Lecture 16 - Electroweak Theory

- Electroweak Unification
- W and Z Boson masses and widths
- Z production at LEP
- Forward-backward asymmetries
- W production at colliders
- Precision tests of Electroweak theory

SU(2) Isospin and U(1) Hypercharge

The combined groups $SU(2)_L \times U(1)$ describe electroweak symmetry

$SU(2)_L$ describes **weak isospin** (I_3) couplings to left-handed fermions (or right-handed antifermions)

U(1) describes **weak hypercharge** (Y) couplings

The weak charged currents are described by SU(2) matrices $\tau^{\pm}$:

$$\tau^+ = \frac{1}{2}(\tau_1 + i\tau_2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad \tau^- = \frac{1}{2}(\tau_1 - i\tau_2) = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$J^+ = \bar{\chi}_{\nu_e} \gamma^\mu \tau^+ \chi_e \qquad J^- = \bar{\chi}_e \gamma^\mu \tau^- \chi_{\nu_e}$$

These are the currents associated with W^+ and W^- exchange

Electroweak Neutral Currents

The third component of weak isospin describes a weak neutral current:

$$J^3 = \bar{\chi}\gamma^\mu \frac{1}{2}\tau_3\chi = \bar{\chi}_{\nu_e}\gamma^\mu\chi_{\nu_e} - \bar{\chi}_e\gamma^\mu\chi_e \qquad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The electromagnetic current couples to charge Q:

$$J^{EM} = \bar{\chi}\gamma^\mu \mathrm{Q}\chi$$

This can be rewritten in terms of I_3 and Y:

$$\mathrm{Q} = \mathrm{I}_3 + \frac{\mathrm{Y}}{2} \qquad J^{EM} = J^3 + \frac{1}{2}J^Y$$

The EM current is a sum of weak isospin and hypercharge

Electroweak Unification

The combined electroweak interaction has a Lagrangian:

$$\mathcal{L} = g\vec{J}.\vec{W} + g'J^Y B$$

$\vec{W}$ is a triplet of vector bosons $W^{1,2,3}$ coupling to weak isospin currents $J^{1,2,3}$ with strength g

B^0 is a singlet vector boson coupling to the weak hypercharge current J^Y with strength g'

The physical vector bosons are:

$$W^{\pm} = \sqrt{\frac{1}{2}}(W^1 \mp iW^2)$$

$$Z^0 = W^3 \cos\theta_W - B^0 \sin\theta_W \qquad A^0 = W^3 \sin\theta_W + B^0 \cos\theta_W$$

where the neutral W^3 and B^0 mix to give Z^0 and photon A^0

The Weinberg Angle

The mixing angle θ_W is measured to be:

$$\sin^2\theta_W = 0.2221$$

The weak isospin, hypercharge and EM couplings are related by:

$$g\sin\theta_W = g'\cos\theta_W = e$$

The electroweak Lagrangian can be written out in terms of the physical weak and electromagnetic currents:

$$\frac{g}{\sqrt{2}}(J^- W^+ + J^+ W^-) + \frac{g}{\cos\theta_W}(J^3 - \sin^2\theta_W J^{EM})Z^0 + eJ^{EM}A^0$$

The first terms describe $W^\pm$ couplings with strength g

The last term describes photon couplings with strength e

The middle term describes the neutral Z^0 coupling.

It has pieces from both J^3 and J^{EM}!

