Lecture 9
Valence Quark Model of Hadrons

- Isospin symmetry
- SU(3) flavour symmetry
- Meson & Baryon states
- Hadronic wavefunctions
- Masses and magnetic moments
- Heavy quark states

Isospin Symmetry

Strong interactions are invariant under isospin rotation

This is a *flavour symmetry* between the light quarks

Coupling of gluons to u and d quarks are the same

Protons and neutrons have the same strong interactions

The u and d quarks are assigned to an **isospin** doublet:

$$u: \;\; \mathrm{I} = \frac{1}{2}, \;\; \mathrm{I}_3 = +\frac{1}{2} \qquad\qquad d: \;\; \mathrm{I} = \frac{1}{2}, \;\; \mathrm{I}_3 = -\frac{1}{2}$$

Described by SU(2) symmetry group with Pauli isospin matrices:

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Lowest States of Mesons and Baryons

The pions are an isospin triplet with I=1:

$$\pi^+ \;\; [1,1] = u\bar{d} \qquad \pi^0 \;\; [1,0] = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \qquad \pi^- \;\; [1,-1] = d\bar{u}$$

The eta meson is an isospin singlet with I=0:

$$\eta \;\; [0,0] = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

The nucleons are an isospin doublet with I=1/2:

$$p \;\; [1/2, +1/2] = uud$$

$$n \;\; [1/2, -1/2] = ddu$$

SU(3) Flavour Symmetry

The u, d and s quarks are assigned to a **flavour** triplet

Strong interactions are *approximately* flavour symmetric

The mass of the s quark breaks the symmetry

Described by an SU(3) flavour symmetry with the same eight λ^a matrices as SU(3) colour symmetry

Assign **strangeness** $S = -1$ to s quark ($S = +1$ to $\bar{s}$ antiquark)

Hypercharge, Y, is sum of strangeness and baryon number B

Charge, Q, is sum of isospin I_3 and hypercharge

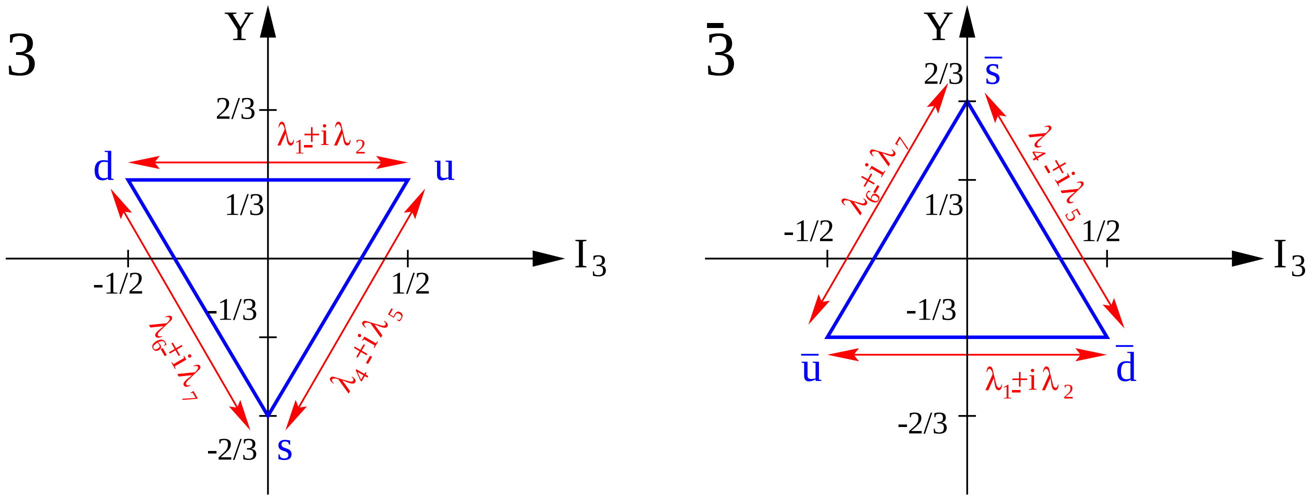
$$\mathrm{Y} = S + B \qquad \mathrm{Q} = \mathrm{I}_3 + \frac{\mathrm{Y}}{2}$$

