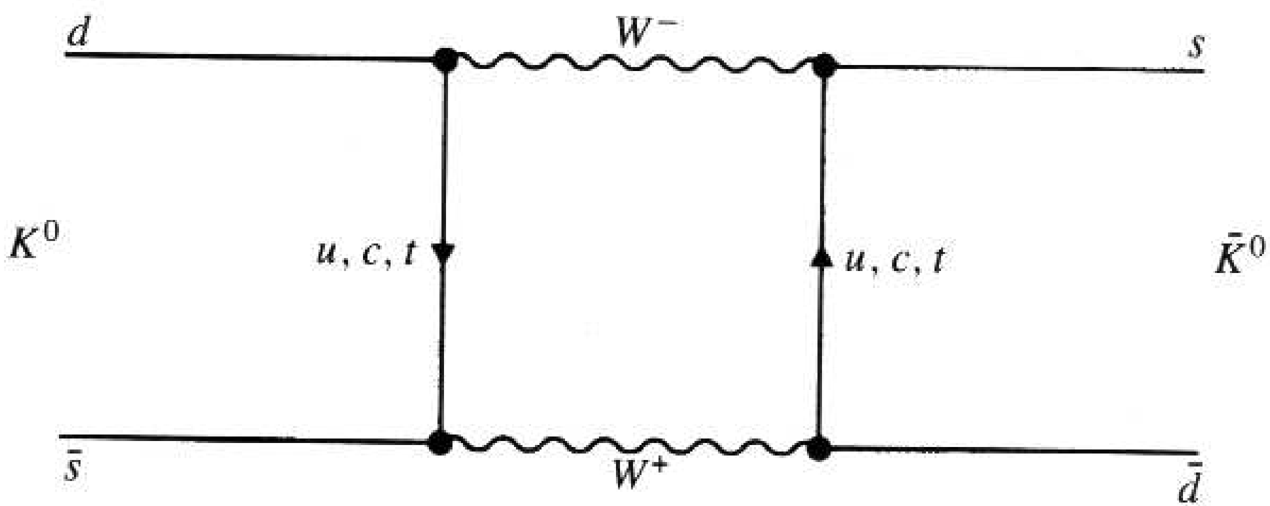
Lecture 14
Mixing and CP Violation

- Mixing of neutral K^0 mesons
- CP violation in K^0 decays
- T violation and CPT conservation
- Observation of charm mixing
- B_d and B_s mixing
- CP violation in B decays

Mixing of Neutral Mesons

A second order weak interaction transforms an initial K^0, D^0 or B^0 into a final $\bar{K}^0$,$\bar{D}^0$ or $\bar{B}^0$:



K^0 and B^0 "box" diagrams contain two W bosons and two u-type quarks

D^0 box diagram contains two W bosons and two d-type quarks

General Description of Mixing

A state that is initially K^0 or $\bar{K}^0$ evolves as a function of time:

$$\psi(t) = a(t)|K^0> + b(t)|\bar{K}^0> \qquad\qquad i\frac{d\psi}{dt} = \mathbf{H}\psi(t)$$

$\mathbf{H}$ is an effective Hamiltonian describing time-dependent mixing:

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma}$$

where $\mathbf{M}$ and Γ are 2×2 mass and decay matrices

Diagonal elements of $\mathbf{H}$ are flavour-conserving, $\Delta S = 0$

Off-diagonal elements of $\mathbf{H}$ are flavour-changing, $\Delta S = 2$
They describe the mixing transitions $K^0 \leftrightarrow \bar{K}^0$

If $\mathbf{H}$ *is diagonal there is no mixing, and the flavour states of neutral mesons are the same as their decay eigenstates*

The Decay Eigenstates K_S and K_L

The matrix **H** has eigenvectors corresponding to the weak decay eigenstates K_L and K_S

$$|K_S >= p|K^0 > +q|\bar{K}^0 > \qquad |K_L >= p|K^0 > -q|\bar{K}^0 >$$

$$\frac{q}{p} = \frac{2\mathbf{M}_{12}^* - \frac{i}{2}\mathbf{\Gamma}_{12}^*}{\Delta m_K - \frac{i}{2}\Delta\Gamma_K} \qquad |q|^2 + |p|^2 = 1$$

The flavour eigenstates have equal mass (CPT theorem):