Masses of W and Z Bosons

The mass of the W boson can be related to $\sin\theta_W$:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \qquad M_W = \sqrt{\frac{e^2\sqrt{2}}{8G_F\sin^2\theta_W}} = \frac{37.4}{\sin\theta_W}\text{GeV}$$

The mass of the Z boson is related to the mass of the W boson:

$$M_Z = \frac{M_W}{\cos\theta_W} = \frac{75}{\sin 2\theta_W}\text{GeV}$$

These are now very accurately measured:

$$M_W = 80.425 \pm 0.038 \text{ GeV} \qquad M_Z = 91.1876 \pm 0.0021 \text{ GeV}$$

The large masses of the W and Z compared to the zero mass of the photon **break** electroweak symmetry

Leads to introduction of Higgs mechanism - see next lecture

Neutral Current Couplings of Z

The Z boson has left-handed couplings from J^3 and J^{EM} and right-handed couplings from J^{EM}:

$$g_L = \mathrm{I}_3 - \mathrm{Q}\sin^2\theta_W \qquad g_R = -\mathrm{Q}\sin^2\theta_W$$

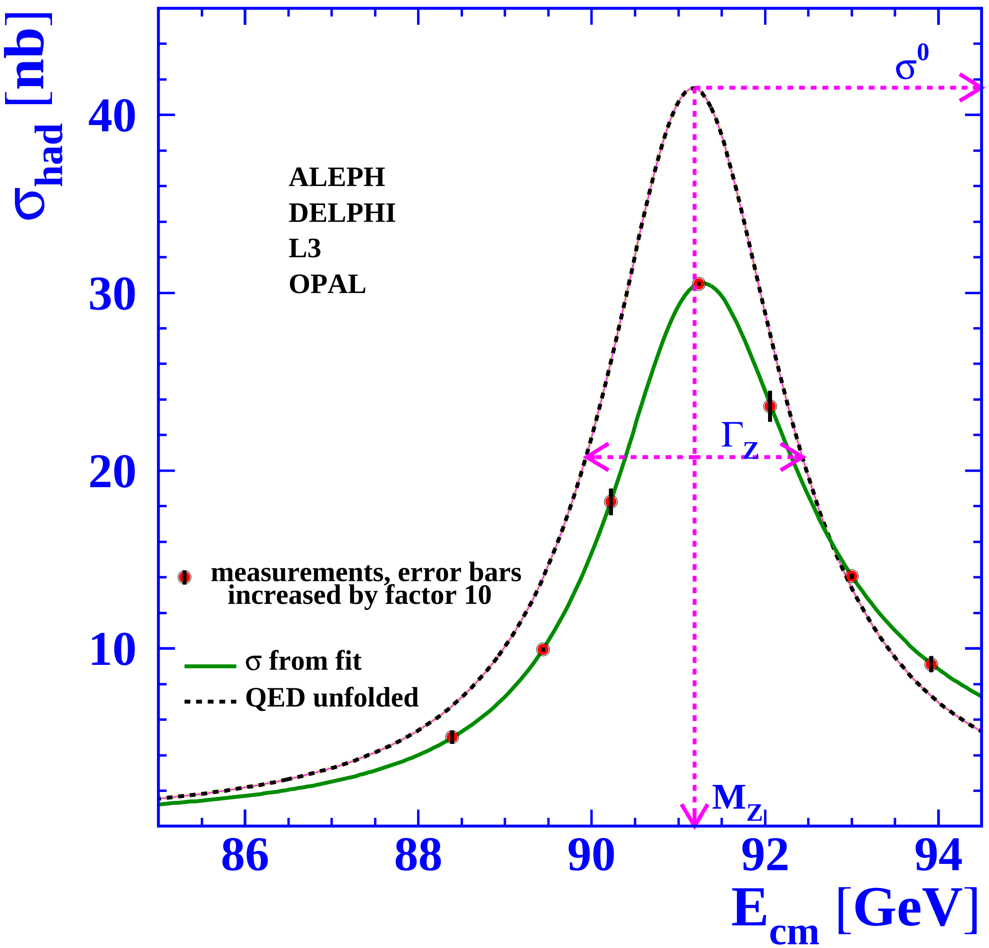
Usually expressed as vector and axial-vector couplings c_V and c_A:

$$c_V = g_L + g_R = \mathrm{I}_3 - 2\mathrm{Q}\sin^2\theta_W \qquad c_A = g_L - g_R = \mathrm{I}_3$$

Lepton	$2c_V$	$2c_A$	Quark	$2c_V$	$2c_A$
ν_e, ν_μ, ν_τ	1	1	u, c, t	$1 - \frac{8}{3}\sin^2\theta_W$	1
e, μ, τ	$-1 + 4\sin^2\theta_W$	-1	d, s, b	$-1 + \frac{4}{3}\sin^2\theta_W$	-1

Electroweak theory predicts the weak neutral current couplings which depend on fermion type!