Isospin and hypercharge are related to the diagonal λ matrices:

$$\mathrm{I}_3 = \frac{1}{2}\lambda^3 \qquad \mathrm{Y} = \frac{1}{\sqrt{3}}\lambda^8$$

SU(3) Multiplets

The three flavours of light quarks and antiquarks can be represented as 2-dimensional SU(3) multiplets of isospin and hypercharge:

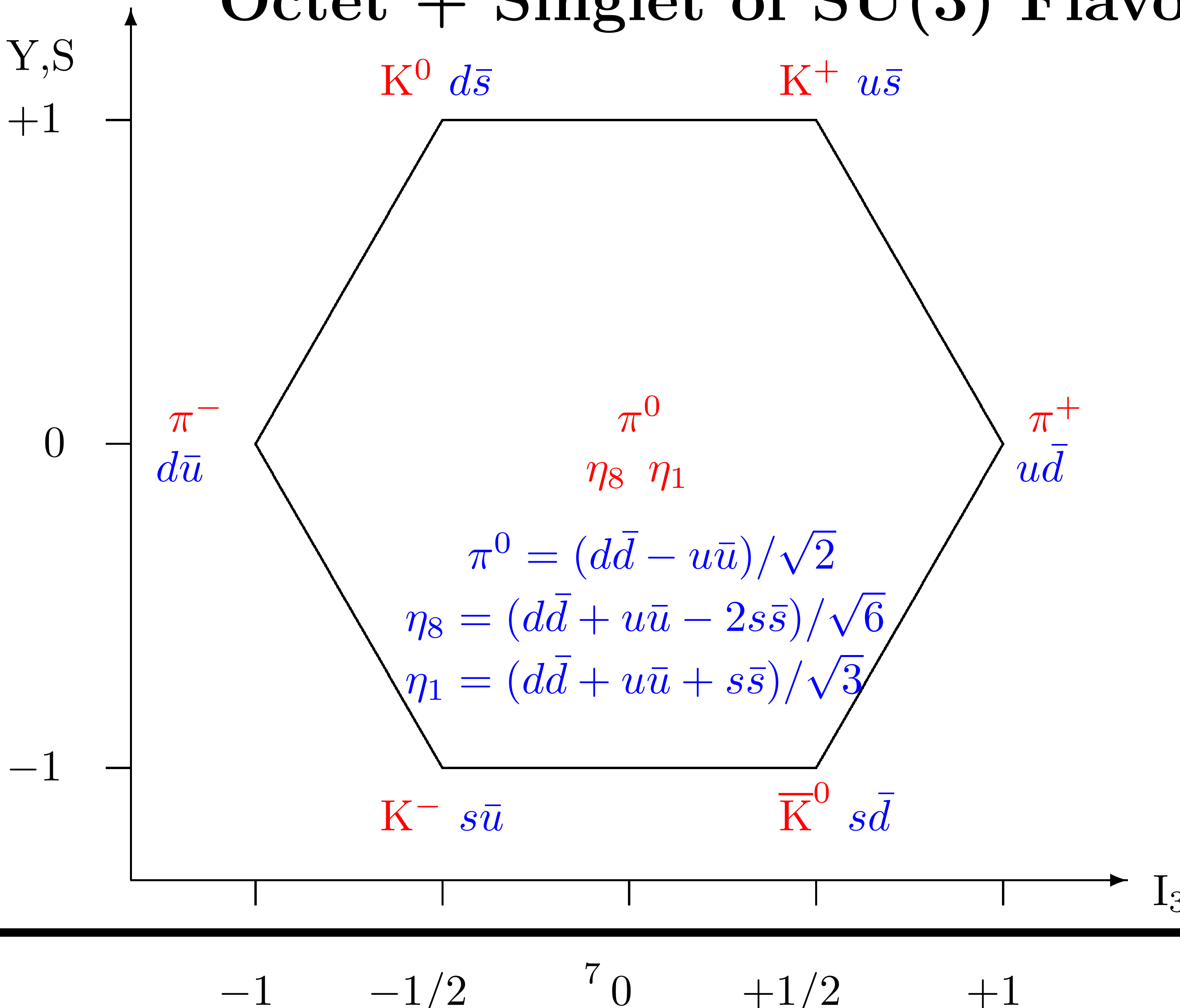


The SU(3) flavour changing operators are shown in red

Mesons are built up from quark-antiquark ($q\bar{q}$) pairs

Baryons are built up from three quark (qqq) states

Pseudoscalar mesons $J^{PC} = 0^{-+}$
Octet + Singlet of SU(3) Flavour



$M_{K^\pm} = 494$ MeV

$M_{K^0} = 498$ MeV

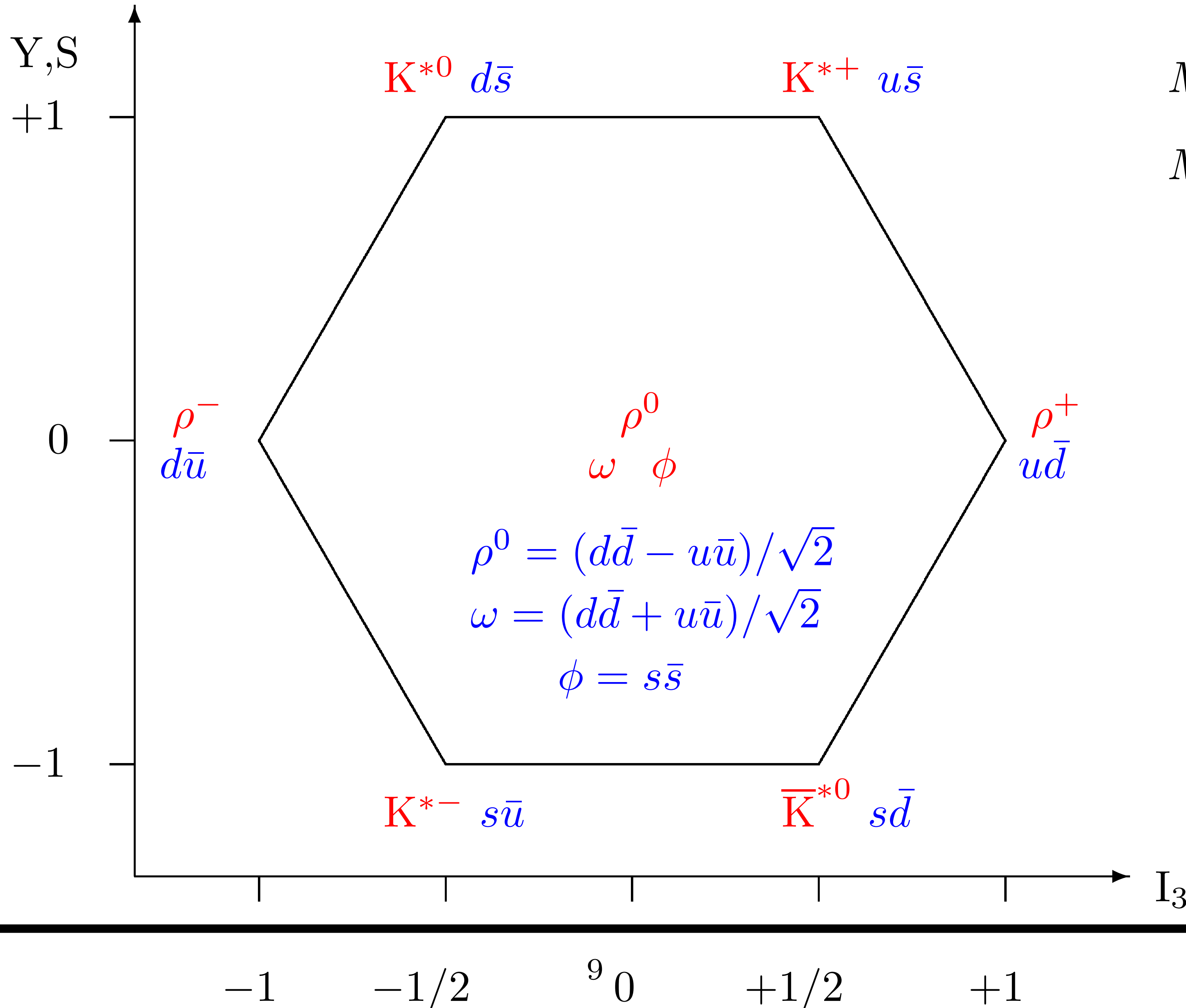
$M_{\pi^\pm} = 140$ MeV

$M_{\pi^0} = 135$ MeV

$M_\eta = 550$ MeV

$M_{\eta'} = 960$ MeV

