Lecture 8

Quantum Chromo Dynamics

QCD is quantum field theory of strong interactions
describes couplings of quarks and gluons through **colour**

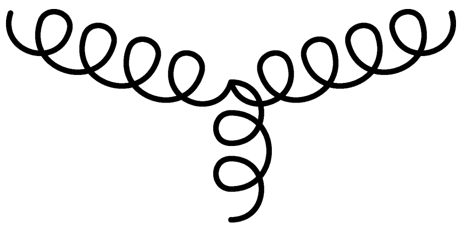
- Feynman Rules for QCD
- SU(3) Group Symmetry
- Colour States of Quarks & Gluons
- Strong Coupling Constant α_s
- Azymptotic Freedom & Confinement
- QCD Potential

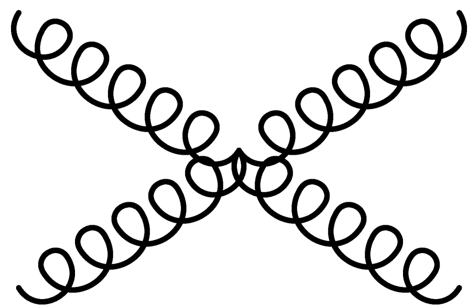
Rules for QCD Feynman diagrams

- A quark has one of three **colour** states (red, green or blue). An antiquark has one of three anticolours.
- A gluon propagator has a colour and an anticolour.
  - There are **eight** possible gluon states
  - *Gluons have strong interactions with each other*
- Colours are conserved at quark-gluon vertices
  - A quark-gluon vertex has a factor $-ig_s\lambda^a\gamma^\mu$. λ^a are **generator matrices** of SU(3) colour symmetry.
  - The quark and gluon colours are combined with the λ^a to give an overall **colour factor**, c_f, in the amplitude.
- The coupling constant $g_s = \sqrt{\alpha_s}$ is a function of q^2:
  - At small q^2 it is $\mathcal{O}(1)$, and QCD is non-perturbative
  - At large q^2 it is smaller, and QCD becomes perturbative

Gluon Self Interactions

Gluons interact with each other through their colour and anticolour states. There can be three or four gluon vertices:





The three gluon vertex has a complicated factor:

$$-g_s f^{abc}\left[g_{\mu\nu}(q_1 - q_2)_\lambda + g_{\nu\lambda}(q_2 - q_3)_\mu + g_{\lambda\mu}(q_3 - q_1)_\nu\right]$$

f^{abc} are *color structure constants* related to the λ^a matrices:

$$\left[\lambda^a, \lambda^b\right] = 2i \sum_c f^{abc} \lambda^c$$

SU(3) Colour Symmetry

An SU(3) group symmetry is related to colour conservation

- Three colour quantum numbers are separately conserved
- Strong interaction coupling g_s is same for all colour states
- Invariance under rotation in three-dimensional colour space $U = e^{-i\alpha_a \cdot \lambda^a}$ is a "non-Abelian" gauge symmetry

There are two types of "colourless" states:

Mesons have symmetric colour-anticolour wavefunctions:

$$\chi_c = \frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$$

Baryons have antisymmetric three-colour wavefunctions:

$$\chi_c = \frac{1}{\sqrt{6}}(rgb - rbg + brg - bgr + gbr - grb)$$

The λ matrices of SU(3)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Colour states of Quarks and Gluons

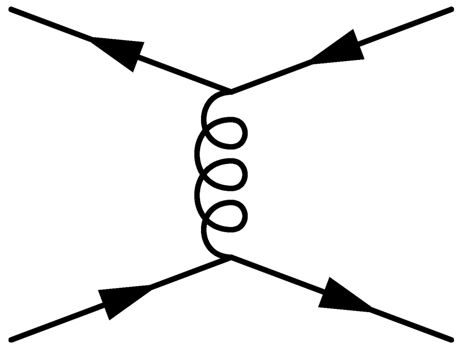
Three quark states: $r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $\quad b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $\quad g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

