} = F^a_{\mu\nu} T^a_{\text{Ad}}
\end{aligned}$$

Remember that $\big(T^a_{\text{Ad}}\big)_{bc} = -if^{abc} \in \mathbb{C}^{N\times N}$ and $tr_{\text{Ad}}\big(T^a_{\text{Ad}} T^b_{\text{Ad}}\big) = N\delta^{ab}$

$$\mathcal{L}_{\text{quarks}} = \sum_{\substack{f \\ \text{flavours}}} \sum_{i,j=1}^{N} \bar{q}^i_f \Big(i\not{D} - m_f\Big)_{ij} q^j_f$$

where $\Big(D_\mu\Big)_{jk} = \partial_\mu \delta_{jk} + igA^a_\mu T^a_{jk}$. The gluon fusion interaction is induced by the covariant derivative.

We need a gauge fixing term to arrive at an invertible gluon propagator.

$$\mathcal{L}_{\text{gauge-fixing}} = -\frac{1}{2\lambda}\big(\partial_\nu A^{\mu c}\big)\big(\partial_\mu A^{\nu c}\big)$$

Note that is is not unique, there are many possible choices. In nonabelian gauge theories we need to add a "ghost" sector which is needed for a consistent quantisation procedure. (It decouples in the case of an abelian gauge theory.).

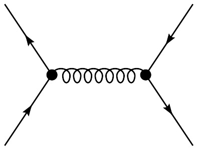
$$\mathcal{L}_{\text{ghost}} = \partial_\mu \bar{\eta}_a \partial^\mu \eta_a + g(\partial_\mu \bar{\eta}_c) f^{abc} A^{b\,\mu} \eta_a$$

- η "Fadeev-Popov ghost" = complex scalar with "wrong" (i.e. fermionic) statistics. (i.e. closed loops get a minus sign).
- Ghost contribution compensates longitudinal degree of freedom in gluon loops in a covariant way.
- Ghosts never appear as "external" particles in scattering amplitudes.
- Ghost sector decouples in QED as $f_{abc} = 0$.
- For a proper derivation path integral methods (original) or BRS methods necessary (→ MQFT lectures).

Asymptotic Freedom in QCD

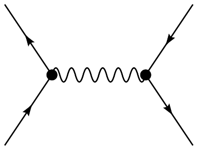
QCD is qualitatively different than classical field theory.

QCD (classical)



$\sim \frac{\alpha_s}{Q^2} \leftrightarrow \frac{\alpha_s}{r}$ potential

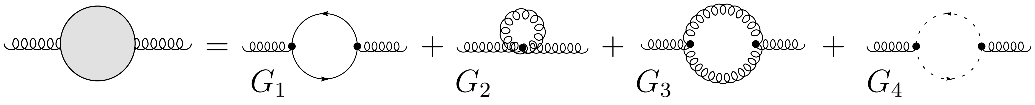
Electrodynamics:



$\sim \frac{\alpha}{Q^2} \leftrightarrow \frac{\alpha}{r}$ potential

However, the quantum effects change qualitatively the low energy behaviour. Let's consider the loop corrections to the gluon propagator. These corrections are given to order $\alpha_s = {}^{g_s^2}/_{4\pi}$ by

$\Sigma^{ab}_{\mu\nu}(p) =$



These diagrams correspond to divergent integrals. We must apply a regularisation and renormalisation procedure to deal with the divergencies. Observables will not depend on the chosen regulator. One approach is *dimensional regularisation* where we evaluate these integrals in dimension $= 4 - 2\varepsilon$.

For the Feynman rule this would amount to ($\overline{\text{MS}}$-scheme):

$$\alpha_s \int \frac{d^4k}{(2\pi)^4} \quad \to \quad \alpha_s^{\overline{\text{MS}}} \frac{\mu^{2\varepsilon}}{(4\pi)^{2\varepsilon}} \frac{1}{\Gamma(1+\varepsilon)} \int \frac{d^nk}{(2\pi)^n}$$

As $\varepsilon \to 0$, divergences occur as $\frac{1}{\varepsilon}$ poles.