Z Production at LEP (1989-1995)



Large samples of Z^0 were collected in e^+e^- collisions at $E_{CM} \approx M_Z$

Note interference between γ (QED) and Z^0 diagrams

Width of the Z^0 boson: $\Gamma_Z = 2.4952 \pm 0.0023$ GeV

Z Couplings to fermions

The couplings of the Z^0 to $f\bar{f}$ are:

$$\Gamma(Z^0 \to f\bar{f}) = \frac{g^2}{48\pi \cos^2\theta_W}(c_V^2 + c_A^2)_f M_Z$$

The couplings to charged leptons are:

$$\Gamma(Z^0 \to \ell\bar{\ell}) = \frac{g^2}{192\pi \cos^2\theta_W} M_Z = 84 \text{ MeV}$$

Sum of all quark couplings gives ratio of hadron jets to leptons:

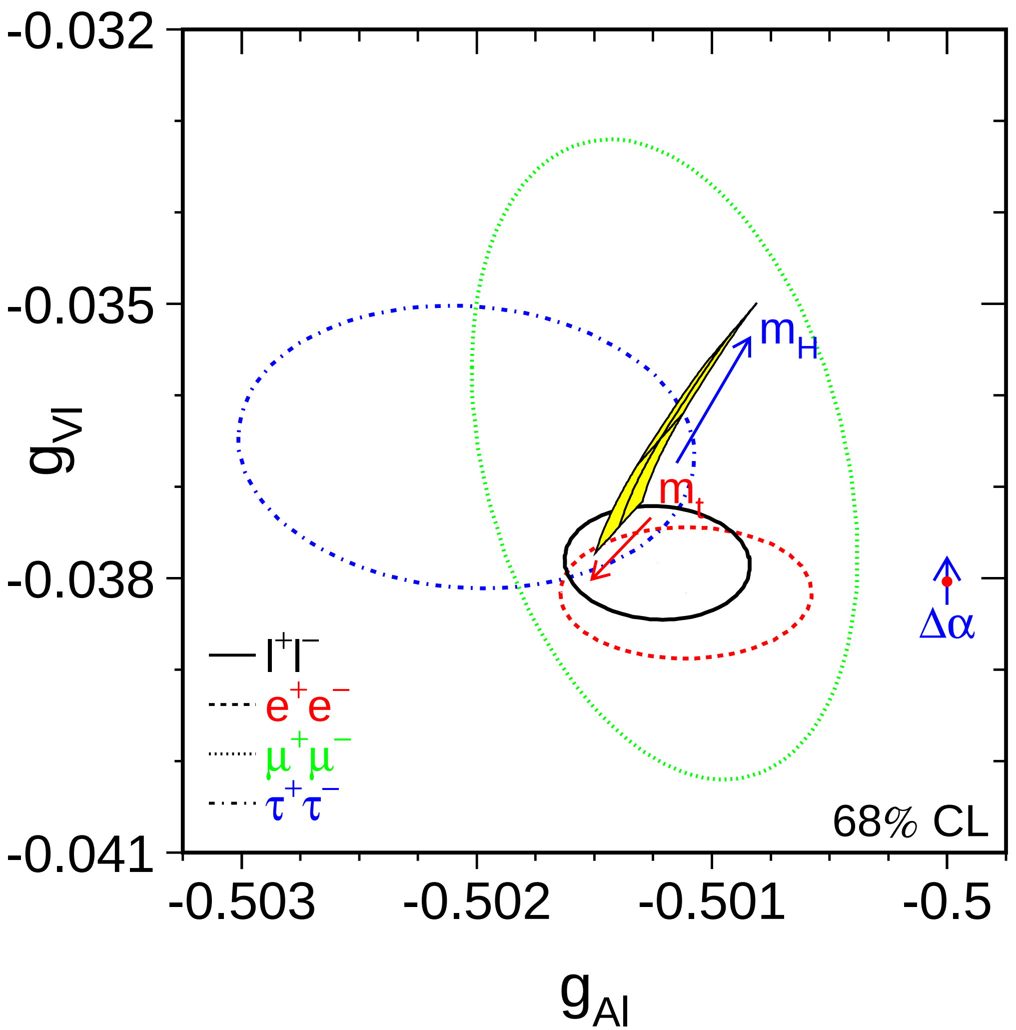
$$R = \frac{\Gamma(Z^0 \to \text{hadrons})}{\Gamma(Z^0 \to \text{leptons})} = 20.767 \pm 0.025$$