Gluon states are **colour octet** of SU(3) (from λ^a):

$$
\begin{aligned}
G_1 &= \tfrac{1}{\sqrt{2}}\left(r\bar{b} + \bar{r}b\right) \qquad & G_2 &= \tfrac{-i}{\sqrt{2}}\left(r\bar{b} - \bar{r}b\right) \\
G_4 &= \tfrac{1}{\sqrt{2}}\left(r\bar{g} + \bar{r}g\right) \qquad & G_5 &= \tfrac{-i}{\sqrt{2}}\left(r\bar{g} - \bar{r}g\right) \\
G_6 &= \tfrac{1}{\sqrt{2}}\left(b\bar{g} + \bar{b}g\right) \qquad & G_7 &= \tfrac{-i}{\sqrt{2}}\left(b\bar{g} - \bar{g}b\right) \\
G_3 &= \tfrac{1}{\sqrt{2}}\left(r\bar{r} - b\bar{b}\right) \qquad & G_8 &= \tfrac{1}{\sqrt{6}}\left(r\bar{r} + b\bar{b} - 2g\bar{g}\right)
\end{aligned}
$$

No **colour singlet** gluon state: $G_0 = \frac{1}{\sqrt{3}}\left(r\bar{r} + g\bar{g} + b\bar{b}\right)$

Quark-Antiquark Scattering



Matrix element for quark-antiquark scattering:

$$-i\mathcal{M} = \left[\bar{u}_3 c_3^\dagger\right] \left[\frac{-ig_s}{2}\lambda^a\gamma^\mu\right] [u_1 c_1] \left[\frac{-ig_{\mu\nu}\delta^{ab}}{q^2}\right] \left[\bar{v}_2 c_2^\dagger\right] \left[\frac{-ig_s}{2}\lambda^b\gamma^\nu\right] [v_4 c_4]$$

$$\mathcal{M} = c_f \frac{\alpha_s}{4q^2} [\bar{u}_3\gamma^\mu u_1] [\bar{v}_2\gamma_\mu v_4]$$

where the colour factor is:

$$c_f = (c_3^\dagger\lambda^a c_1)(c_2^\dagger\lambda^a c_4)$$

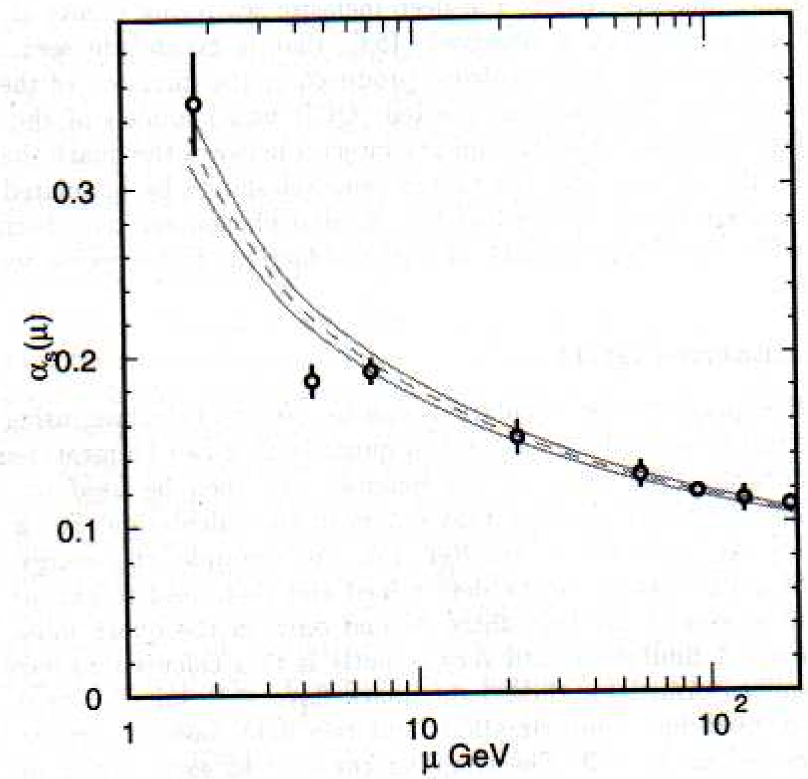
Colour factors in qq and $q\bar{q}$ Scattering

