

Electromagnetism - Lecture 9

Dielectric Materials

- Electric Dipole Moments of Atoms
- Polarization Vector \mathbf{P}
- Local Electric Field
- Electric Displacement Vector \mathbf{D}
- Linearity, Isotropy and Homogeneity
- Polar Molecules

Polarizability of Atoms

In an insulator the electrons are bound to the atomic lattice and cannot move freely ... but in the presence of an **external electric field \mathbf{E}** each atom becomes slightly **polarized**

The atomic dipole moment is:

$$\mathbf{p} = Ze\mathbf{a} = \alpha\epsilon_0\mathbf{E}_L$$

where the separation between the centroid of the electron cloud and the atomic nucleus is small compared to the Bohr radius:

$$|\mathbf{a}| \approx 10^{-15}m = 10^{-5}a_0$$

α is known as the atomic or molecular *polarizability*

\mathbf{E}_L is the **local electric field** inside the material which is not identical to the external field \mathbf{E}

Polarization Vector & Susceptibility

The atomic dipole moment can be expressed as an integral over the atomic charge density:

$$\mathbf{p} = \int_{atom} \mathbf{r} \rho d\tau$$

The **dipole moment density** is the **polarization vector \mathbf{P}** :

$$\mathbf{P} = \frac{d\mathbf{p}}{d\tau} = \langle \mathbf{r} \rho \rangle_{atom}$$

The polarization vector is proportional to the external field \mathbf{E} :

$$\mathbf{P} = \chi_E \epsilon_0 \mathbf{E}$$

where χ_E is the **electric susceptibility** of the material

Notes:

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Polarization Flux

The *polarization flux* through a surface has units of charge:

$$\mathbf{P} \cdot d\mathbf{S} = \frac{d(\mathbf{p} \cdot \hat{\mathbf{n}})}{dl} = -Q_P$$

The *polarization charge* Q_P is equivalent to *induced surface charges* $\sigma_P = \mp |\mathbf{P}|$ on the surfaces perpendicular to the external field

Integrating the polarization flux over a closed surface

$$\oint_A \mathbf{P} \cdot d\mathbf{S} = \oint_A \frac{d(\mathbf{p} \cdot \hat{\mathbf{n}})}{dl} = - \int_V \rho_P d\tau$$

where ρ_P is the *polarization charge density*

Using the divergence theorem:

$$\nabla \cdot \mathbf{P} = -\rho_P$$

The distributions σ_P and ρ_P represent the effect of the atomic polarization with *equivalent* macroscopic charge distributions

Local Electric Field

Inside the material the polarization vector \mathbf{P} modifies the local electric field \mathbf{E}_L compared to the external electric field \mathbf{E}

$$\mathbf{E}_L = \mathbf{E} + \frac{\mathbf{P}}{3\epsilon_0} = \left(1 + \frac{\chi_E}{3}\right) \mathbf{E}$$

The factor 1/3 will be derived in a later lecture

The polarization vector can be expressed microscopically in terms of \mathbf{E}_L and macroscopically in terms of \mathbf{E} :

$$\mathbf{P} = N_A \alpha \epsilon_0 \mathbf{E}_L \quad \mathbf{P} = \chi_E \epsilon_0 \mathbf{E}$$

where N_A is Avogadro's number and α is the atomic polarizability

$$\chi_E = N_A \alpha \left(1 + \frac{\chi_E}{3}\right) \quad \chi_E = \frac{N_A \alpha}{(1 - N_A \alpha / 3)}$$

For small polarizability $N_A \alpha \ll 1$: $\chi_E \approx N_A \alpha$ and $\mathbf{E}_L \approx \mathbf{E}$

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Electric Displacement Vector

Gauss's Law is modified to include polarization effects:

$$\oint_A \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \int_V (\rho_C + \rho_P) d\tau \quad \nabla \cdot \mathbf{E} = \frac{(\rho_C + \rho_P)}{\epsilon_0}$$

where ρ_C are free charges (if any).