The couplings to neutrinos give the *invisible* width:

$$\Gamma(Z^0 \to \nu\bar{\nu}) = 166 \text{ MeV} \qquad \Gamma(Z^0 \to \mathit{invisible}) = 499 \pm 1.5 \text{ MeV}$$

The total width requires 2.988 ± 0.023 neutrinos with $m_\nu < M_Z/2$

Universality of Z couplings to leptons



e^+e^-, $\mu^+\mu^-$, $\tau^+\tau^-$ couplings are same

Note the sensitivity of these couplings to the Higgs mass!

Interference between photon and Z

$e^+e^- \to f\bar{f}$ is described by sum of Z^0 and photon diagrams:

$$\mathcal{M}_Z = \frac{\sqrt{2}G_F M_Z^2}{s - M_Z^2}[g_R^f \bar{f}_R \gamma^\mu f_R + g_L^f \bar{f}_L \gamma^\mu f_L][g_R^e \bar{e}_R \gamma^\mu e_R + g_L^e \bar{e}_L \gamma^\mu e_L]$$

$$\mathcal{M}_\gamma = \frac{e^2}{s}[\bar{f}\gamma^\mu f][\bar{e}\gamma^\mu e]$$

The cross-sections depend on fermion helicities:

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to \mu_L^- \mu_R^+) = \frac{\alpha^2}{4s}(1+\cos\theta)^2 |1 + r g_L^\mu g_L^e|^2$$

$$\frac{d\sigma}{d\Omega}(e_L^- e_R^+ \to \mu_R^- \mu_L^+) = \frac{\alpha^2}{4s}(1-\cos\theta)^2 |1 + r g_R^\mu g_L^e|^2$$

where r is ratio of coefficients of Z^0 and photon amplitudes

Forward-Backward Asymmetries

Combining all four possible helicity combinations:

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-) = \frac{\alpha^2}{4s}[A_0(1+\cos^2\theta) + A_1\cos\theta]$$

$$A_0 = 1 + 2Re[r]c_V^2 + |r|^2(c_V^2 + c_A^2)^2 \qquad A_1 = 4Re[r]c_A^2 + 8|r|^2 c_V^2 c_A^2$$

