Lecture 11 - Weak Interactions

- Weak Charged Currents and β Decay
- Weak Neutral Currents
- Lepton-Neutrino Scattering
- Muon and Tau Decays
- Lepton Universality
- Lepton Flavour Conservation

Weak Charged Currents

The exchange of a heavy W^+ or W^- boson describes weak interactions with **charged currents**

The fermion currents are either:

- A charged lepton changing into a neutrino (or vice-versa)
  These must be of the same flavour!
- An up-type quark changing into a down-type quark

Weak interaction strength is related to the **Fermi constant** G_F:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \qquad G_F = 1.16 \times 10^{-5} \mathrm{GeV}^{-2}$$

where g is a dimensionless coupling constant

Description of β Decay

Quark level:

$$u \to d e^+ \nu_e \qquad d \to u e^- \bar{\nu}_e$$

Hadron level ($m_p = 938\text{MeV}$, $m_n = 940\text{MeV}$):

$$n \to p e^- \bar{\nu}_e \qquad \text{is} \qquad \textbf{allowed}$$

Free neutron is unstable and decays with $\tau_n = 886s$

$$p \to n e^+ \nu_e \qquad \text{is} \qquad \textbf{forbidden}$$

Free proton is stable; decay forbidden by $m_p < m_n$

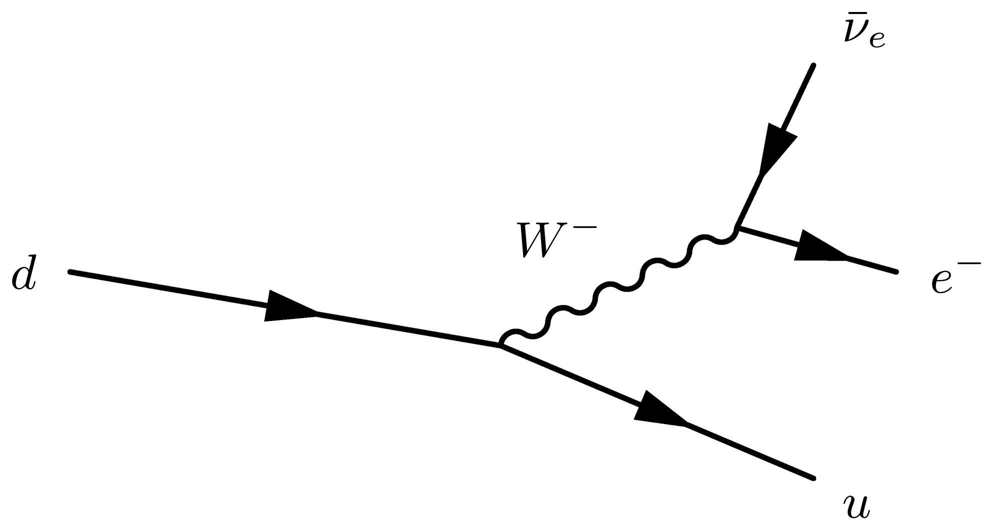
Nuclear level:

$$^{14}O \to ^{14}N^* + e^+ + \nu_e \qquad \beta^+ \quad \text{decay}$$

$$^{60}Co \to ^{60}Ni^* + e^- + \bar{\nu}_e \qquad \beta^- \quad \text{decay}$$

Which type is allowed depends on energy available between nuclei

Feynman diagram for β Decay



- Propagator for W boson is $-ig_{\mu\nu}/(M_W^2 - q^2)$
- Dimensionless coupling constant is g for leptons
  modified by CKM factors for quarks (V_{ud})
- The charged current operator is $\gamma^\mu \frac{1}{2}(1 - \gamma^5)$ instead of γ^μ
  Vector minus Axial-vector (V-A)

Amplitude for β Decay

The matrix element "factorizes" into lepton and quark currents:

$$\mathcal{M} = \left(\frac{g}{\sqrt{2}} \bar{u}_d \gamma^\mu \frac{1}{2}(1-\gamma^5) u_u \right) \frac{1}{M_W^2 - q^2} \left(\frac{g}{\sqrt{2}} \bar{u}_{\nu_e} \gamma^\mu \frac{1}{2}(1-\gamma^5) u_e \right)$$

