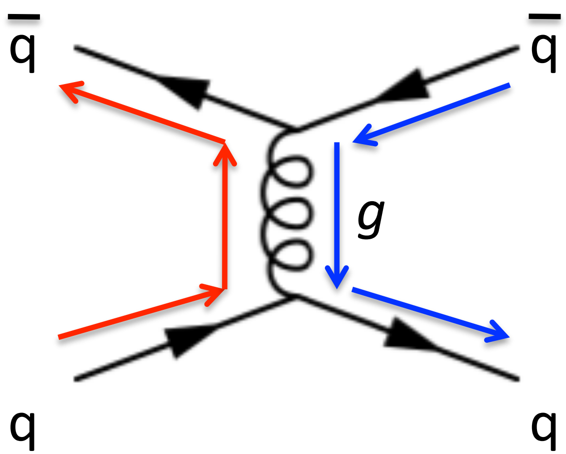
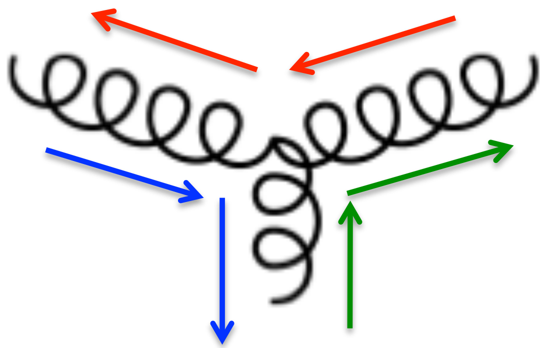
Lecture 8 – Quantum ChromoDynamics (QCD)

The theory of strong interactions between quarks and gluons





Coupling is to three “colour” charges r(ed), b(lue) and g(reen)

Gluons carry colour-anticolour charges and have self-interactions

Colour Symmetry

- Strong couplings are the same for all colour states
- Each of the three colours is separately conserved
- Colour symmetry is described by an SU(3) group

  The generators of SU(3) are eight 3x3 matrices λ^a
- Strong interactions are invariant under a rotation in SU(3) space

$$U = e^{-i\alpha_a \lambda^a}$$

Technically this is known as a "non-Abelian" symmetry

Quark states: $r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ $b = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $g = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

SU(3) Matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Gluon Colour States

Naively there are 9 states: $r\bar{r}$, $b\bar{b}$, $g\bar{g}$, $r\bar{b}$, $b\bar{r}$, $r\bar{g}$, $g\bar{r}$, $b\bar{g}$, $g\bar{b}$

In SU(3) these are arranged into a **colour octet** (allowed for gluons)**:**

$$G_1 = 1/\sqrt{2}\,[\,r\bar{b} + b\bar{r}\,] \qquad G_2 = 1/\sqrt{2}\,[\,r\bar{b} - b\bar{r}\,]$$

$$G_4 = 1/\sqrt{2}\,[\,r\bar{g} + g\bar{r}\,] \qquad G_5 = 1/\sqrt{2}\,[\,r\bar{g} - g\bar{r}\,]$$

$$G_6 = 1/\sqrt{2}\,[\,b\bar{g} + g\bar{b}\,] \qquad G_7 = 1/\sqrt{2}\,[\,b\bar{g} - g\bar{b}\,]$$

$$G_3 = 1/\sqrt{2}\,[\,r\bar{r} - b\bar{b}\,] \qquad G_8 = 1/\sqrt{6}\,[\,r\bar{r} + b\bar{b} - 2\,g\bar{g}\,]$$

and a **colour singlet** which is symmetric (forbidden for gluons) :

$$G_0 = 1/\sqrt{3}\,[\,r\bar{r} + b\bar{b} + g\bar{g}\,]$$

Feynman Rules for QCD

- An incoming (outgoing) quark has a spinor and a colour

  $u\, c \;\; (\bar{u}\, c^{\dagger})$

- An incoming (outgoing) antiquark has $\bar{v}\, c^{\dagger}$ ($v\, c$)
- An incoming (outgoing) gluon has a polarization vector and a colour-anticolour state $\varepsilon^{\mu}\, G_{\alpha}$ ($\varepsilon^{\mu *}\, G_{\alpha}{}^{*}$)
- A quark-gluon vertex has a strong coupling and a colour factor