Vector mesons $J^{PC} = 1^{--}$



$M_{K^{*\pm}} = 892$ MeV

$M_{K^{*0}} = 896$ MeV

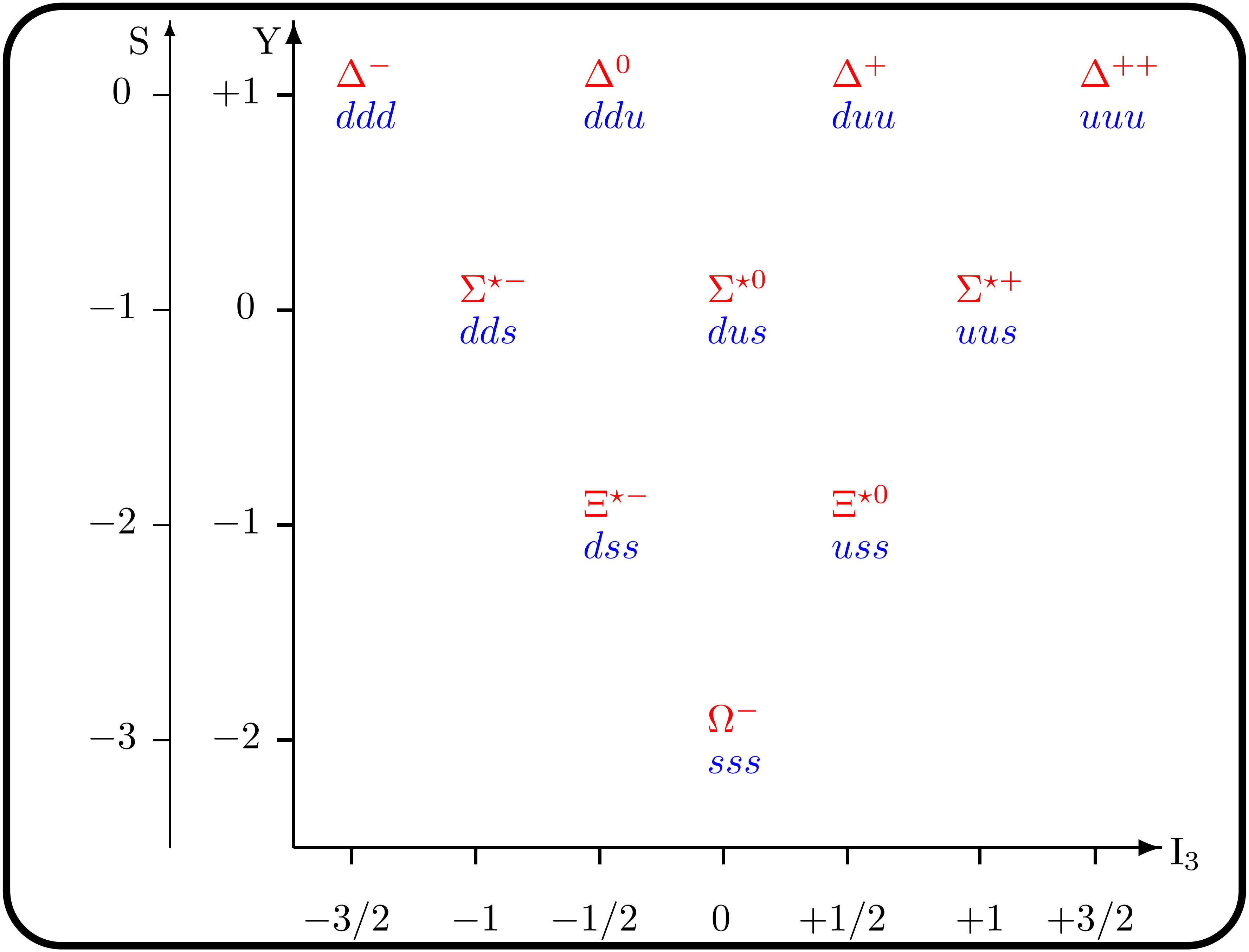
$M_{\rho^\pm} = 776$ MeV

$M_{\rho^0} = 767$ MeV

$M_\omega = 783$ MeV

$M_\phi = 1019$ MeV

Baryon Decuplet $J = 3/2$



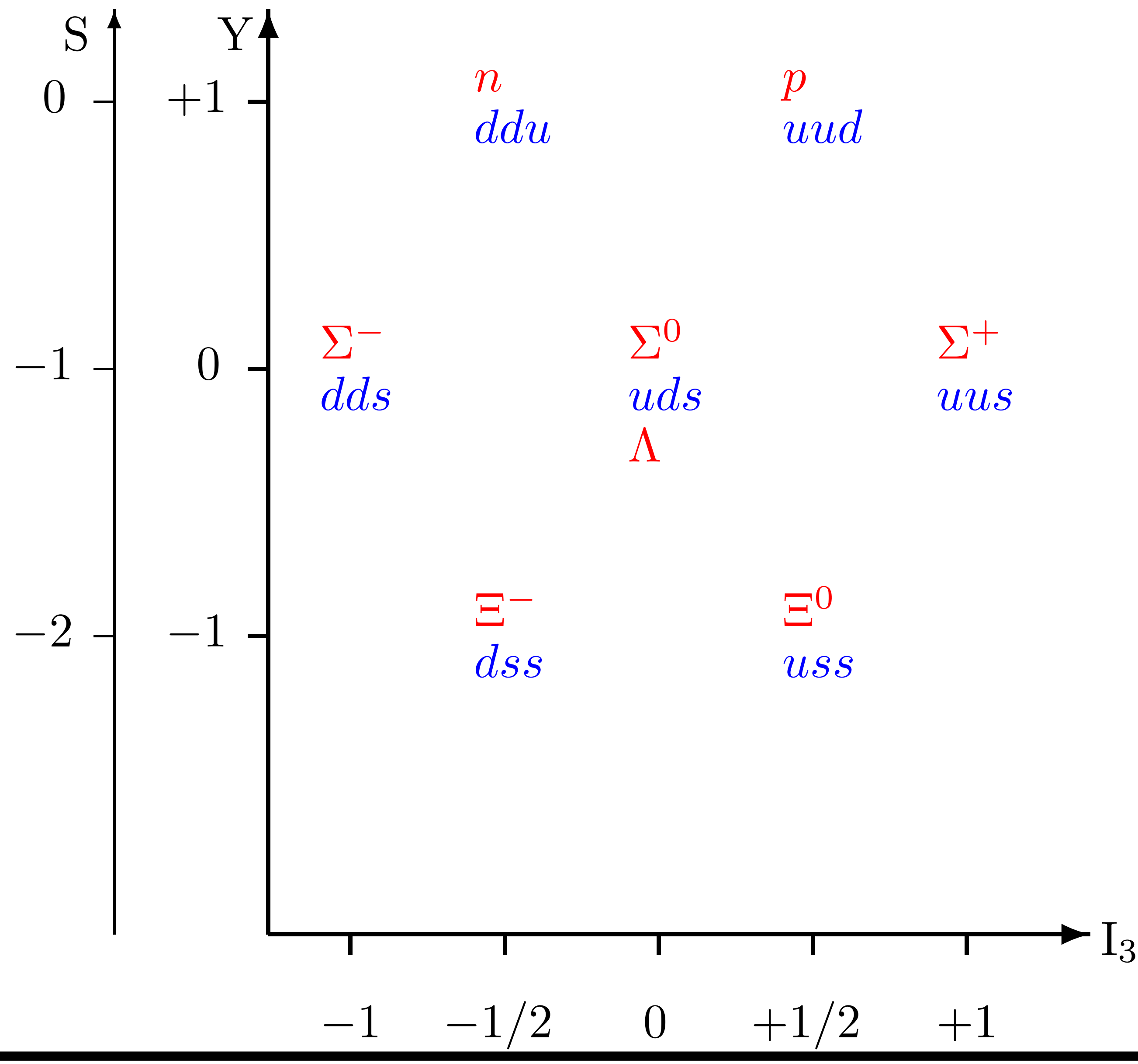
$M_\Delta =$ 1232 MeV

$M_{\Sigma^*} =$ 1383 MeV

$M_{\Xi^*} =$ 1532 MeV

$M_\Omega =$ 1672 MeV

Baryon Octet $J = 1/2$



$M_n = 940$ MeV
$M_p = 938$ MeV

$M_\Sigma = 1193$ MeV
$M_\Lambda = 1116$ MeV

$M_\Xi = 1318$ MeV

Baryon Wavefunctions

The overall wavefunction of a system of identical fermions is antisymmetric (A) under the interchange of any two fermions

$$\psi[\Delta^{++}] = uuu(\uparrow\uparrow\uparrow) = \chi_c \chi_f \chi_S \chi_L$$

where $\chi_{c,f,S,L}$ are the color, flavour, spin and orbital parts