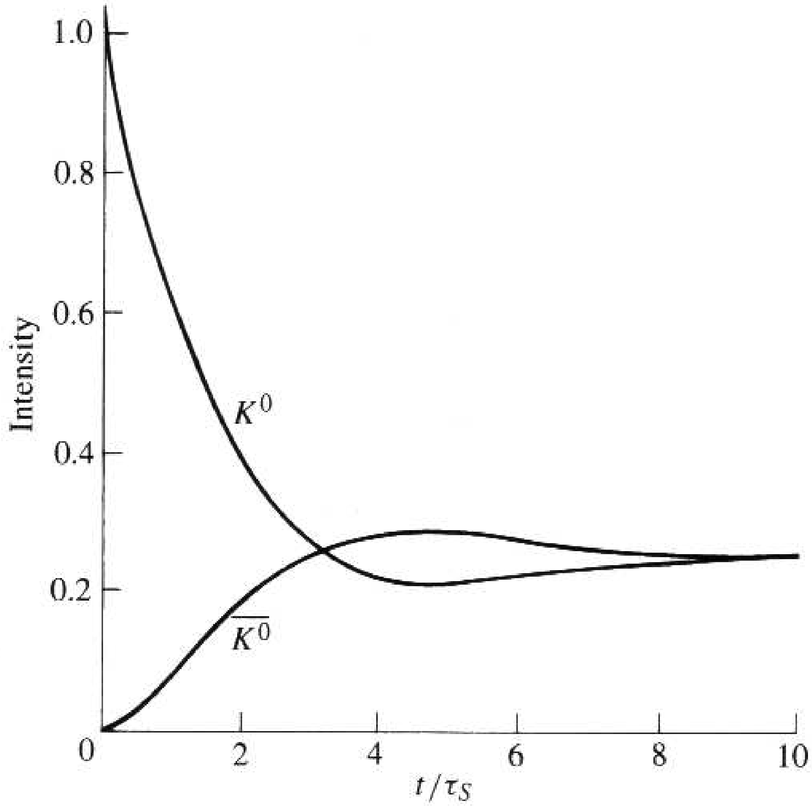
$$M(K^0) = M(\bar{K}^0) = 498\text{MeV}$$

The weak eigenstates have different masses and lifetimes:

$$\Delta m_K = m_L - m_S = (3.52 \pm 0.01) \times 10^{-12}\text{MeV} = 0.53 \times 10^{10} s^{-1}$$

$$\tau_L = 51ns \qquad \tau_S = 0.09ns \qquad \Delta\Gamma_K = 1.1 \times 10^{10} s^{-1}$$

Time evolution of K^0 states



$$A_S(t) = A_S(0)e^{-(\Gamma_S/2+im_S)t}$$

$$A_L(t) = A_L(0)e^{-(\Gamma_L/2+im_L)t}$$

$$\tau_1 = \frac{0.5}{\Delta m_K} \approx \tau_S$$

$t \gg \tau_S$ is pure K_L

$$|\psi_{K^0}(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{(\Gamma_L+\Gamma_S)}{2}t}\cos\Delta m_K t\right)$$

$$|\psi_{\bar{K}^0}(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} - 2e^{-\frac{(\Gamma_L+\Gamma_S)}{2}t}\cos\Delta m_K t\right)$$

CP Eigenstates K_1 and K_2

The combined operation of Charge Conjugation and Parity:

$$CP|K^0>=|\bar{K}^0> \qquad CP|\bar{K}^0>=|K^0>$$

CP eigenstates are:

$$K_1 = \frac{1}{\sqrt{2}}[K^0 + \bar{K}^0] \qquad CP = +1$$

$$K_2 = \frac{1}{\sqrt{2}}[K^0 - \bar{K}^0] \qquad CP = -1$$

If CP is conserved $K_1 \rightarrow 2\pi$ and $K_2 \rightarrow 3\pi$

$$CP|\pi^+\pi^- >= CP|\pi^0\pi^0 >= +1$$

$$CP|\pi^+\pi^-\pi^0 >= CP|\pi^0\pi^0\pi^0 >= -1$$

$\tau_S < \tau_L$ explained by more phase space for $K_1 \rightarrow 2\pi$ than $K_2 \rightarrow 3\pi$