  $g_s\, \lambda^{\alpha}\, \gamma^{\mu}$

  (λ^{α} is the SU(3) generator matrix corresponding to the gluon state)

- A gluon propagator has a factor $g_{\mu\nu}\, \delta^{\alpha\beta} / q^2$
- A quark propagator has a factor $(\gamma^{\mu} q_{\mu} + m)/(q^2 - m^2)$

Gluon-gluon interactions

Three gluon vertex

There is also a four gluon vertex (see P.288 of Griffiths)

This vertex has a complicated factor:

$$g_s f^{\alpha\beta\gamma} \left[g_{\mu\nu} (q_1 - q_2)_\lambda + g_{\nu\lambda} (q_2 - q_3)_\mu + g_{\lambda\mu} (q_3 - q_1)_\nu \right]$$

where the gluons have four momenta q_1, q_2, q_3 (all into vertex) and colour-anticolour states α, β, γ

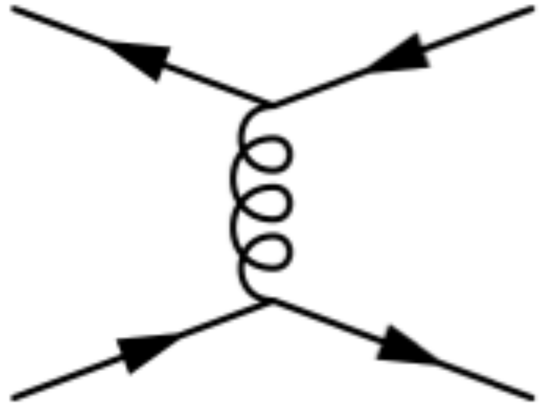
The "colour structure constants" $f^{\alpha\beta\gamma}$ are defined by anticommutators of the λ matrices:

$$[\lambda^\alpha, \lambda^\beta] = 2i\, f^{\alpha\beta\gamma} \lambda^\gamma$$

They are just numbers (see P.287 of Griffiths)

Quark-(Anti)quark Scattering

Not directly observable.
No free quarks or antiquarks.
Colour states not detectable.



$$M = c_f \frac{\alpha_s}{4q^2} (\bar{u}_3 \gamma^\mu u_1)(\bar{v}_2 \gamma^\mu v_4)$$

$c_f = (c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4)$
is a colour factor
(also just a number)

Observables are averaged over initial state colours and summed over final states colours

quark states	gluon states	c_f
$rr \leftrightarrow rr$	G_7, G_8	$+2/3$
$r\bar{r} \leftrightarrow r\bar{r}$	G_7, G_8	-2/3
$rb \leftrightarrow rb$	G_7, G_8	-1/3
$rb \leftrightarrow br$	G_1, G_2	$+1$
$r\bar{r} \leftrightarrow b\bar{b}$	G_1, G_2	-1
$r\bar{b} \leftrightarrow r\bar{b}$	G_7, G_8	$+1/3$

In this table G_7 is my G_3

Quark-(Anti)quark states

Overall colour factors are classified by their colour symmetry.

Griifiths P.289-294
Halzen & Martin P.67-69
Perkins 3rd edition Appendix J
N.B. there are differences in signs and factors of 2 between these!

Quark-antiquark octet states
e.g. $(r\bar{b} - b\bar{r})$ (correspond to G_1-G_8)

$c_f = +1/3$

Quark-quark symmetric states
sextet, e.g. r r , (r b + b r)

$c_f = +2/3$

Quark-antiquark singlet state
$r\bar{r} + b\bar{b} + g\bar{g}$ (corresponds to G_0)