Neglecting lepton masses the lepton current gives:

$$|\bar{u}_{\nu_e} \gamma^\mu \frac{1}{2}(1-\gamma^5) u_e|^2 = 8 E_e E_\nu (1 + \cos\theta)$$

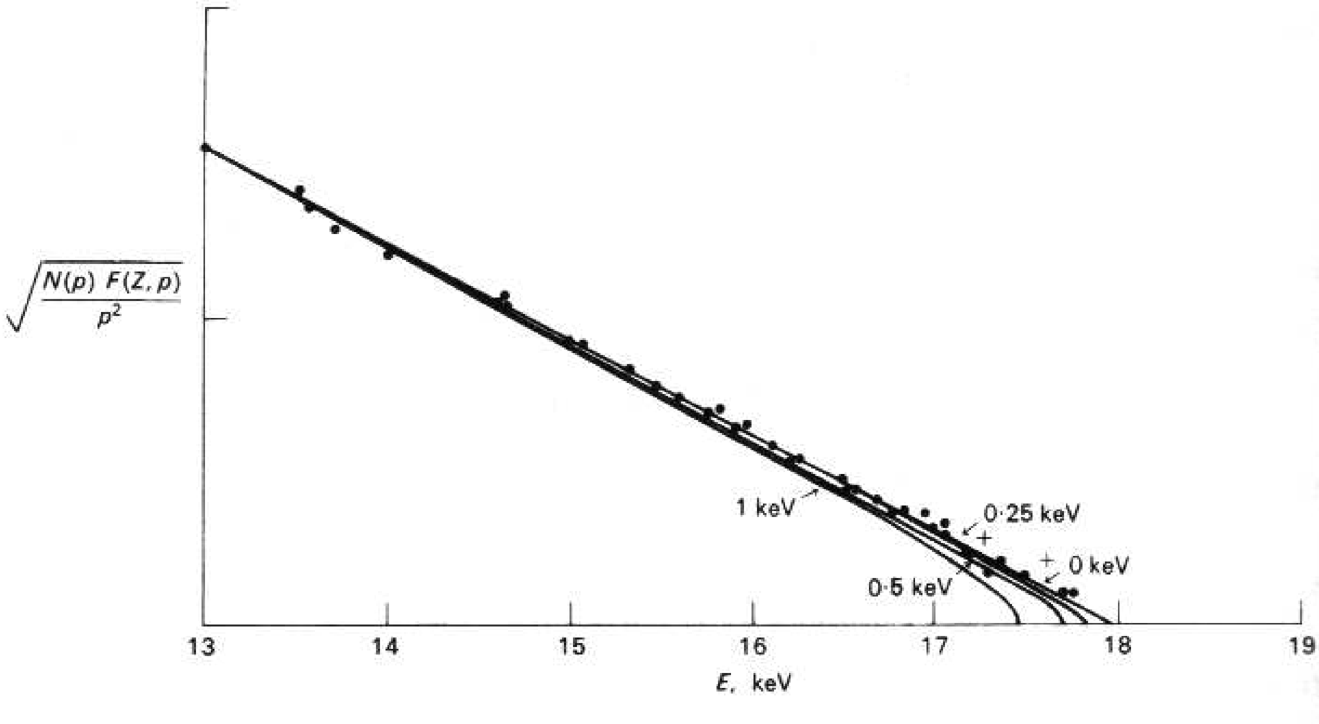
where θ is the opening angle between e and ν

The quark current squared $|\bar{u}_d \gamma^\mu \frac{1}{2}(1-\gamma^5) u_u|^2$ is parametrised by a "hadronic form factor", e.g. $F(n \to p)$, which is a function of q^2

In nuclear β decay use "matrix elements", e.g. $^{60}Co \to ^{60} Ni^*$

At low q^2 can replace g^2/M_W^2 with Fermi constant G_F

Tritium β decay (Kurie plot)



Spectrum: $\frac{d\Gamma}{dE_e} = \frac{G_F^2}{\pi^3} E_e^2 (E_0 - E_e)^2$

Total rate: $\Gamma = \frac{1}{\tau} = \frac{G_F^2 E_0^5}{30\pi^3}$

neglecting lepton mass and nuclear matrix element

Shape near endpoint E_0 determines $m(\nu_e) < 2\text{eV}$

Weak Neutral Currents

Described by the exchange of a heavy Z^0 boson

Z^0 couples to all types of fermion

but there are no flavour-changing neutral currents!