Weak and CP Eigenstates

If $|p/q| = 1$ can identify $K_S = K_1$, $K_L = K_2$

$$|K_S >= \frac{(p+q)}{\sqrt{2}}|K_1 > + \frac{(p-q)}{\sqrt{2}}|K_2 >$$

$$|K_L >= \frac{(p-q)}{\sqrt{2}}|K_1 > + \frac{(p+q)}{\sqrt{2}}|K_2 >$$

Writing $p = 1 + \epsilon$, $q = 1 - \epsilon$, where ϵ is in general complex:

$$K_L = \frac{1}{\sqrt{1+|\epsilon|^2}}[\epsilon K_1 + K_2] \qquad K_S = \frac{1}{\sqrt{1+|\epsilon|^2}}[K_1 + \epsilon K_2]$$

If the weak states are not identical to the CP eigenstates there should be some decays $K_S \to 3\pi$ and $K_L \to 2\pi$

$\epsilon \neq 0$ measures CP violation in K^0 decays

Observation of CP violation

In 1964 $K_L \to 2\pi$ decays were observed:

$$\eta_{+-} = \frac{K_L \to \pi^+\pi^-}{K_S \to \pi^+\pi^-} = \epsilon + \epsilon'$$

$$\eta_{00} = \frac{K_L \to \pi^0\pi^0}{K_S \to \pi^0\pi^0} = \epsilon - 2\epsilon'$$

ϵ represents CP violation in the mixing amplitude

ϵ' represents *direct* CP violation between $\Delta I = 1/2$ and $\Delta I = 3/2$

After 40 years the magnitudes and phases are now measured:

$$|\epsilon| = (2.232 \pm 0.007) \times 10^{-3} \qquad \phi_\epsilon = (43.5 \pm 0.05)^\circ$$

$$\left|\frac{\epsilon'}{\epsilon}\right| = (1.66 \pm 0.26) \times 10^{-3} \qquad \phi_{\epsilon'} = (42.3 \pm 1.5)^\circ$$

T violation in Semileptonic Decays

$K^0 \to \pi^- \ell^+ \nu$ and $\bar{K}^0 \to \pi^+ \ell^- \nu$ are *flavour-specific* decays which obey the $\Delta Q = \Delta S$ rule

The semileptonic charge asymmetry in K_L decays measures ϵ:

$$\frac{\Gamma(K_L \to \pi^- \ell^+ \nu) - \Gamma(K_L \to \pi^+ \ell^- \nu)}{\Gamma(K_L \to \pi^- \ell^+ \nu) + \Gamma(K_L \to \pi^+ \ell^- \nu)} = 2\mathrm{Re}(\epsilon) = (3.27 \pm 0.12) \times 10^{-3}$$

Can test T violation in mixing using initial K^0 and $\bar{K}^0$ beams:

$$\Gamma(K^0 \to \bar{K}^0 \to \pi^+ \ell^- \nu) \neq \Gamma(\bar{K}^0 \to K^0 \to \pi^- \ell^+ \nu)$$

Agrees with expectation if T,CP are violated but CPT is conserved

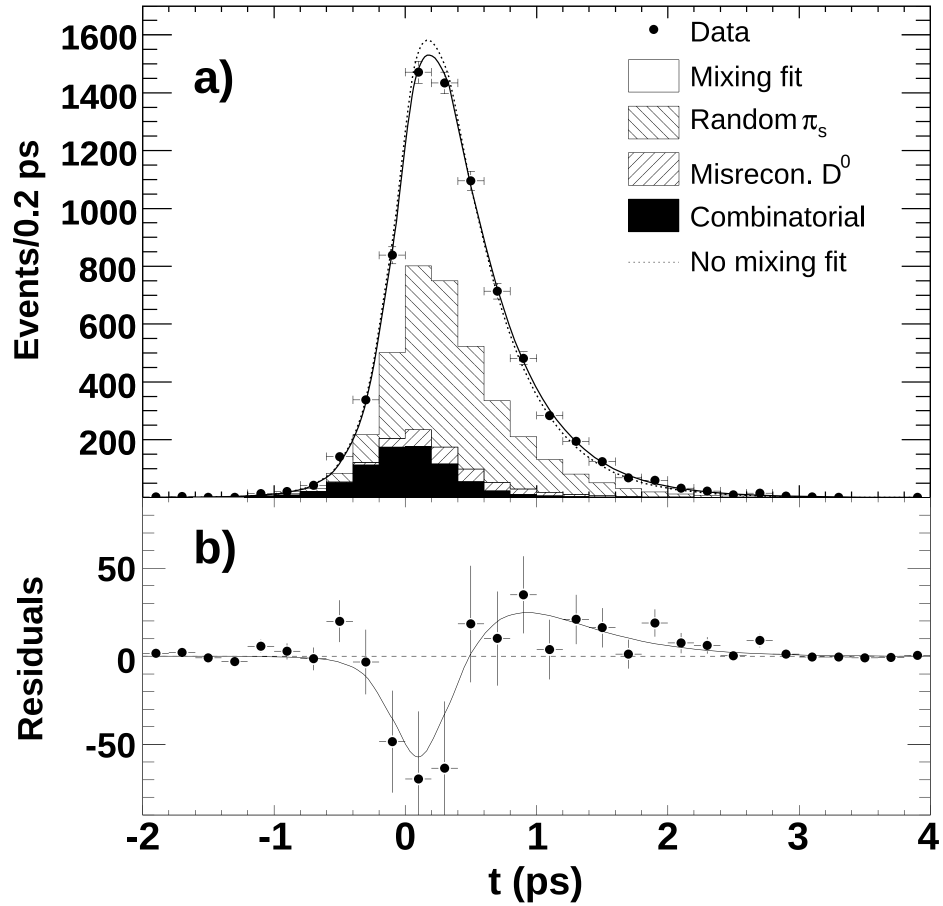
A test of CPT conservation is the comparison of:

$$\Gamma(K^0 \to \pi^- \ell^+ \nu) = \Gamma(\bar{K}^0 \to \pi^+ \ell^- \nu)$$

CPT violation parameter $Re(\delta) = (2.9 \pm 2.7) \times 10^{-4}$

Observation of D^0 Mixing (2007)

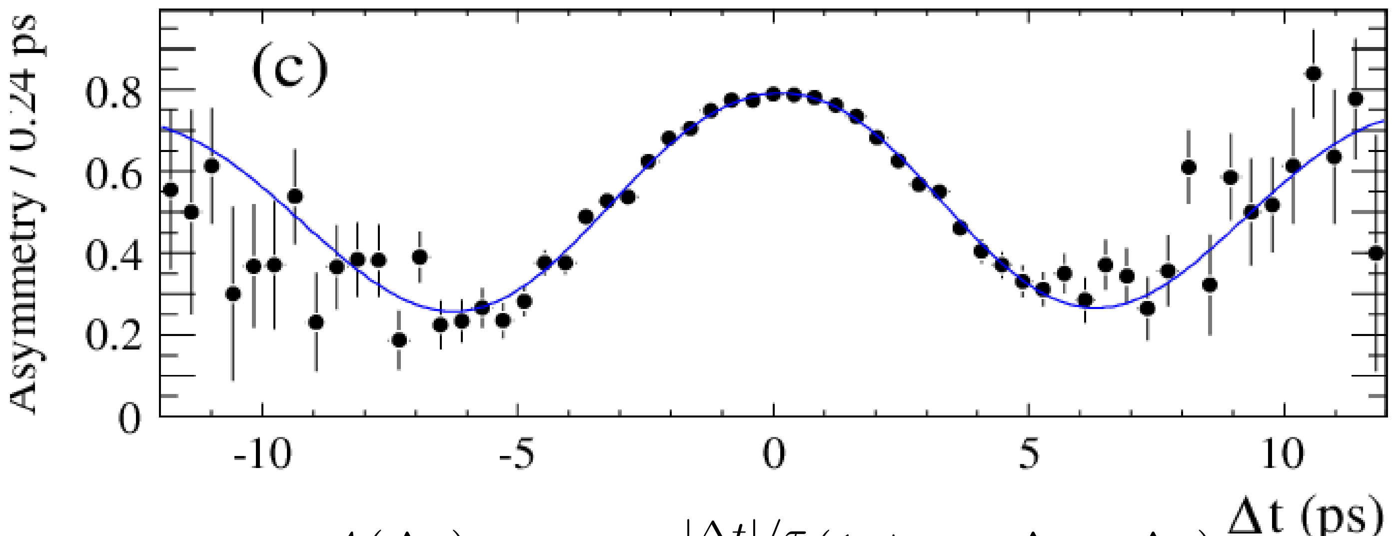
BaBar measurement of decay time distribution of "wrong-sign" $D^0 \to K^+\pi^-$ decays