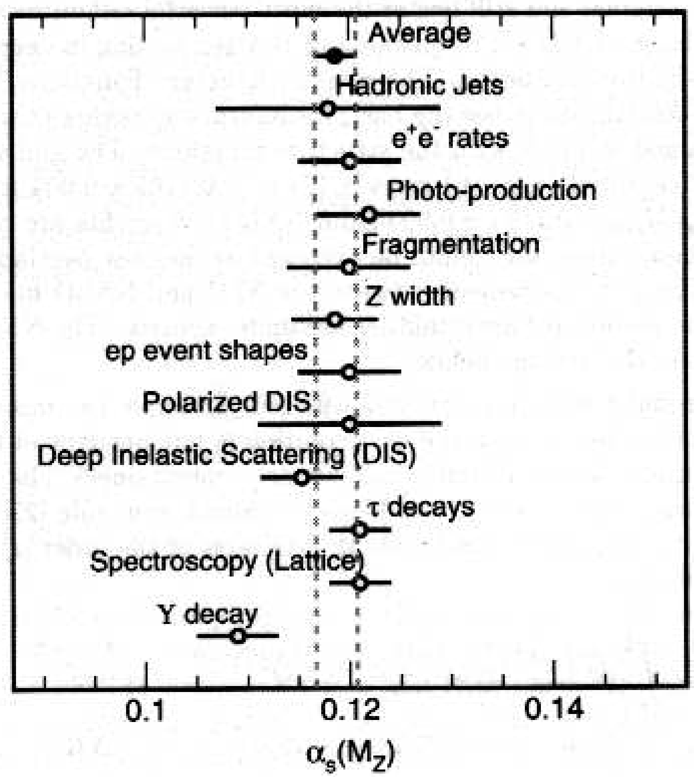
quark states	gluon states	c_f
$rr \leftrightarrow rr$	G_7, G_8	+2/3
$r\bar{r} \leftrightarrow r\bar{r}$	G_7, G_8	-2/3
$rb \leftrightarrow rb$	G_7, G_8	-1/3
$rb \leftrightarrow br$	G_1, G_2	+1
$r\bar{r} \leftrightarrow b\bar{b}$	G_1, G_2	-1
$r\bar{b} \leftrightarrow r\bar{b}$	G_7, G_8	+1/3

... and similarily for the other colour combinations by replacing $r \to b$, $r \to g$ and $b \to g$ and assigning the relevant gluon states

See Halzen & Martin Pp.67-69 for detailed calculations of c_f

Strong Coupling Constant α_s





Hard to measure running of α_s at low mass scale μ!

α_s is measured to a few % at $\mu = M_Z$

Description of Running of α_s

In strong interactions the running of α_s is due to:

- Screening of colour by quark-antiquark ($f\bar{f}$) pairs
- *Anti-screening* of colour by gluons

Anti-screening of gluons dominates $\Rightarrow$ α_s decreases with q^2

$$\alpha_s(q^2) = \frac{12\pi}{(33 - 2N_f) ln\left(\frac{q^2}{\Lambda^2_{QCD}}\right)} \qquad \text{(QCD)}$$

$N_f \leq 6$ is the number of available quark flavours at a given q^2

$\Lambda_{QCD} = 217 \pm 25$ MeV is the QCD scale parameter

Reminder - in QED screening of electric charge by $f\bar{f}$ pairs:

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \ln\left(\frac{q^2}{\mu^2}\right)} \qquad \text{(QED)}$$

Azymptotic Freedom

At low $q^2 \approx \Lambda^2_{QCD}$ quarks and gluons are tightly bound inside meson and baryons

QCD calculations in this region are non-perturbative

Usually solved by numerical methods (Lattice QCD)
Example: baryon and meson masses

At high $q^2 \gg \Lambda^2_{QCD}$ quarks and gluons are **azymptotically free**

QCD calculations in this region are perturbative

Can be solved by summing diagrams in powers of α_s (like QED)
Example: Gluon emission and hard scattering in DIS

It is difficult to understand transition between two regions
Example: fragmentation of partons into jets of hadrons

Confinement

Quarks and gluons are **confined** inside hadrons and cannot be directly observed as free partons

There are many models of confinement:

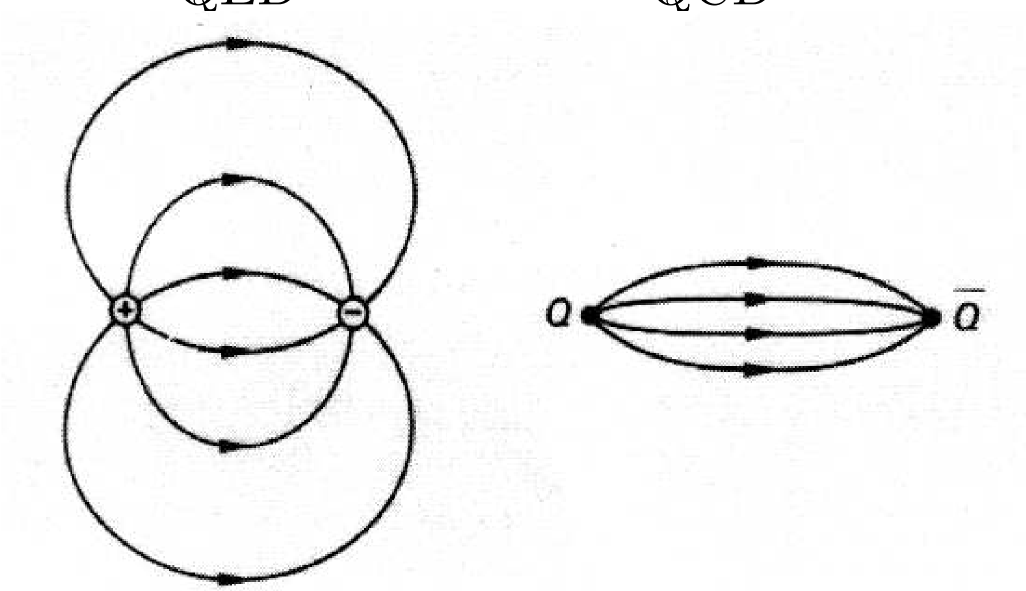
- The valence quark model of hadrons (next lecture)
  Ignores effects of sea quarks and gluons at low x
- The colour flux-tube model
- The QCD potential model

None of these models provides a rigorous approach to non-perturbative QCD but they are conceptually useful

Colour Flux-tube Model

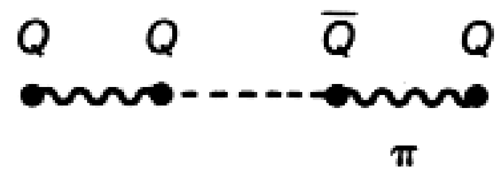
Colour field lines compressed
into flux-tube between quarks