The Δ^{++} has symmetric (S) flavour and spin, so the color wavefunction must be antisymmetric (see previous lecture)

	χ_c	χ_f	χ_S	χ_L	ψ
Δ^{++}	A	S	S	S	A

The proton wavefunction has parts $\chi_f \chi_S$ (overall S):

$$uud(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) + udu(\downarrow\uparrow\uparrow + \uparrow\uparrow\downarrow - 2\uparrow\downarrow\uparrow) + duu(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow - 2\downarrow\uparrow\uparrow)$$

There are no J=1/2 baryon states uuu, ddd or sss!

Hadronic Masses & Constituent Quarks

In renormalised QCD, quark masses are quoted in the $\overline{MS}$ scheme:

$$m_u \approx m_d \approx 1\text{MeV} \qquad\qquad m_s \approx 100\text{MeV}$$

These are too small to account for the hadron masses!

Valence quark model of hadrons uses **constituent quarks**:

$$m_u = m_d = \frac{m_N}{3} \approx 300\text{MeV} \qquad\qquad m_s \approx 500\text{MeV}$$

There are some "semi-empirical" mass formulae:

$$M(\Sigma^*) - M(\Delta) = M(\Xi^*) - M(\Sigma^*) = M(\Omega) - M(\Xi^*) = 150\text{MeV}$$

$$3M(\Lambda) + M(\Sigma) = 2M(N) + 2M(\Xi)$$

The hyperfine splitting between J=0 and J=1 mesons is:

$$M(q\bar{q}) = m_q + m_{\bar{q}} + a\left[\vec{\sigma_1} \cdot \vec{\sigma_2} / m_q m_{\bar{q}}\right]$$

Anomalous Magnetic Moments

Magnetic moments of valence constituent quarks:

$$\mu = 2\mu_q S_z \quad \text{where} \quad \mu_u = \frac{2e}{3m_u} \approx 2\mu_N \quad \mu_d = -\frac{e}{3m_d} \approx -\mu_N$$

Starting from the proton flavour/spin wavefunction (see above):

$$\mu_p = \frac{1}{3}[\mu_d + 2(2\mu_u - \mu_d)] = \frac{1}{3}(4\mu_u - \mu_d)$$

Paired quarks of the same flavour and opposite spin cancel

The neutron magnetic moment follows from isospin symmetry:

$$\mu_n = \frac{1}{3}(4\mu_d - \mu_u)$$

The anomalous magnetic moments are correctly predicted!

$$\mu_p = 2.79\mu_N \approx 3\mu_N \qquad \mu_n = -1.86\mu_N \approx -2\mu_N$$

Heavy Quark States

The c and b quarks can form hadrons with **charm** or **beauty**

The t quark does not form hadrons due to its short lifetime

Lowest lying charm states are D mesons with masses $\approx 2\text{GeV}$

$$D^+(c\bar{d}) \quad D^0(c\bar{u}) \quad \bar{D}^0(\bar{c}u) \quad D^-(\bar{c}d) \quad D_s^+(c\bar{s}) \quad D_s^-(\bar{c}s)$$