Bottom plot shows difference from no-mixing

Mixing of B_d mesons

Measured by B factories (BaBar & Belle) using
coherent production $e^+e^- \to \Upsilon(4S) \to B^0\bar{B}^0$
Δt is the *difference* between the two B^0 decay times



$$A(\Delta t)_{f/\bar{f}} \propto e^{-|\Delta t|/\tau}(1 \pm \cos \Delta m_d \Delta t)$$

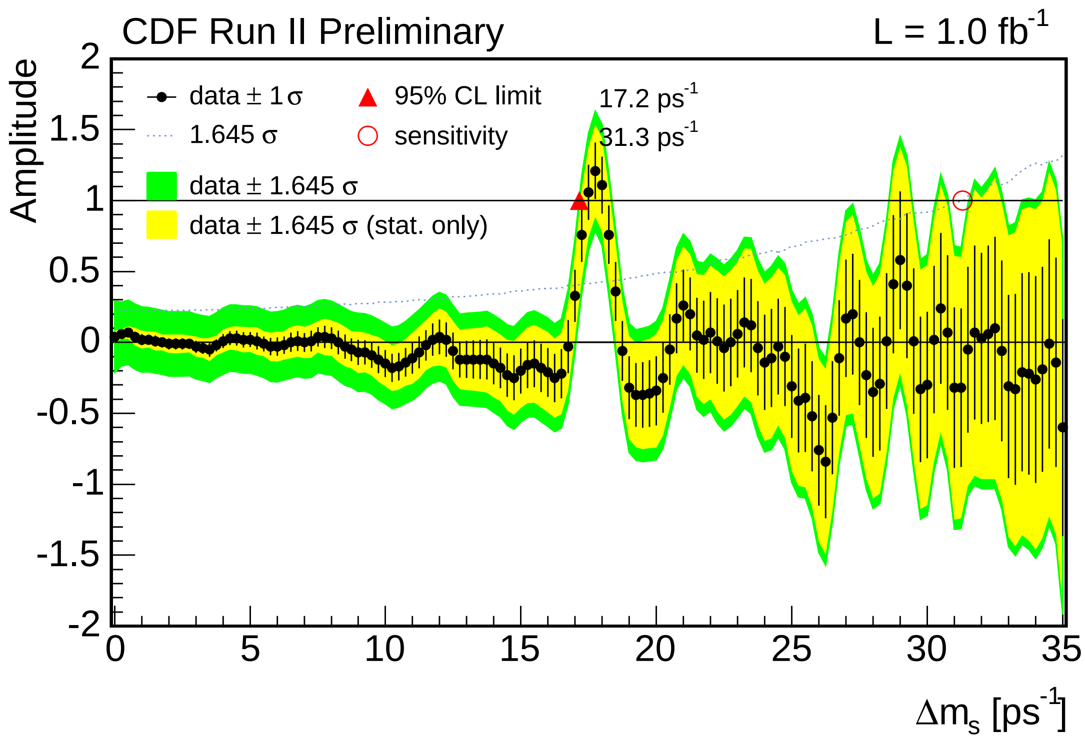
$$\Delta m_d = (0.508 \pm 0.004) ps^{-1} \qquad \tau_{B_d} = (1.53 \pm 0.01) ps$$

$$M_{12} \propto (V_{tb}V_{td}^*)^2 \qquad \frac{q}{p} = \frac{V_{tb}^* V_{td}}{V_{tb}V_{td}^*}$$

Mixing of B_s mesons

Measured at Fermilab in 2006 using $p\bar{p}$ collisions

Plot shows scan of predicted amplitude vs oscillation frequency



$$\Delta m_s = (17.8 \pm 0.1) ps^{-1} \qquad \tau_{B_s} = (1.47 \pm 0.06) ps$$

From the ratio of the two B mixing results: $|\frac{V_{td}}{V_{ts}}| = 0.211 \pm 0.007$

CP Violation in B Decays

There are three types of CP violation that can be observed:

- CP violation in **mixing**, due to the weak eigenstates being different from the CP eigenstates, $|q/p| \neq 1$.

$$A_{SL}(b \to clv) = -0.0012 \pm 0.0010$$

- **Direct** CP violation in decay amplitudes $A(B \to f)$ and $\bar{A}(\bar{B} \to \bar{f})$, due to $|A/\bar{A}| \neq 1$. This does not require mixing, and is seen in both charged and neutral B decays.

$$A_{CP}(B^0 \to K^{\pm}\pi^{\mp}) = -0.093 \pm 0.015$$

- CP violation in the **interference** between mixing and decay amplitudes $A(B^0 \to f)$ and $\bar{A}(\bar{B}^0 \to f)$.
  This requires $\mathrm{Im}\lambda \neq 0$, where $\lambda = q\bar{A}/pA$.