QED QCD

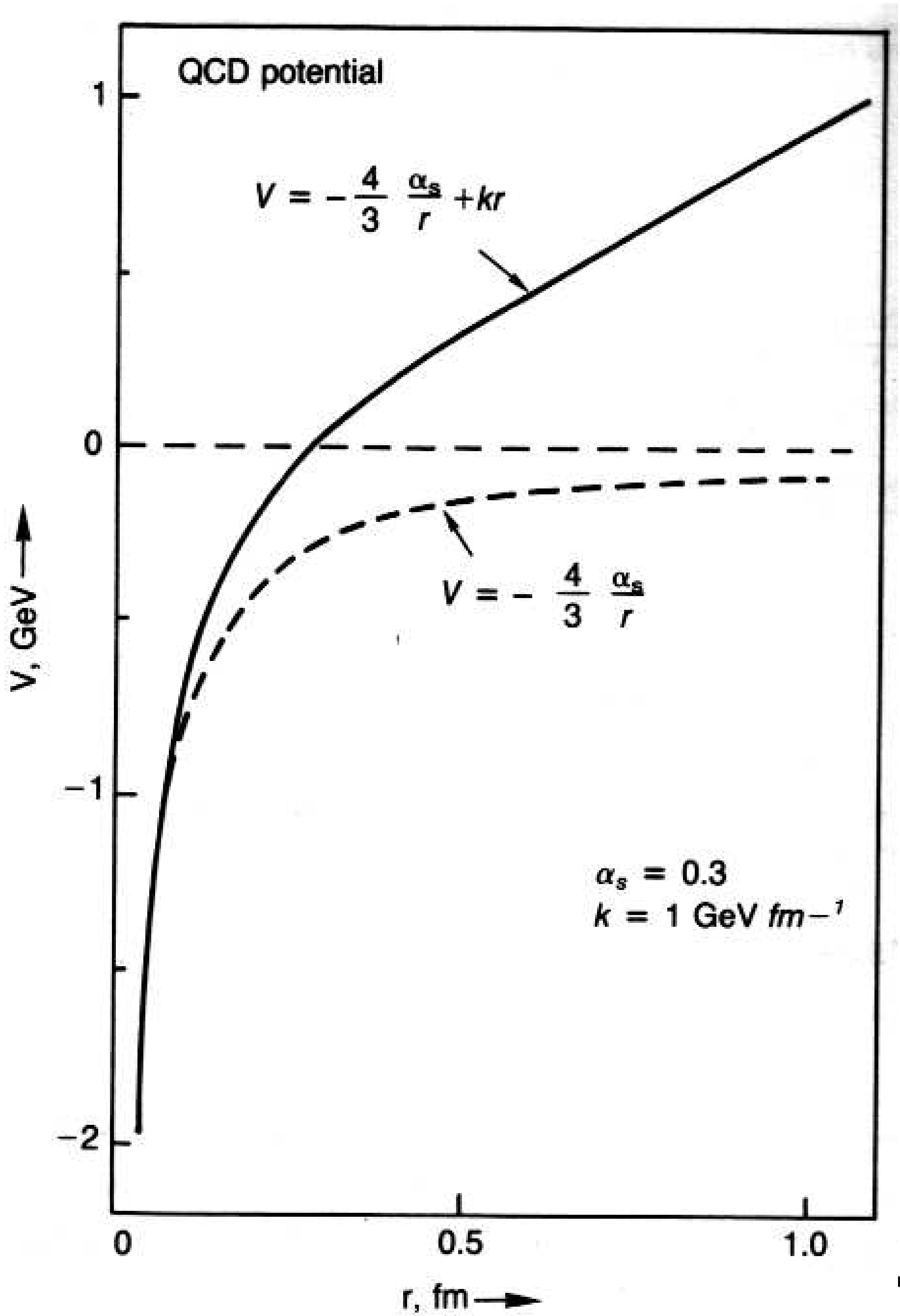


Energy stored is like
in an elastic string

Can break into $q\bar{q}$ pair:



Potential $V = kr \Rightarrow$ need infinite energy to separate quarks



QCD potential

$$V_{q\bar{q}}(r) = -\frac{4}{3}\frac{\alpha_S}{r} + kr$$

Mesons are $q\bar{q}$
bound states in a
QCD potential well