Hamiltonian Dynamics

[PHYS11012]

R. Horsley

email: rhorsley@ph.ed.ac.uk

room: JCMB 4419

tel: (0131)-650-6481

Monday 10:00-10:50 lecture (LTC)

Thursday 10:00-10:50 lecture (LTC)

Friday 16:10-18:00 workshop (5327, wk 2 onwards)

[wk 1 16:10-17:00 lecture (LTC)]

TA: Matthew Thomas

12 January 2026

Contents

Syllabus

- Review of Lagrangian dynamics
  Lagrangian equations of motion, generalised coordinates, constants of motion, symmetries and Noether's theorem
- Hamiltonian dynamics
  Legendre transformations, Hamilton's equations, conservative systems, phase space, Liouville's Theorem
- Qualitative dynamics
  Classification of the behaviours of 1 and 2 dimensional continuous time autonomous systems via linearisation of the system. Fixed points and limit cycles, simple Preditor–Prey systems, Rayleigh's equation, Hamiltonian systems (eg vertical pendulum)
- Canonical Transformations
  Variational principles, point transformations and generating functions, sympletic structure, Poisson brackets, infinitesimal canonical transformations, relation to symmetries and constants of motion (eg momentum, angular momentum, energy)
- Hamilton–Jacobi theory
  Hamilton–Jacobi equation, Hamilton's principal function, Hamilton's characteristic function, separation of variables, connection with quantum mechanics
- Action–Angle variables
  Constants of motion and integrability, libration and rotation, phase diagrams, frequencies and periods of motion, the harmonic oscillator and Kepler problem
- Canonical Perturbation theory
  Simple minded time dependent and time independent perturbation theory, adiabatic invariants, the harmonic oscillator and vertical pendulum, the KAM theorem (descriptive)

A highly formal course developing tools/language necessary for the study of mechanical systems with a finite number of degrees of freedom.

Books

- H. Goldstein, C. Poole and J. Safko,
  Classical Mechanics, (Addison–Wesley)
- L. N. Hand and J. D. Finch,
  Introduction to Dynamics, (CUP)
- T. W. B. Kibble and F. H. Berkshire,
  Classical Mechanics, (Imperial College Press)
- I. Percival and D. Richards,
  Introduction to Dynamics, (CUP)

Further material:

- Integrability and chaos: the classical uncertainty, J. Masoliver and A. Ros, arXiv:1012.4384 [nlin.CD].

Generating Function	Derivatives		K	Trivial case
$F_1(\vec{q},\vec{Q},t)$	$p_i=\frac{\partial F_1}{\partial q_i}$	$P_i=-\frac{\partial F_1}{\partial Q_i}$	$H+\frac{\partial F_1}{\partial t}$	$F_1=\vec{q}\cdot\vec{Q}\Rightarrow(\vec{Q},\vec{P})=(\vec{p},-\vec{q})$
$F_2(\vec{q},\vec{P},t)$	$p_i=\frac{\partial F_2}{\partial q_i}$	$Q_i=\frac{\partial F_2}{\partial P_i}$	$H+\frac{\partial F_2}{\partial t}$	$F_2=\vec{q}\cdot\vec{P}\Rightarrow(\vec{Q},\vec{P})=(\vec{q},\vec{p})$
$F_3(\vec{p},\vec{Q},t)$	$q_i=-\frac{\partial F_3}{\partial p_i}$	$P_i=-\frac{\partial F_3}{\partial Q_i}$	$H+\frac{\partial F_3}{\partial t}$	$F_3=\vec{p}\cdot\vec{Q}\Rightarrow(\vec{Q},\vec{P})=(-\vec{q},-\vec{p})$
$F_4(\vec{p},\vec{P},t)$	$q_i=-\frac{\partial F_4}{\partial p_i}$	$Q_i=\frac{\partial F_4}{\partial P_i}$	$H+\frac{\partial F_4}{\partial t}$	$F_4=\vec{p}\cdot\vec{P}\Rightarrow(\vec{Q},\vec{P})=(\vec{p},-\vec{q})$

Table 1: Properties of the four basic Canonical Transformations $(\vec{q},\vec{p})\to(\vec{Q},\vec{P})$.

Conversion factors from SI units to Heaviside–Lorentz units

$$\vec{E}=\sqrt{\epsilon_0}\vec{E}_{SI}\,,\qquad \vec{B}=c\sqrt{\epsilon_0}\vec{B}_{SI}\,,\qquad \rho=\frac{1}{\sqrt{\epsilon_0}}\rho_{SI}\,,\qquad \vec{j}=\frac{1}{\sqrt{\epsilon_0}}\vec{j}_{SI}\,,\qquad [e=\frac{1}{\sqrt{\epsilon_0}}e_{SI}]\,,$$

($c=1/\sqrt{\epsilon_0\mu_0}$) giving Maxwell's equations as

$$\begin{aligned}\vec{\nabla}\cdot\vec{E} &= \rho\\ \vec{\nabla}\times\vec{E}+\frac{1}{c}\frac{\partial\vec{B}}{\partial t} &= 0\\ \vec{\nabla}\cdot\vec{B} &= 0\\ \vec{\nabla}\times\vec{B} &= \frac{1}{c}\vec{j}+\frac{1}{c}\frac{\partial\vec{E}}{\partial t}\end{aligned}$$

and

$$\vec{F}=e\left[\vec{E}+\frac{\dot{\vec{r}}}{c}\times\vec{B}\right]\,.$$

Also with

$$\phi=\sqrt{\epsilon_0}\phi_{SI}\,,\qquad \vec{A}=c\sqrt{\epsilon_0}\vec{A}_{SI}$$

then

$$\vec{E}=-\vec{\nabla}\phi-\frac{1}{c}\dot{\vec{A}},\qquad \vec{B}=\vec{\nabla}\times\vec{A},$$

and

$$V=e\left(\phi-\frac{\dot{\vec{r}}}{c}\cdot\vec{A}\right)\,.$$