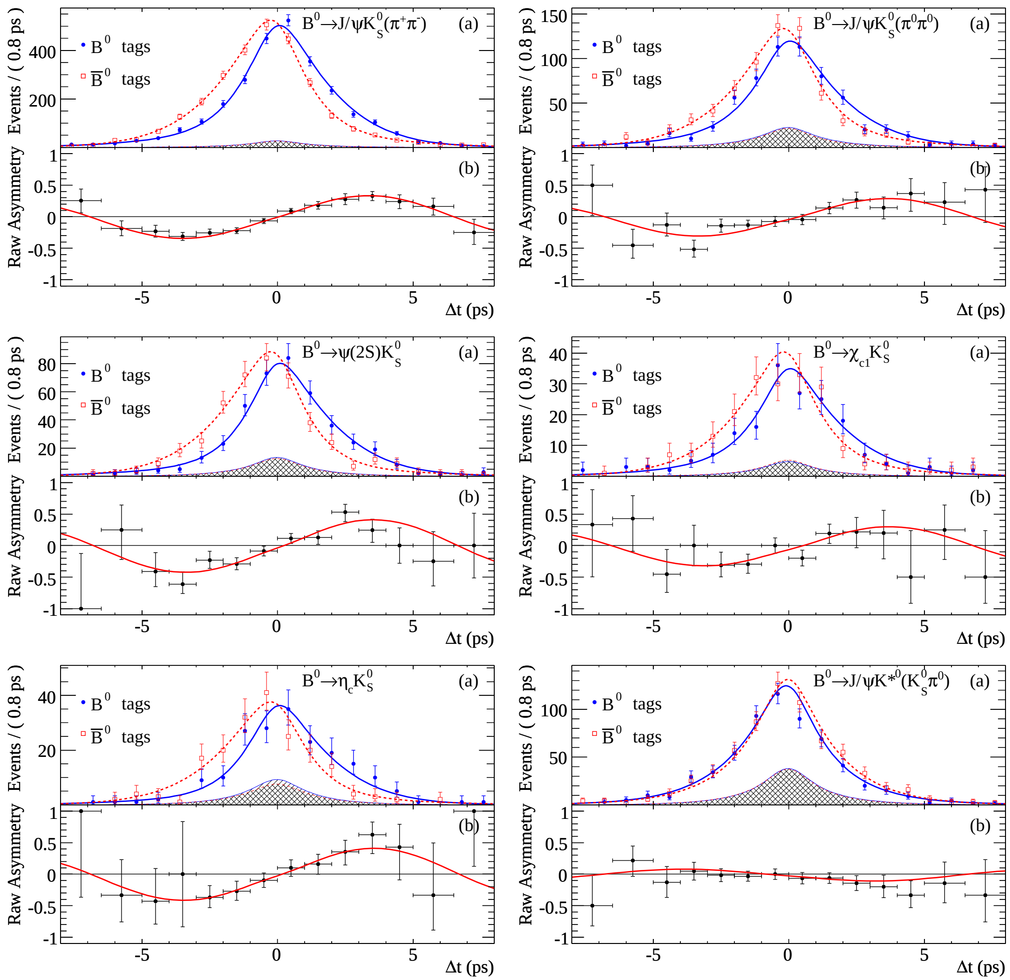
CP Violation in $b \to c\bar{c}s$ Decays

Top left:
$B \to J/\psi K_S$

$|\lambda| = 0.982 \pm 0.025$

$Im(\lambda) = \sin 2\beta$

$\beta = 23^\circ$ is
phase of V_{td}



$$A_{CP}(B \to J/\psi K_S) = (1 - |\lambda|^2)\cos \Delta m_d \Delta t - 2Im(\lambda)\sin \Delta m_d \Delta t$$

The CKM parameters ρ and η

Unitarity triangle: $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$

Normalised sides are $|V_{ub}/V_{cb}|$, $|V_{td}/V_{ts}|$ and 1

Angles are α, β (phase of V_{td}) and γ (phase of V_{ub})

CP violation requires non-zero complex phase η