Introduction to the Standard Model

Lecture 21

Conclusion

The Standard Model is a spontaneously broken gauge theory:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow[\text{Higgs}]{\text{mechanism}} SU(3)_L \otimes U(1)_{\text{em}}$$

The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Dirac}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}} \Big(+ \mathcal{L}_{\text{gauge-fixing}} + \mathcal{L}_{\text{ghost}} \Big)$$

The particle content is derived from experiment

$$\text{Leptons} \quad \left.\begin{array}{c} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{\text{L}}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_{\text{L}}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_{\text{L}} \\ e_{\text{R}},\ \mu_{\text{R}},\ \tau_{\text{R}} \\ \nu_{e\text{R}},\ \nu_{\mu\text{R}},\ \nu_{\tau\text{R}} \end{array}\right\} \quad \text{no colour}$$

The right handed quark sector has been included recently due to obsevations of neutrino oscillations which are only possible for massive neutrinos.

$$\text{Quarks} \quad \begin{array}{l} \begin{pmatrix} u \\ d' \end{pmatrix}_{\text{L}}, \begin{pmatrix} c \\ s' \end{pmatrix}_{\text{L}}, \begin{pmatrix} t \\ b' \end{pmatrix}_{\text{L}} \\ u_{\text{R}},\ c_{\text{R}},\ t_{\text{R}} \\ d'_{\text{R}},\ s'_{\text{R}},\ b'_{\text{R}} \end{array} \qquad \begin{array}{l} \text{with the colour triplets} \\ \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = v \begin{pmatrix} d \\ s \\ b \end{pmatrix} \end{array}$$

where $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$ are the electroweak eigenstates, $\begin{pmatrix} d \\ s \\ b \end{pmatrix}$ are the mass eigenstates and $V \in U(3)$ is the CKM matrix.

The only yet to be determined parameter is the Higgs mass (or the Higgs self-coupling).

The Standard Model has 18 free parameters to be fixed by experiment. If the neutrino fermions are of Dirac type, means equivalent to the quark sector, there are in addition 3 neutrino masses, 3 mixing angles and 1 mixing phase.

$$\begin{array}{rcl} \text{gauge couplings:} & g, g', g_s & \leftrightarrow \quad \alpha, \alpha_s, \sin\theta_W \\ \text{Higgs sector:} & \lambda, v & \leftrightarrow \quad M_z^2,\ M_H^2 \sim \\ \text{Yukawa sector:} & \text{9 fermion masses} & \\ & \text{CKM-Matrix} & \rightarrow \quad \text{3 angles} + \text{1 phase} \end{array}$$

Comment: In principle there is another term allowed by gauge symmetry:

$$\mathcal{L}_{\text{CP}} = \theta F^a_{\mu\nu} \varepsilon^{\mu\nu\sigma\rho} F^a_{\sigma\rho} \qquad \text{where} \theta = 0$$

This terms then would lead to CP violation and is experimentally shown to be about $\theta \sim 0$.

Measured Values of the Standard Model parameters

$$\begin{array}{rcl}
m_\gamma & < & 6 \times 10^{-17}\ eV \\
m_g & = & 0 \\
m_Z & = & 91.1876\ \pm\ 0.0021\ GeV \\
\left(\Gamma_Z\right. & = & \left. 2.4952\ \pm\ 0.0023\ GeV\right) \\
\left(M_W\right. & = & \left. 80.403\ \pm\ 0.0029\ GeV\right)
\end{array}$$

$$\begin{array}{rcl}
m_H & \geq & 114.4\ MeV\ \ (95\%)(\text{LEP II search}) \\
m_e & = & 0.51099892\ \pm\ 0.00000004\ MeV \\
m_\mu & = & 0.1134289264\ \pm\ 0.0000000030\ MeV \\
m_\tau & = & 1776.99\ {}^{+\,0.29}_{-\,0.26}\ MeV
\end{array}$$

$$\begin{array}{rcl}
N_\nu & = & 2.994\ \pm\ 0.0012 \\
v & = & 246\ GeV
\end{array}$$