$c_f = -8/3$

↑

Applies to gluon exchange in a meson

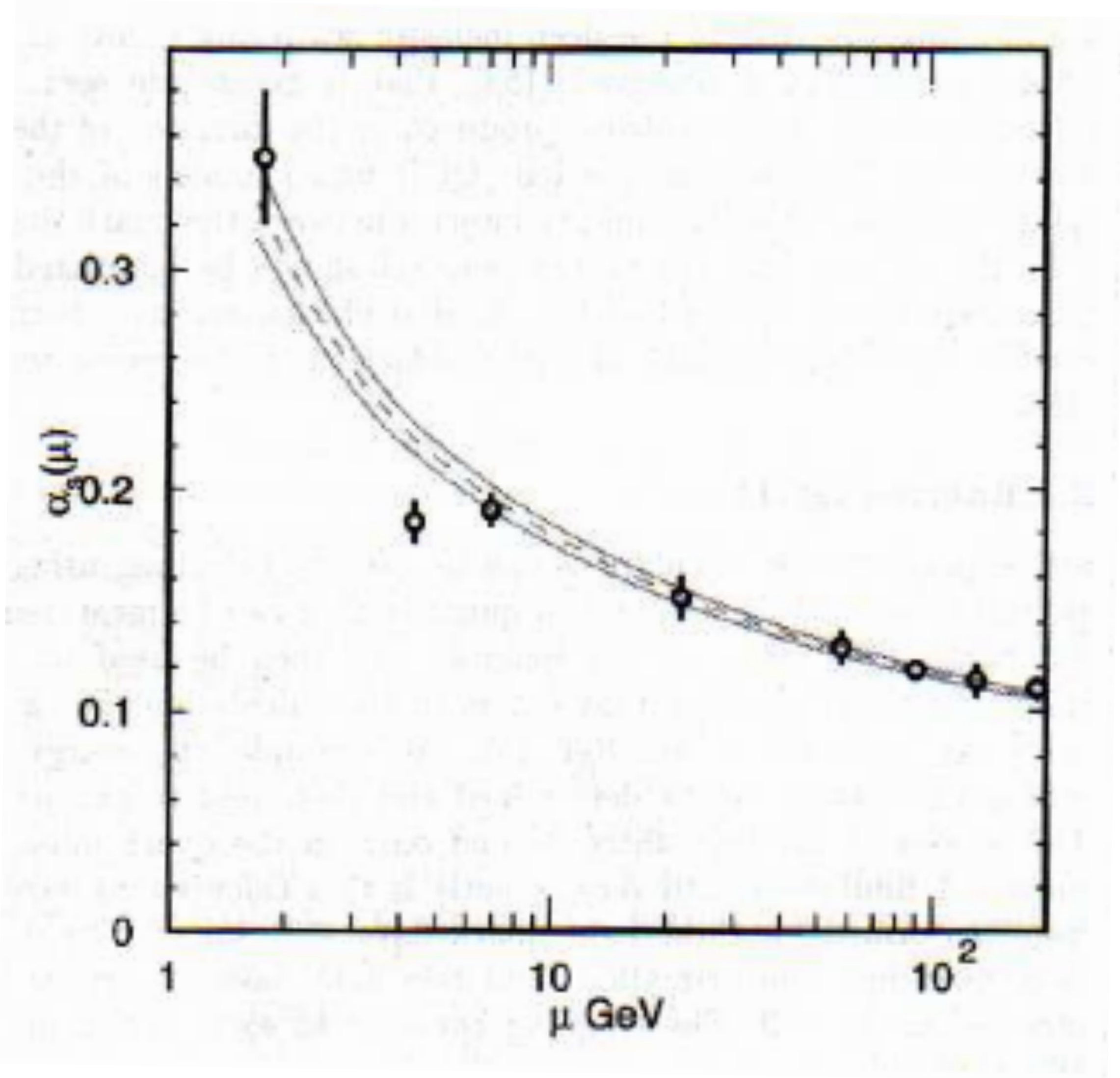
Quark-quark antisymmetric states
triplet, e.g. (r b − b r)

$c_f = -4/3$

↑

Applies to gluon exchange in a baryon

Strong Coupling α_s



$\alpha_s = g_s^{\ 2}$ is a steep function of energy scale μ (renormalization scale)

α_s~ 1 at low energy

α_s~ 0.1 at $\mu = M_Z$

Description of Running of α_s

Reminder – running of α_{EM} was attributed to screening of electric charge by fermion-antifermion pairs:

$$\alpha_{EM}(q^2) = \alpha(\mu^2) \left(1 - \alpha(\mu^2) \frac{z_f}{3\pi} \ln (|q|^2/\mu^2) \right)^{-1}$$

Running of α_s is attributed to:

1) Screening of colour charge by quark-antiquark pairs
2) Anti-screening of colour charge by gluons

$$\alpha_s(q^2) = 12\pi \left((33 - 2N_f) \ln (|q|^2/\Lambda^2) \right)^{-1}$$

$\Lambda_{QCD} \sim 220$ MeV

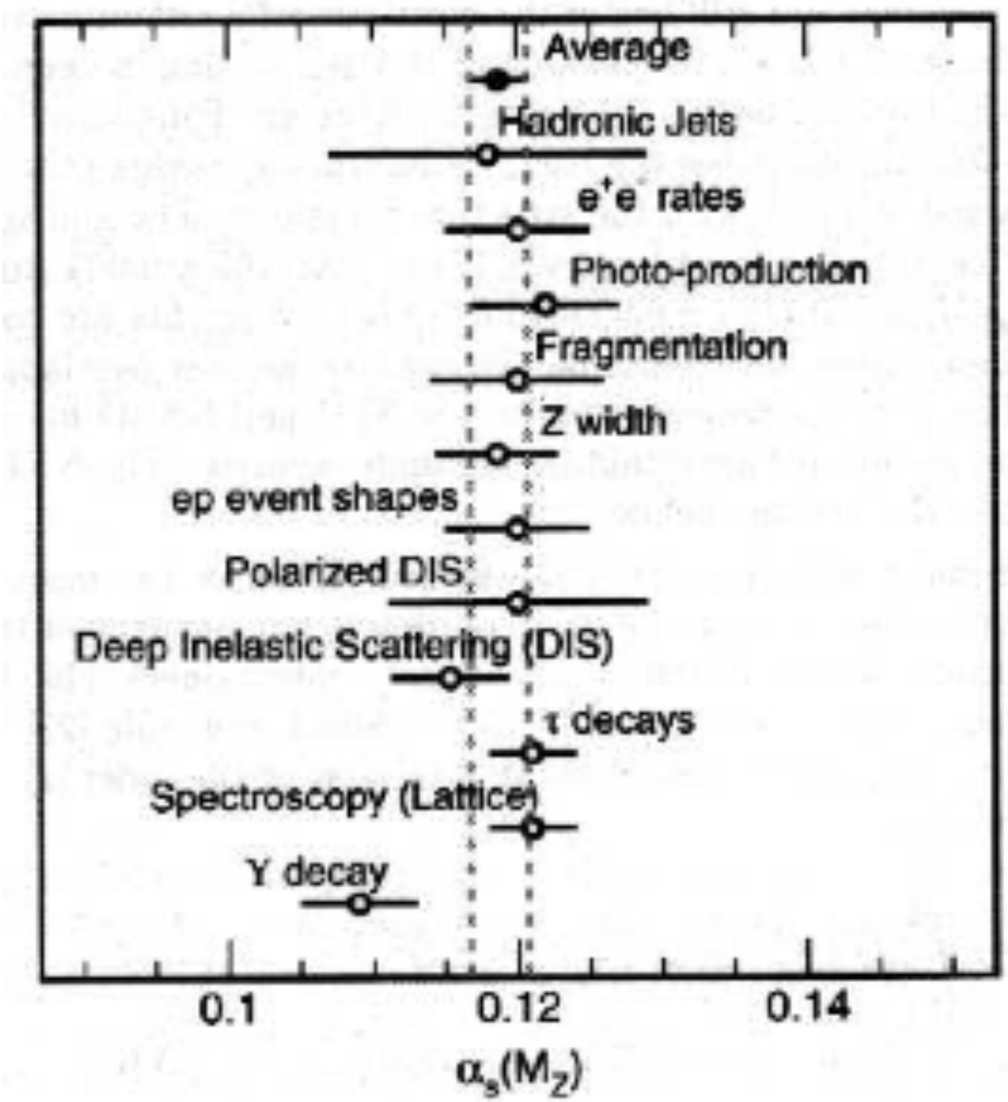
$$\ln(\Lambda^2) = \ln(\mu^2) - 12\pi \left((33 - 2N_f) \alpha_s(\mu^2) \right)^{-1}$$

$N_f = 2\text{-}6$ is the number of "active" quark flavours (depends on q^2)

Anti-screening by gluons dominates

Leads to a decrease of α_s as a function of q^2

Measurements of $\alpha_s (M_Z)$



Hadronic jets (Lecture 10)

$e^+e^- \rightarrow$ hadrons (Lecture 10)

Z width (Lecture 15)

ep event shapes & Deep Inelastic Scattering (Lecture 7)

Tau decays (Lecture 11)