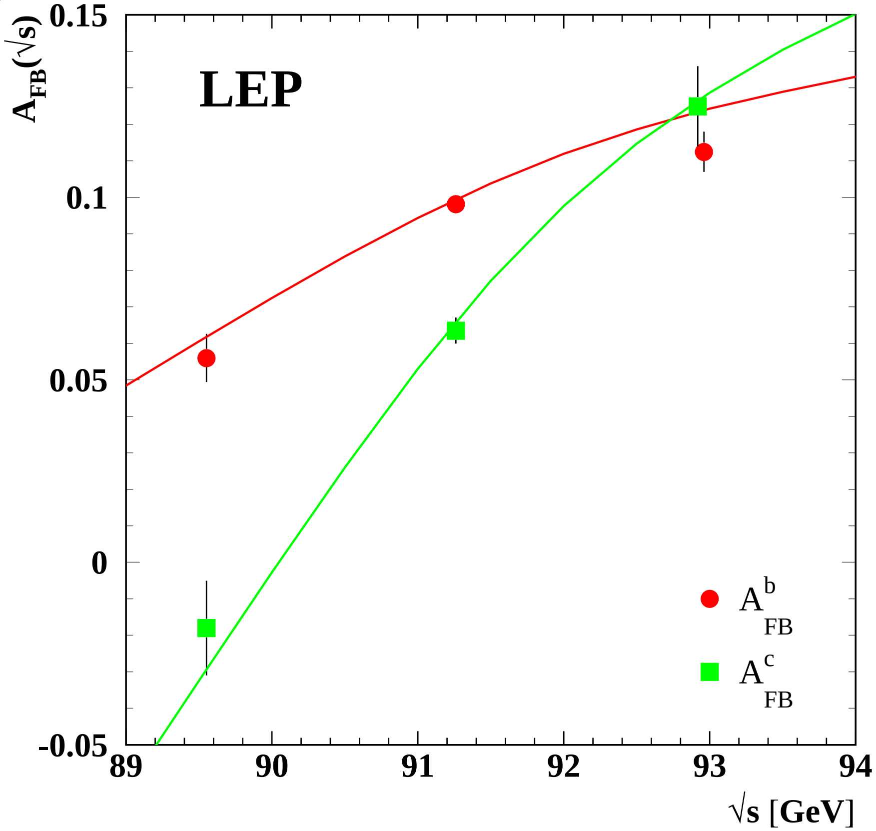
The $\cos\theta$ term gives a forward-backward asymmetry

$$A_{FB} = \frac{F-B}{F+B} \qquad F = \int_0^1 \frac{d\sigma}{d\Omega} d\Omega \qquad B = \int_{-1}^0 \frac{d\sigma}{d\Omega} d\Omega$$

$$A_{FB} = \frac{3A_1}{8A_0} \qquad A_{FB}(s \ll M_Z) = \frac{3}{2}Re(r)c_A^2 = -\frac{3c_A^2 G_F s}{\sqrt{2}e^2}$$

A_{FB} is small at low s where $|r| \ll 1$ and photon dominates

$A_{FB} \approx 0$ at $\sqrt{s} = M_Z$ where Z^0 dominates



Forward-Backward Asymmetries at $\sqrt{s} \approx M_Z$

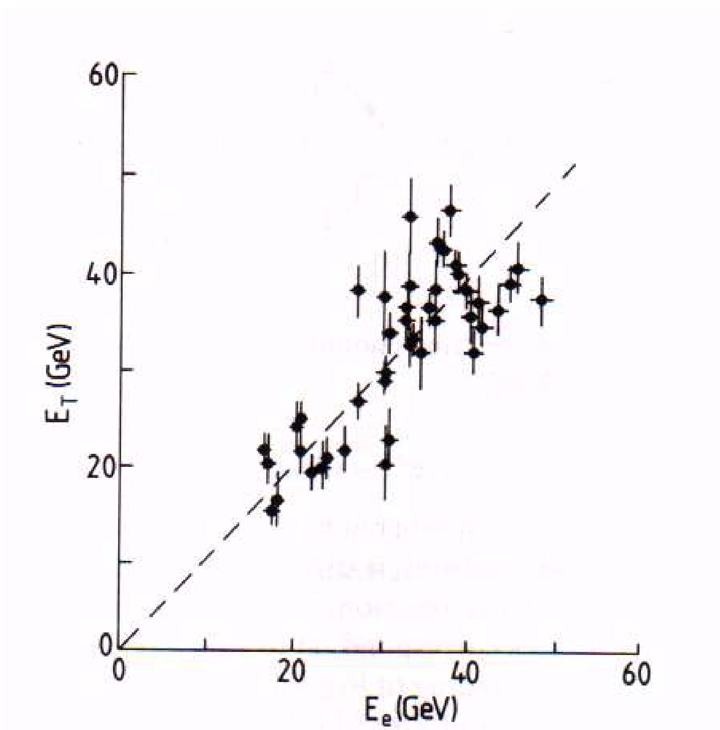
The heavy quark A_{FB} are sensitive to virtual H, t diagrams