Lowest lying beauty states are B mesons with masses $\approx 5\text{GeV}$

$$B^+(\bar{b}u) \quad B^0(\bar{b}d) \quad \bar{B}^0(b\bar{d}) \quad B^-(b\bar{u}) \quad B_s^0(\bar{b}s) \quad \bar{B}_s{}^0(b\bar{s})$$

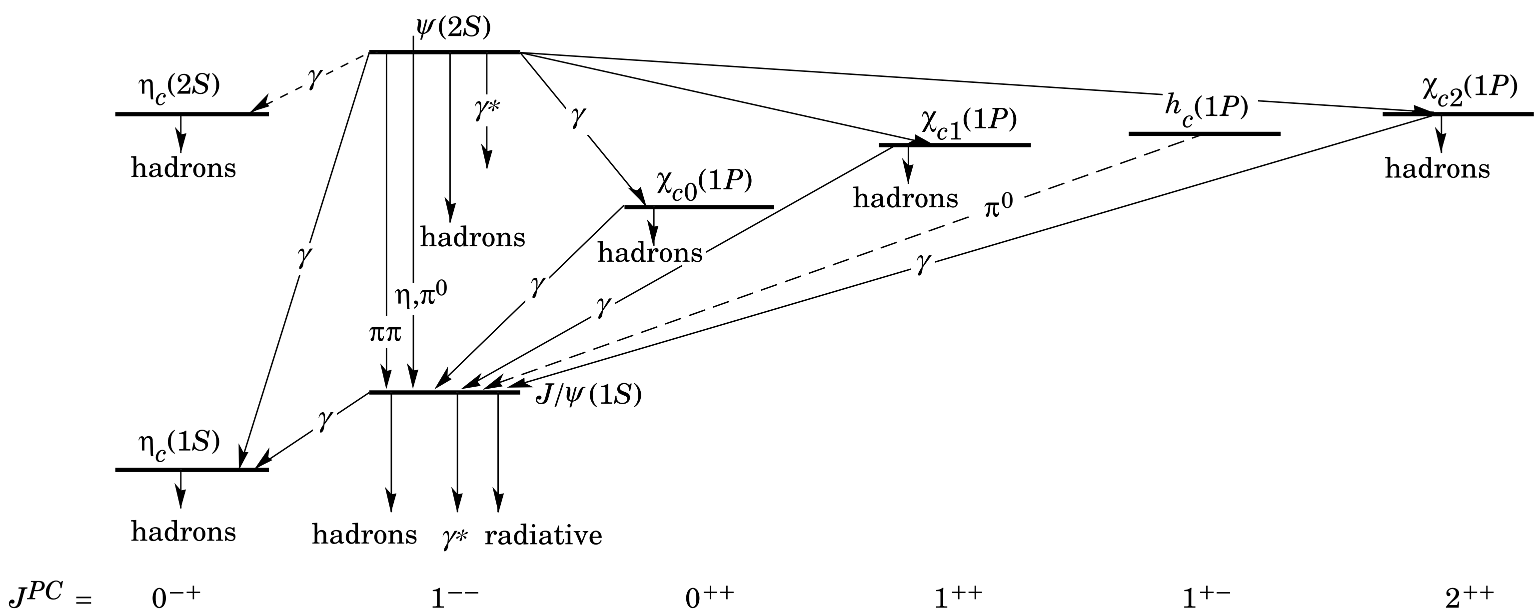
There are bound states of charmonium $(c\bar{c})$ and bottomonium $(b\bar{b})$

$$M(J/\psi) = 3.1\text{GeV} \qquad M(\Upsilon(1S)) = 9.5\text{GeV}$$

Charmonium Spectroscopy

There is a spectroscopy of excited states of hadrons with higher l, n (just like atomic physics)

Example of $c\bar{c}$ charmonium states ($n = 1, 2$ and $l = 0, 1$)



In spectroscopic notation the J/ψ is the ${}^3S_1$ state with $J^{PC} = 1^{--}$