Neutral current amplitude for $\nu e \to \nu e$ scattering:

$$\mathcal{M} = \frac{G}{\sqrt{2}} \left(\bar{u}_\nu \gamma^\mu \frac{1}{2}(1 - \gamma^5) u_\nu \right) \left(\bar{u}_e \gamma^\mu (c_V^e - c_A^e \gamma^5) u_e \right)$$

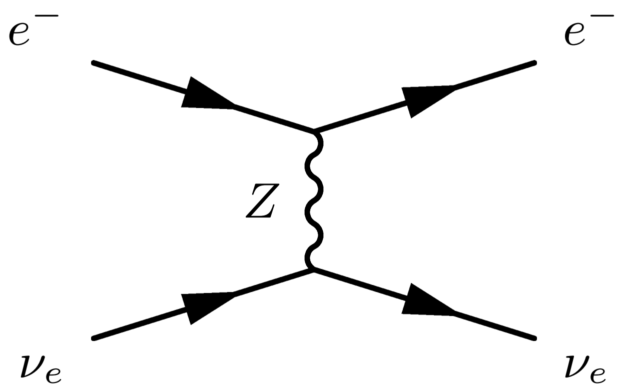
c_V and c_A are vector and axial-vector couplings

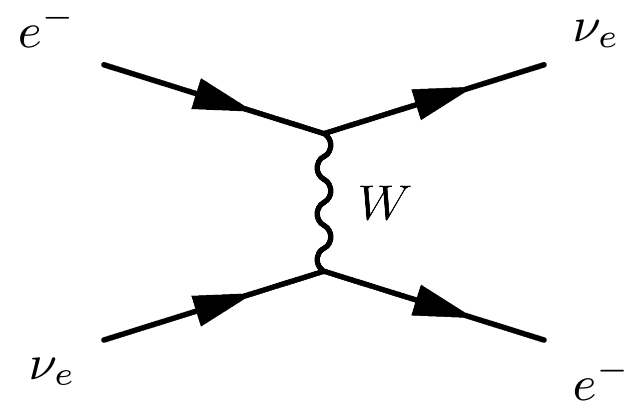
which depend on fermion type:

Lepton	$2c_V$	$2c_A$	Quark	$2c_V$	$2c_A$
ν_e, ν_μ, ν_τ	1	1	u, c, t	0.38	1
e, μ, τ	-0.06	-1	d, s, b	-0.68	-1

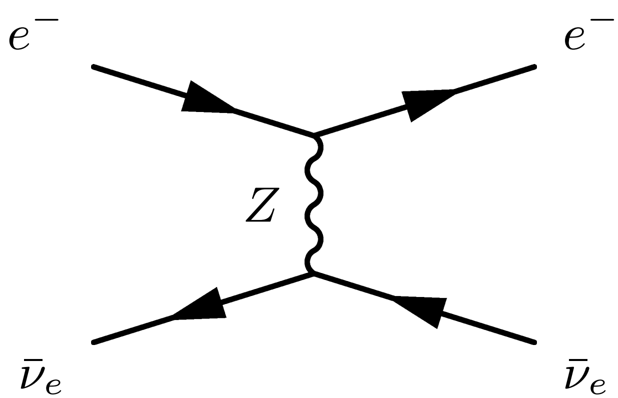
Electron Neutrino Scattering

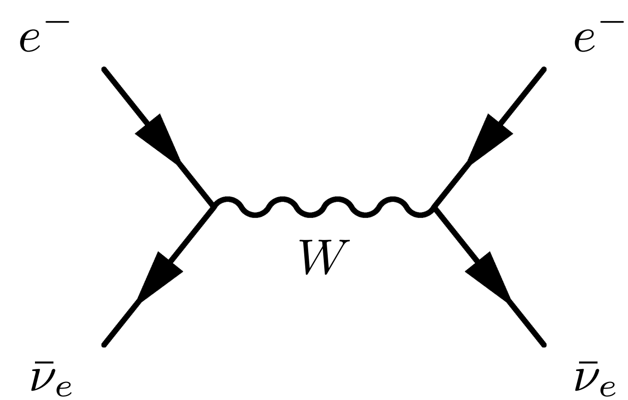
- Neutrino-Electron scattering $\nu_e e^- \to \nu_e e^-$:





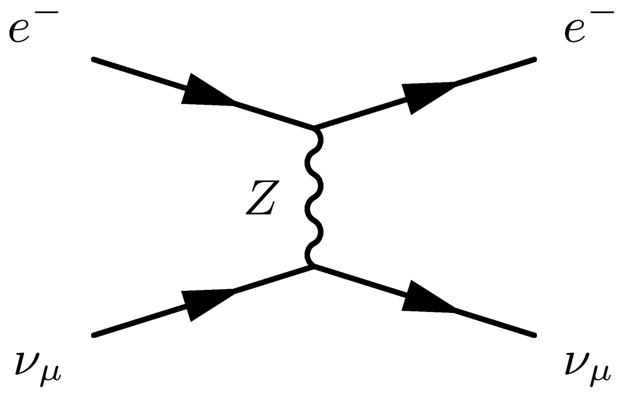
- Antineutrino-Electron scattering $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$:

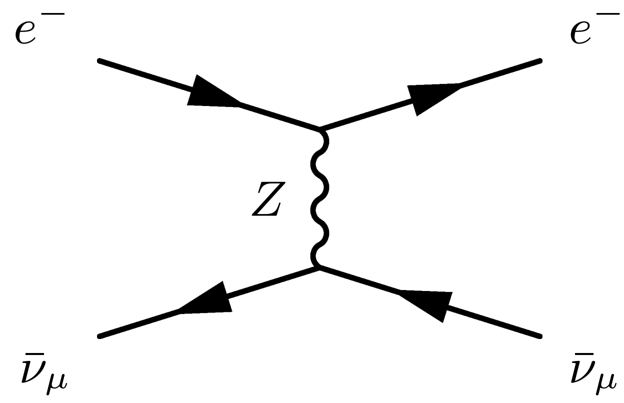




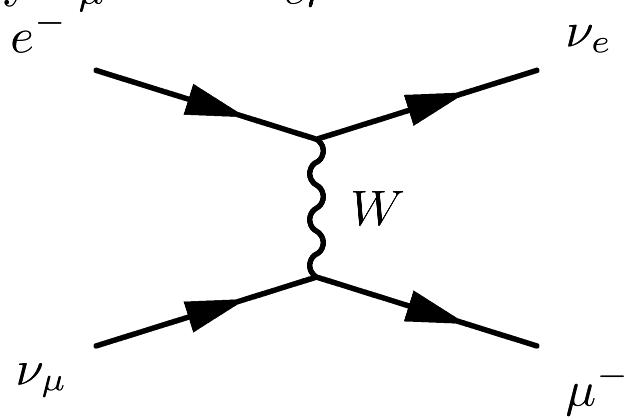
Muon Neutrino Scattering

- Muon neutrino-Electron scattering $\nu_\mu e^- \to \nu_\mu e^-$:





- Inverse muon decay $\nu_\mu e^- \to \nu_e \mu^-$:



ν_μ scattering cross-sections

- Charged current exchange diagram $\nu_\mu e^- \to \nu_e \mu^-$:

$$|\mathcal{M}|^2 = 16 G_F^2 s^2 \qquad \frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{4\pi^2} \qquad \sigma = \frac{G_F^2 s}{\pi}$$

- Neutral current scattering of muon neutrinos:

$$\frac{d\sigma}{dy} = \frac{G_F^2 s}{4\pi}\left[(c_V + c_A)^2 + (c_V - c_A)^2(1-y)^2\right]$$

$$\sigma(\nu_\mu e^- \to \nu_\mu e^-) = \frac{G_F^2 s}{3\pi}(c_V^2 + c_V c_A + c_A^2)$$

- Neutral current scattering of muon antineutrinos:

$$\sigma(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-) = \frac{G_F^2 s}{3\pi}(c_V^2 - c_V c_A + c_A^2)$$