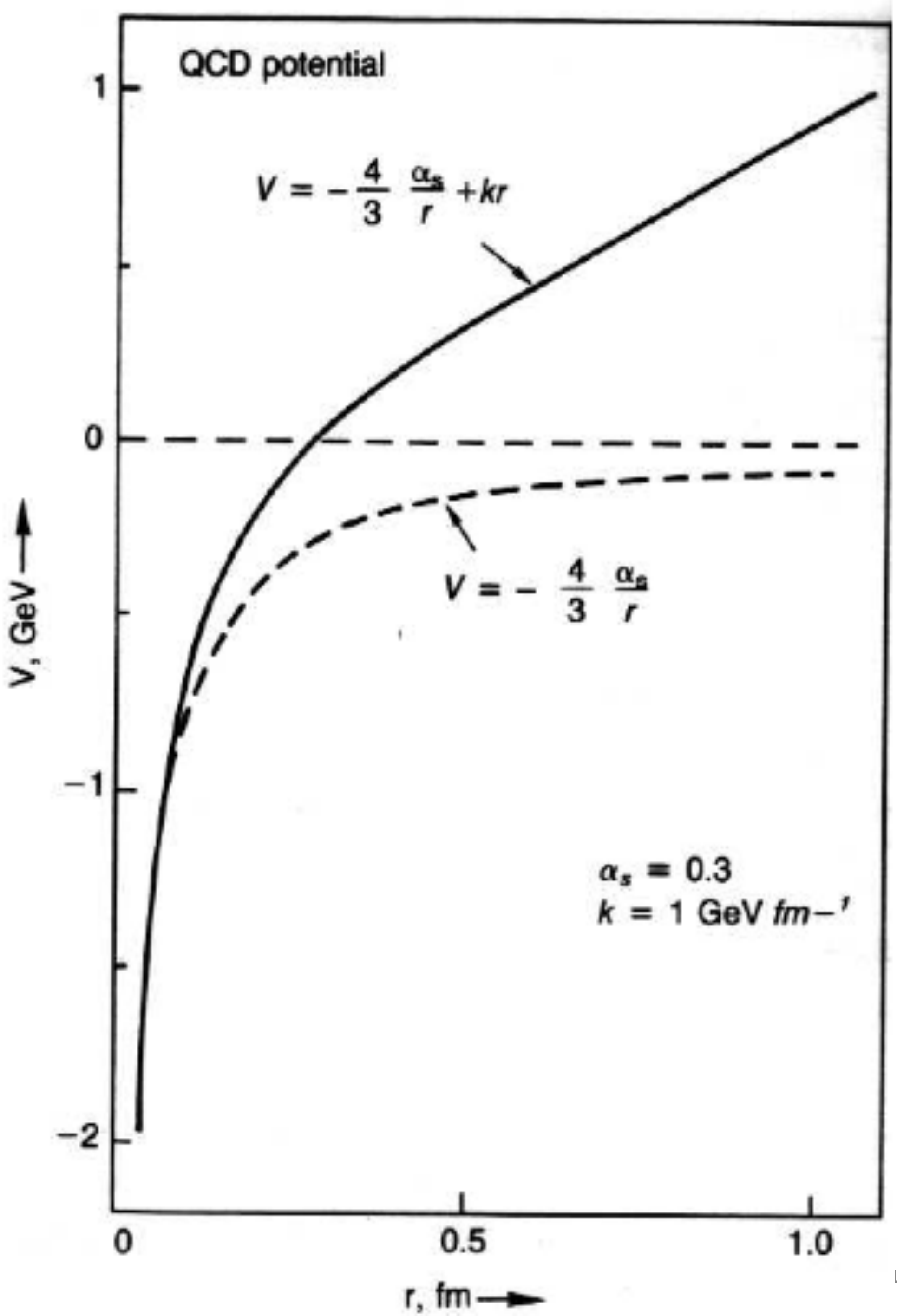
Hadron spectroscopy & decays (Lectures 9 & 11)

Azymptotic Freedom

- At large $q^2 >> \Lambda^2$ the strong coupling α_s is significantly < 1
  - In this limit it is possible to calculate strong amplitudes perturbatively
  - Sum over expansion in powers of α_s : leading order, next-to-leading order (NLO), next-to-next-to-leading order (NNLO) …
- Large q^2 corresponds to short distances << 1 fm
  - Cannot describe mesons and baryons perturbatively
  - Can describe high energy collisions perturbatively
  - Heavy quark decays (Lecture 11) are somewhere between these limits
  - Fragmentation of partons to form hadronic jets (Lecture 10) is another example of an intermediate case
- Deep inside hadrons the quarks and gluons do behave like free particles. Hence the validity of the parton model for high energy proton collisions.