Leptons	$A_{FB}(M_Z)$	Quarks	$A_{FB}(M_Z)$
$e^+e^- \rightarrow e^+e^-$	$(1.45\pm 0.25)\%$	$e^+e^- \rightarrow s\bar{s}$	$(9.8\pm 1.1)\%$
$e^+e^- \rightarrow \mu^+\mu^-$	$(1.69\pm 0.13)\%$	$e^+e^- \rightarrow c\bar{c}$	$(7.04\pm 0.36)\%$
$e^+e^- \rightarrow \tau^+\tau^-$	$(1.88\pm 0.17)\%$	$e^+e^- \rightarrow b\bar{b}$	$(10.01\pm 0.17)\%$

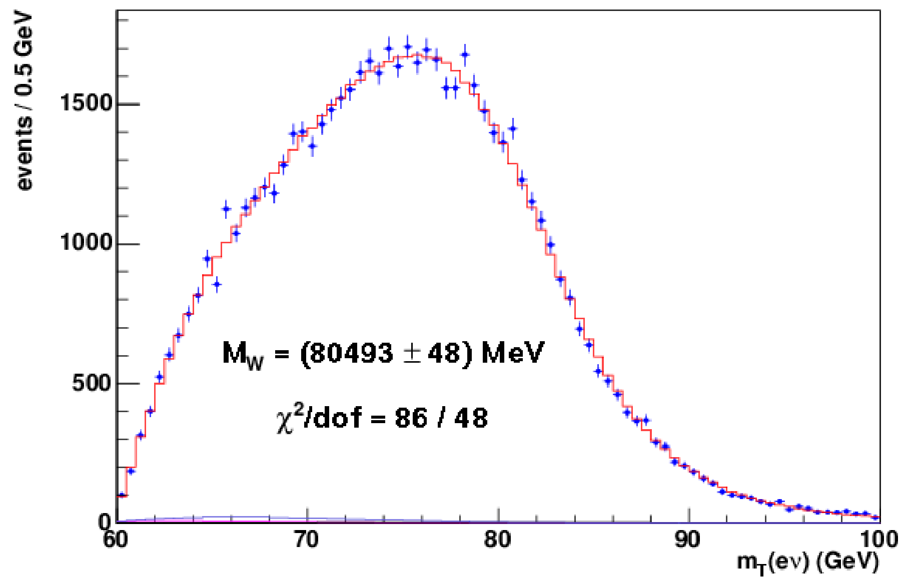
W Boson Production

The W and Z bosons were discovered in $p\bar{p}$ collisions at CERN

$W \to \ell\nu$ decays to high p_T lepton and high missing E_T of ν



UA1 experiment (1983)