ν_e scattering cross-sections

ν_e scattering is superposition of charged and neutral currents

- Scattering of electron neutrinos:

$$\frac{d\sigma}{dy}(\nu_e e^- \to \nu_e e^-) = \frac{G_F^2 s}{\pi}\left[g_L^2 + g_R^2(1-y)^2\right]$$

- Scattering of electron antineutrinos:

$$\frac{d\sigma}{dy}(\bar{\nu}_e e^- \to \bar{\nu}_e e^-) = \frac{G_F^2 s}{\pi}\left[g_R^2 + g_L^2(1-y)^2\right]$$

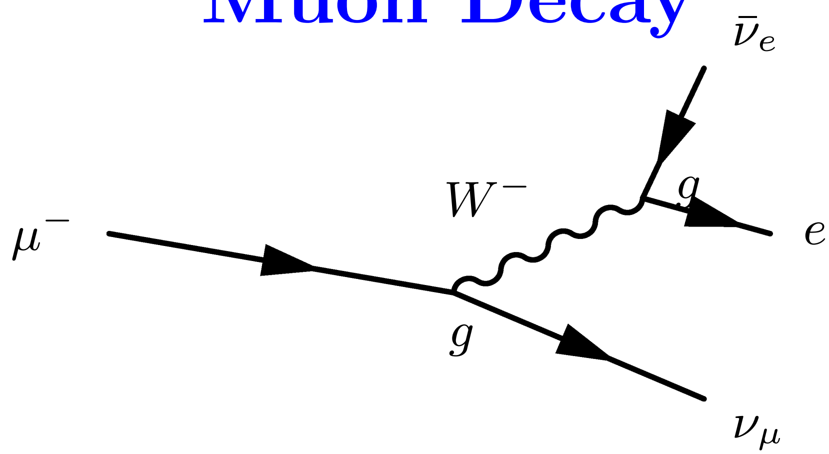
where $g_L = 1 + (c_V + c_A)/2$ and $g_R = (c_V - c_A)/2$

- Total cross-sections:

$$\sigma(\nu_e e^-) = \frac{G_F^2 s}{\pi}(g_L^2 + g_R^2/3) \qquad \sigma(\bar{\nu}_e e^-) = \frac{G_F^2 s}{\pi}(g_R^2 + g_L^2/3)$$

In the lab frame $\sigma(\nu_e e^-) \approx 10^{-41} E_\nu$ cm^2 where E_ν is in GeV

Muon Decay



Matrix element:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\bar{u}_{\nu_\mu} \gamma^\mu \frac{1}{2}(1-\gamma^5) u_\mu \right) \left(\bar{u}_e \gamma^\mu \frac{1}{2}(1-\gamma^5) u_{\nu_e} \right)$$

Michel spectrum:

$$\frac{d\Gamma}{dE_e} = \frac{G_F^2}{12\pi^3} m_\mu^2 E_e^2 \left(3 - \frac{4E_e}{m_\mu} \right)$$

Total decay rate:

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \qquad \tau_\mu = \frac{1}{\Gamma_\mu} = 2.197\mu s$$

Tau Decays

The τ lepton is heavy enough to decay to many final states ($m_\tau = 1.777$ GeV)

$$\tau^- \to e^- \nu_\tau \bar{\nu}_e \qquad \tau^- \to \mu^- \nu_\tau \bar{\nu}_\mu$$

$$\tau^- \to d\bar{u}\nu_\tau \qquad \tau^- \to s\bar{u}\nu_\tau$$

Naive branching fractions (with factor $\times 3$ for quark color):

$$\mathcal{B}(\tau \to e) = \mathcal{B}(\tau \to \mu) = 20\% \qquad \mathcal{B}(\tau \to \text{hadrons}) = 60\%$$