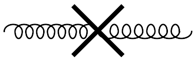
The physics is unchanged under a reparameterisation of the coupling constant and fields

$$\left.\begin{array}{rcl} A^\mu & \to & \sqrt{Z_3}A^\mu \\ g_s & \to & Z_g g_s \end{array}\right\} \quad \text{rescaling doesn't change physics}$$

where the constants Z_3, Z_g are defined perturbatively

$$
\begin{aligned}
Z_3 &= 1 + Z_3^{(1)} + Z_3^{(2)} + \dots \\
Z_3^{(1)} &= \alpha_s \big(Z_3^{(1,1)} \frac{1}{\varepsilon} + Z_3^{(1,0)} \big) \\
Z_3^{(2)} &= \alpha_s^2 \big(Z_3^{(2,2)} \frac{1}{\varepsilon^2} + \frac{1}{\varepsilon} Z_3^{(2,1)} + Z_3^{(2,0)} \big) \\
&\vdots
\end{aligned}
$$

$Z_3^{(k)}$ are constants which can be chosen freely.
These renormalisation constants lead to additional Feynman graphs like

$G_5 =$ 

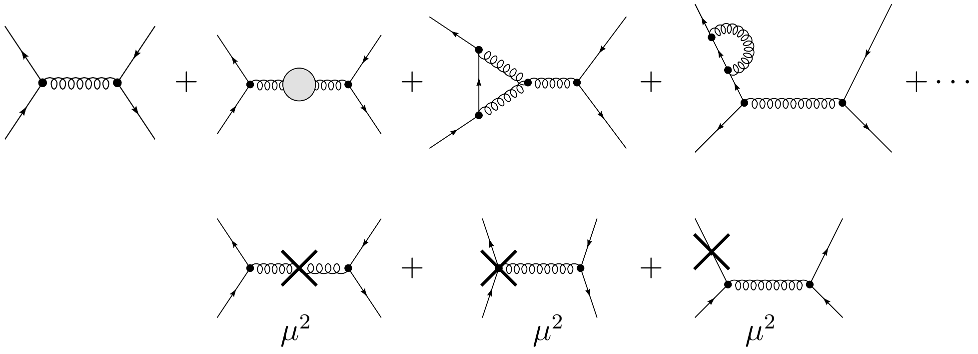
Evaluating the divergent part of $G_1 + \dots + G_4$ gives (this is beyond the scope of this course)

$$\Sigma^{ab}_{\mu\nu}(p)\Big|_{\text{div}} = \frac{\alpha_s}{4\pi}\frac{r_3}{\varepsilon}\delta^{ab}\Big(T_R N_f \frac{4}{3} - C_A \frac{13}{6}\Big)\Big(- p^2 g^{\mu\nu} + p^\mu p^\nu\Big)$$

As it is a constant, we choose $Z_3^{(1)}$ to cancel this (unphysical) contribution at a certain scale $\mu^2 = -p^2 > 0$.

For the coupling at one loop we find, schematically,

$\alpha_s(Q^2) \sim$



The latter are divergences which come from the renormalisation constants. The are adjusted so to cancel the divergences and with an experimental value at a certain scale.

$$
\begin{aligned}
\alpha_s(Q^2) =& \alpha_s + \alpha_s^2 \frac{b}{\varepsilon}\big(Q^2\big)^{-\varepsilon} - \alpha_s^2 \frac{b}{\varepsilon}\big(\mu^2\big)^{-\varepsilon} \\
=& \alpha_s \Big(1 - \alpha_s b \log\big(\frac{Q^2}{\mu^2}\big) + \mathcal{O}(\alpha^2)\Big)
\end{aligned}
$$

Quantum corrections in this way lead to a scale-dependent coupling

$$\frac{1}{\alpha_s(Q^2)} = \frac{1}{\alpha_s} + b \log\big(\frac{Q^2}{\mu^2}\big); \quad b = \frac{11 C_A - 4 T_R N_f}{12\pi}$$

This is the *renormalised coupling* and it depends logarithmically on the scale of the process. The same is true for QED but b has a different sign.