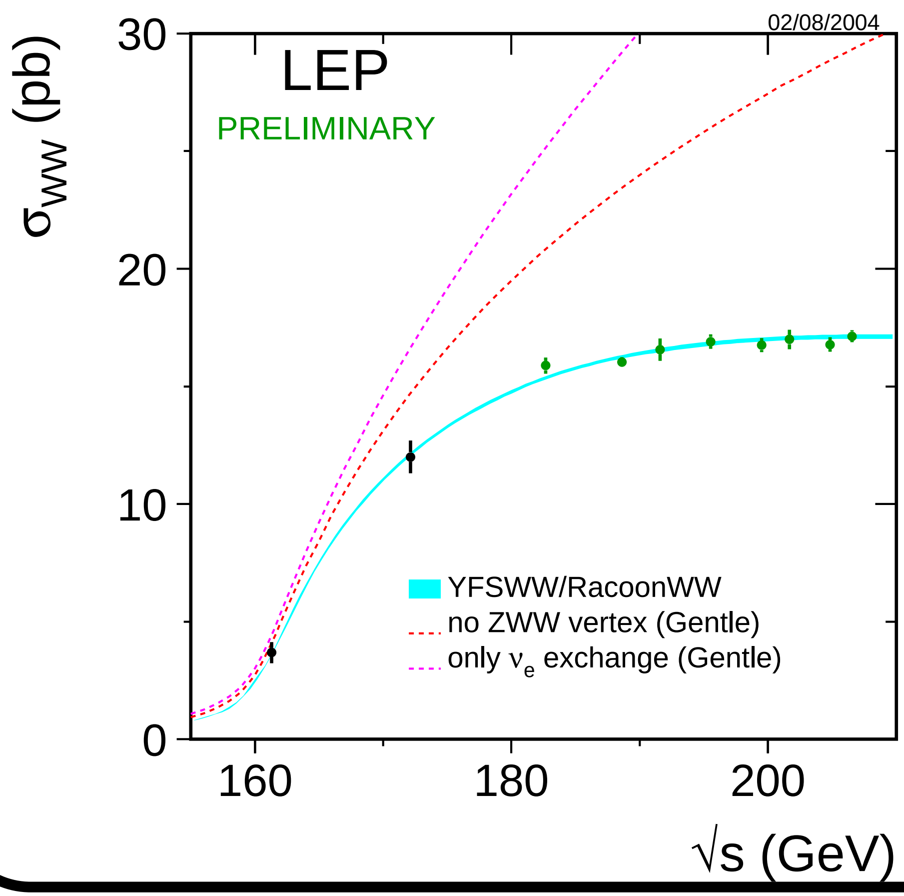
CDF Experiment (2007)

$M_W = 80.493 \pm 0.048$ GeV

$\Gamma_W = 2.115 \pm 0.105$ GeV

W^+W^- Pair Production at LEP-2

From 1996-2001 LEP-2 produced W pairs at $\sqrt{s} \approx 200$ GeV.



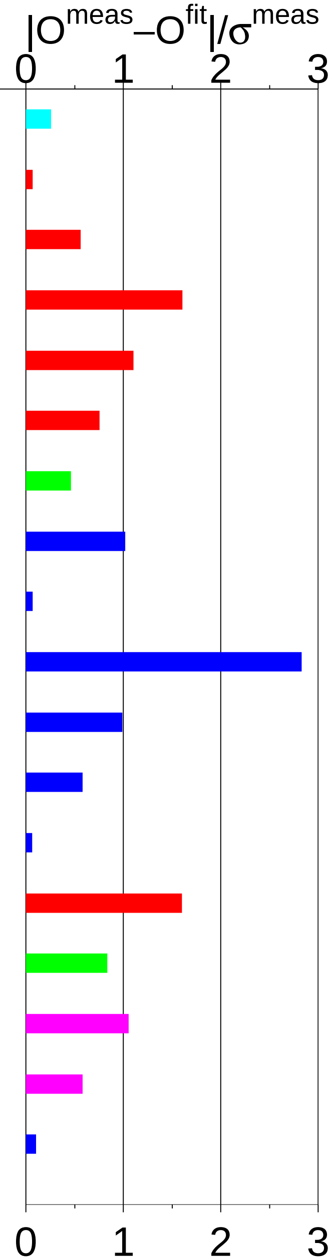
Cross-section is a sum of:

ν_e exchange (t-channel)

+ virtual photon (s-channel)

+ Z^0 with trilinear gauge coupling $Z^0W^+W^-$

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ_{had}^0 [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21562
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.389
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.5



Summary of electroweak measurements

http://lepewwg.web.cern.ch/LEPEWWG/

Good agreement with electroweak theory (apart from A_{FB}^b?)