Using $\nabla \cdot \mathbf{P} = -\rho_P$ this can be rewritten as:

$$\nabla \cdot \mathbf{E} = \frac{\rho_C}{\epsilon_0} - \frac{\nabla \cdot \mathbf{P}}{\epsilon_0} \quad \nabla \cdot (\epsilon_0 \mathbf{E} + \mathbf{P}) = \rho_C$$

The electric displacement vector \mathbf{D} is:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad \nabla \cdot \mathbf{D} = \rho_C$$

Note that \mathbf{D} and \mathbf{P} have units Cm^{-2} , whereas \mathbf{E} is in Vm^{-1}

Dielectric Constant

The linear relationships between \mathbf{D} , \mathbf{E} and \mathbf{P} :

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} \quad \mathbf{P} = \chi_E \epsilon_0 \mathbf{E}$$

can be expressed in terms of a **dielectric constant** ϵ_r

$$\mathbf{D} = \epsilon_r \epsilon_0 \mathbf{E} \quad \epsilon_r = 1 + \chi_E$$

For most dielectrics $\chi_E \ll 1$ and ϵ_r is slightly above 1

This reflects the small atomic polarizability α of most materials

*General advice - wherever ϵ_0 appears in electromagnetism,
it should be replaced by $\epsilon_r \epsilon_0$ for insulators*

LIH Dielectrics

Implicit in the idea of a dielectric *constant* for a material are certain assumptions:

- Linearity - the polarization is proportional to the magnitude of \mathbf{E} . *This breaks down at high electric fields.*
- Isotropy - the polarization is independent of the direction of \mathbf{E} . *Some crystals are anisotropic, particularly under mechanical stress. Known as the piezoelectric effect.*
- Homogeneity - the polarization is the same everywhere in the material.

Materials satisfying these conditions are known as LIH dielectrics

Most insulators are LIH dielectrics

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Polar Dielectrics

Some molecules have an **intrinsic dipole moment** $\mathbf{p} = e\mathbf{a}$, where $|\mathbf{a}| \approx 10^{-10}$ is the molecular size. These polar dielectrics have large dielectric constants.

An example is water with $\epsilon_r = 81$ at room temperature

In the absence of an external field \mathbf{E} the molecular dipoles are randomly oriented and $\mathbf{P} = 0$

In the presence of an external \mathbf{E} the molecular dipoles align with the field to generate a large \mathbf{P}

Susceptibility of Polar Dielectrics

The alignment of the dipoles with the external field is disrupted by thermal motion:

$$N(\theta)d\theta \propto e^{-U/kT} \sin \theta d\theta \quad U = -\mathbf{p} \cdot \mathbf{E} = -pE \cos \theta$$

Expanding the exponent under the assumption that $U \ll kT$ and integrating over the orientation angle θ :

$$\mathbf{P} = \frac{N_A |\mathbf{p}|^2}{3kT} \mathbf{E}$$

The susceptibility χ_E decreases with increasing temperature

$$\chi_E = N_A \left(\alpha + \frac{|\mathbf{p}|^2}{3\epsilon_0 kT} \right)$$

where the first term is the usual LIH dielectric result, and the second term applies to polar dielectrics.

Energy Storage in Dielectrics

The capacitance of a capacitor increases if the gap is filled with a dielectric material:

$$C = \epsilon_r C_0$$

Hence the energy stored in a capacitor increases:

$$U = \frac{1}{2}CV^2 = \epsilon_r U_0$$

The energy density of the electric field also increases:

$$\frac{dU_E}{d\tau} = \frac{1}{2}\epsilon_r\epsilon_0 E^2 = \frac{1}{2}\mathbf{D}\cdot\mathbf{E}$$

*These are very large effects for polar dielectrics...
a lot of electric energy can be stored in a water bath!*

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