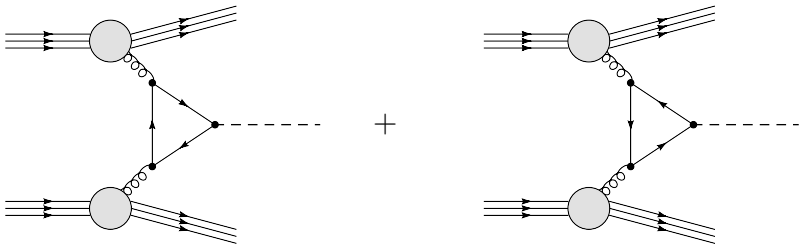
$$\begin{array}{rcll}
m_u & \approx & 1.5 - 3\ MeV & \\
m_d & \approx & 3 - 7\ MeV & \\
m_s & \approx & 95\ \pm\ 25\ MeV & \\
m_c & = & 1.25\ \pm\ 0.09\ GeV & \\
m_b & = & 4.2\ \pm\ 0.07\ GeV & \\
m_t & = & 174.2\ \pm\ 3.3\ GeV & \text{direct observation} \\
 & = & 172.3\ {}^{+\,10.2}_{-\,7.6}\ GeV & \text{fit to data}
\end{array}$$

From Standard Model tested at the one-loop level. Note that the Standard Model is a *renormalisable* theory, means that divergencies in the loop expansions can be systematically removed by a renormalisation procedure and that there is formally no dependence on the UV cut-off. This leads to a high predictive power of the theory.

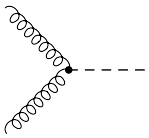
Finding the Higgs boson at hadron colliders

The Higgs boson is the last missing particle. It is experimentally restricted to have a mass in a relatively narrow range: $114.4\ GeV \leq M_H \leq 200\ GeV$.

The LHC has the largest cross-section from the *gluon fusion process*



Only an internal top quark contributes sizably in the Standard Model.
In the limit $M_{\text{top}} \to \infty$, gluon fusion can be described by an effective ggH-vertex:



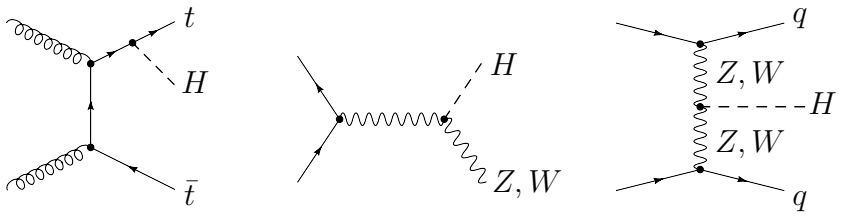
Using this effective vertex and the parton model, one finds for the cross-section

$$\sigma = \int_0^1 dx_1 \int_0^1 dx_2 f_{g/P}(x_1) f_{g/P}(x_2) \delta(x_1 x_2 S - M_H^2) \frac{\alpha_g^2}{576\pi} \frac{M_H^2}{v^2} = L_{gg}(M_H) \big(\frac{\alpha_g^2}{576\pi} \frac{M_H^2}{v^2} \big)$$

where $\sqrt{s} = 14000\, GeV$ is the center of mass energy of the two protons.

The luminosity function, $L_{gg}(M_H^2)$ for $M_H = 125\, GeV$ is $\approx 10^7$ picobarn.
With $\alpha_s \approx 0.1$, $v \approx 125$ GeV the cross section is defined. The yearly integrated luminosity of the LHC will be between 10 and 100 fb$^-$1. One obtains then of the order of 10^5 to 10^6 events per year.

Other ppossible production mechanisms for the Higgs boson are



The possible decay channels for the Higgs boson are: $H \to \gamma\gamma,\ H \to WW, ZZ\ H \to \Sigma\Sigma$.

The branching ratio, $H \to b\bar{b}$, is large for $M_H < 2m_z$ but there is too much background.

Higgs discovery is expected for 2008+x years, where $x \sim 2 - 3$ years depending on machine and detector performance.

The Standard Model is expected to be a low energy limit of a more fundamental theory to be revealed at still higher energies.