Confinement

- At small $q^2 \sim \Lambda^2$ the strong coupling α_s is large (and diverging)
  - In this limit it is not possible to calculate strong amplitudes perturbatively
- Corresponds to distances ~ 1 fm
  - This is the size of mesons and baryons
- Non-perturbative calculations of strong interactions are done using numerical methods (Lattice QCD)
- The quarks and gluons are no longer free particles inside hadrons
  - What is the mechanism that confines them?
  - Why are mesons and baryons the only allowed bound states?
  - Why are strong interactions between hadrons short range (~1fm) even though the gluon is massless?
- Consider a set of models of confinement …



QCD Potential

Short distance part (1/r term)
from quark-antiquark gluon exchange

$$V(q\bar{q}) = -\frac{4}{3}\frac{\alpha_s}{r} + k\,r$$

Long distance part (k r term)
is modelled on an elastic spring

k is known as the string tension

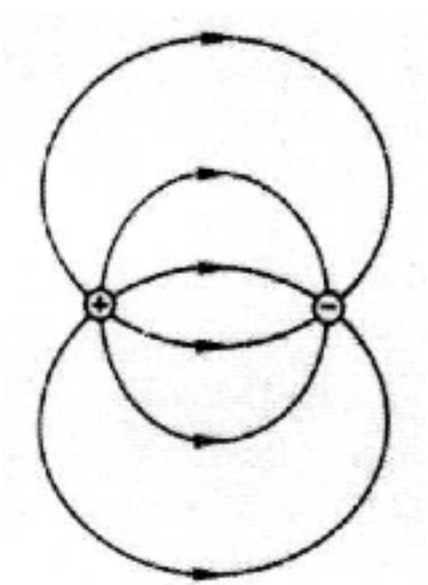
This model provides a good description
of the bound states of heavy quarks:
charmonium ($c\bar{c}$)
bottomonium ($b\bar{b}$)

Colour Flux-tube Model

QED

Field lines extend out to infinity with strength $1/r^2$

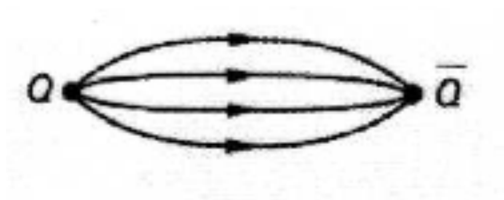
Electromagnetic flux conserved to infinity



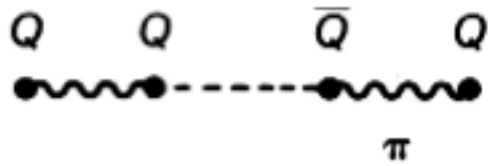
QCD

Field lines are compressed into region between quark and antiquark

Colour flux is confined within a tube. No strong interactions outside the flux-tube .



Breaking a flux tube requires the creation of a quark-antiquark pair



Like breaking a string! Requires energy to overcome string tension

Valence Quark Model

- Mesons are quark-antiquark bound states
  - Symmetric colour singlet state:

$$1/\sqrt{3}\ [\, r\,\bar{r} + b\,\bar{b} + g\,\bar{g}\,]$$

  - Colour singlet does not couple to a gluon (no G_0 gluon state)
- Baryons are three quark bound states
  - Antisymmetric colour singlet state:

$$1/\sqrt{6}\ [\, r\,g\,b - r\,b\,g + b\,r\,g - b\,g\,r + g\,b\,r - g\,r\,b\,]$$

  - Also does not couple to gluon because colour singlet
- Gluon exchanges only occur inside mesons and baryons
- Model ignores sea quarks and gluons (they don't matter at low q^2)
- Are there other types of colour singlet bound states?
  - Some evidence for glueballs (gg, ggg) as predicted by Lattice QCD
  - Hybrid mesons ($q\,\bar{q}\,g$), four quark states ($q\bar{q}\,q\bar{q}$), Pentaquarks ($qqq\,q\bar{q}$) ?