The decay to an electron can be related to muon decay:

$$\frac{\Gamma_{\tau\to e}}{\Gamma_\mu} = \left(\frac{m_\tau}{m_\mu}\right)^5$$

This agrees with the measured τ lifetime $\tau_\tau = 291 fs$

Lepton Universality

Couplings of W and Z bosons are the same for all families of leptons

This can be tested in τ decays:

$$\mathcal{B}(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = (17.84 \pm 0.06)\%$$

$$\mathcal{B}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = (17.36 \pm 0.06)\%$$

small difference is due to the muon mass

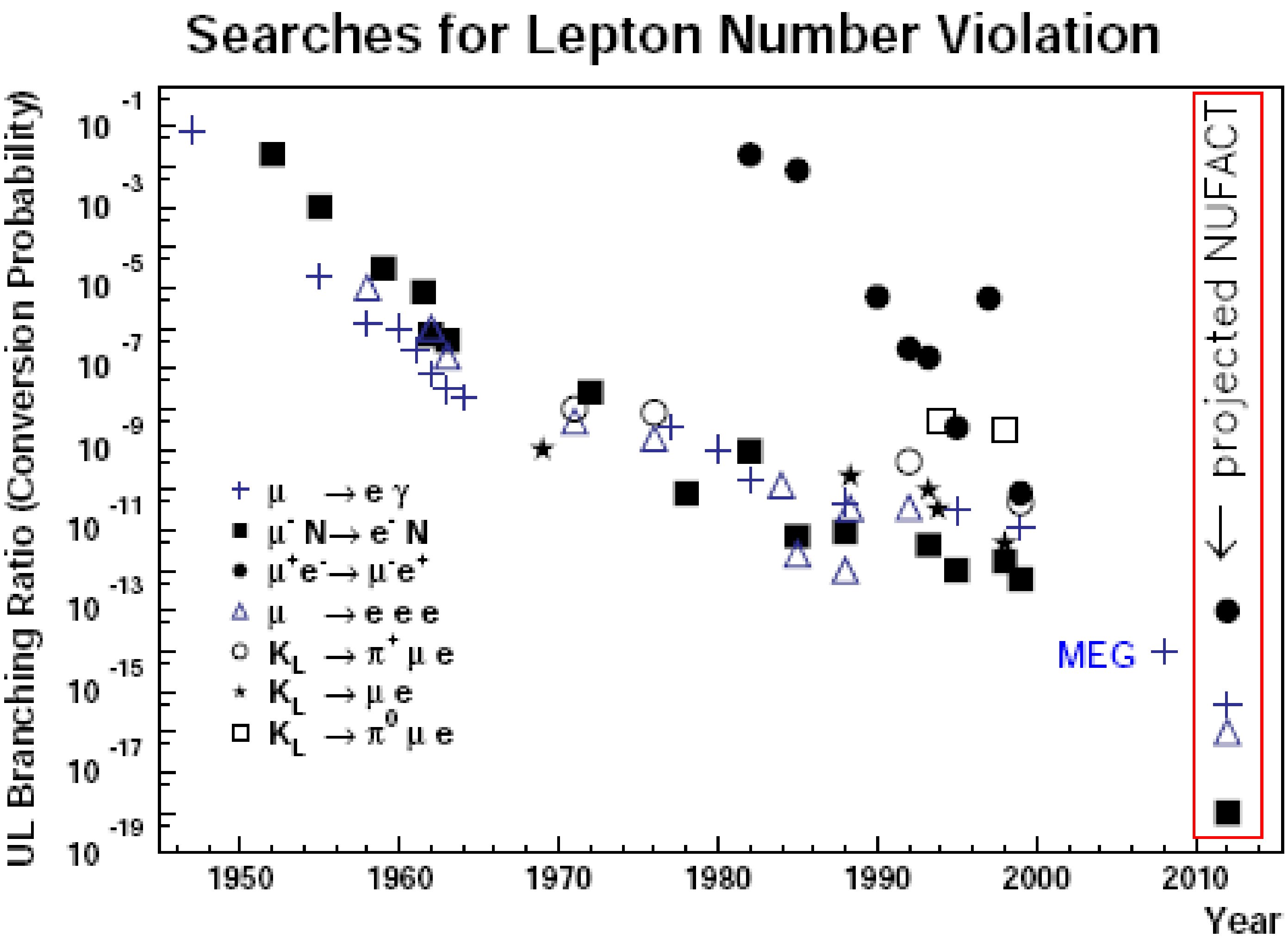
Other tests include:

Michel spectrum shape in μ and τ decays

$W \to \ell\nu$ and $Z \to \ell^+\ell^-$ at high energy colliders

Heavy quark decays $b \to c\ell\nu$ and $c \to s\ell\nu$

50 years of $\mu \to e$ flavour violation searches



25 years of τ flavour violation searches