A higher derivative/Lee-Wick Standard Model

Roman Zwicky (Southampton)
January 26 2010, Mainz
Overview

- general introduction -- hierarchy problem - Higgs composite or fund.?

- Lee-Wick QFT -- toy model $L \sim \varphi^3$
  1. Higher derivative formulation
  2. Auxiliary field formulation

- tree-level unitarity -- an instructive example

- The Lee-Wick Standard Model (construction principle)
  - absence of quadratic divergences
  - fermion sector
  - electroweak precision observables (oblique formulation)
  - LHC phenomenology I & II

- Ideas behind unitarity -- acausality (brief)

- Epilogue formal & pheno aspects
Introduction...

ca ’69 renormalization theory not popular with all field theorists
prior to: Wilson view renormalization (effective theories)
‘t Hooft & Veltman renormalization spont.-broken gauge theories
strong interaction as a gauge theory known as QCD

ca ’07 omnipresent hierarchy problem ...
inspires most new physics models (not flavour ...)
Grinstein O’Connel Wise ... apply Lee-Wick field theories to SM
⇒ only logarithmically divergent (cure to hierarchy problem)
Lee-Wick field theories

a theory with (unobservable) ghosts renders QED finite

Field theory laboratory

.. only candidate: non-trivial acausal QFT

Curious (LW)SM phenomenology
Grinstein, O’Connel, Wise 2007

$|V_{tb}| > 1$ !!
interference effects
“solution” to hierarchy problem

unitary?
Hierarchy Problem

If SM viewed as an effective theory (Wilson)
\[ \mathcal{L}_{\Lambda_{EW}}^{\text{eff}} = m_H^2 h_0^2 \]

scalar mass term is relevant operator (grows at low energy)
\[ \Rightarrow \Lambda_{EW} \sim m_H \text{ appears accidental/unnatural} \]

Viewpoint from perturbation theory -- 1-loop corrections
\[ \Lambda_{\text{cut-off}} = 10 \text{ TeV} \]
\[ \Delta m_H = -2 \text{ TeV} \]
\[ \text{top} \]
\[ \text{W,Z} \]
\[ \text{higgs} \]
\[ 0.7 \text{ TeV} \]
\[ 0.5 \text{ TeV} \]

“Cure”: something ought to happen \( E_{\text{new}} \approx O(1) \Lambda_{EW} \)
EWPO often \( \Rightarrow E_{\text{new}} \approx O(10) \Lambda_{EW} \) “little hierarchy problem”
In fact: why there really ought to be something ..... 

\[ W_L - W_L \text{ scattering} \]

\[ A_0 = \frac{g_2^2}{16\pi m_W^2} s \]

\[ s_{\text{crit}} \sim (1.2 \text{ TeV})^2 \]

\[ \delta_H A_0 = \frac{g_2^2}{16\pi m_W^2} s \]

"perturbative unitarity"

S-wave

Unitarity bound

\[ \text{Re}[A_0] \leq \frac{1}{2} \]

Standard (Model) solution
Beyond the SM centered around the Higgs mechanism of SSB
⇒ W,Z masses; hierarchy problem?

**fundamental particle**
small width

**composite particle**
large width

SSB SU(2)$_L$
Higgs-mechanism

$<q_R Q_L> \sim (\Lambda_{EW})^3$

need cancelation mechanism

hierarchy problem

strong dynamics

quantum corrections cut off by analogue of $\Lambda_{QCD}$

**Supersymmetry**
opposite statistics partner

**Examples**

**Technicolour**
Higgs sector ⇒ Gauge theory

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes G_{TC}$
... time to enter Lee-Wick field theories

Toy Model: $\varphi^3$

A. Higher derivative formulation
B. Auxiliary field formulation
A. Higher derivative formulation

Add higher derivative term to $L$

$$L_{\text{hd}} = -\frac{1}{2}\phi \partial^2 \phi - \frac{1}{2}m^2 \phi^2 - \frac{1}{2M^2}(\partial^2 \phi)^2 - \lambda \phi^3$$

Propagator:

$$\hat{D}(p) = \frac{1}{p^2 - p^4/M^2 - m^2} \sim \frac{1}{(p^2 - m_{\text{ph}}^2)} - \frac{1}{(p^2 - M_{\text{ph}}^2)}$$

- two poles -- one “wrong” sign residue (PV ghost)
- improved convergence diagrams
B. Auxiliary field formulation

Introduce auxiliary field $\Phi$
(equivalent Lagrangian)

$$\mathcal{L}_{\text{eff}} = \begin{pmatrix} \phi \\ \phi_{\text{LW}} \end{pmatrix} \left[ \partial^2 \begin{pmatrix} 1 & 0 \\ 0 & \mathbf{1} \end{pmatrix} + \begin{pmatrix} m^2 & -m^2 \\ -m^2 & m^2 + M^2 \end{pmatrix} \right] \begin{pmatrix} \phi \\ \phi_{\text{LW}} \end{pmatrix} + \lambda(\phi - \phi_{\text{LW}})^3$$

shift to physical fields couplings & masses
Improved convergence: e.g. self energy

\[ \Sigma(p^2) \sim \int \frac{1}{k^2 - m^2} \ominus \int \frac{1}{k^2 - M^2} = \int \frac{(m^2 - M^2)}{(k^2 - m^2)(k^2 - M^2)} \]

```
\[ \phi \] + [\text{\(\phi_{LW}\)}}
```

“only” log-divergent

Hasty impression (AF-formalism):

P₁: double number of fields
P₂: power counting improves: \#(inter lines) (-2)
Higher derivative (HD) formulation:

- construction principle of LWSM
- power counting (divergences)

Auxiliary field (AF) formulation:

- explicit calculations
- contact with experiment

use them both ⇒ check your results 😊
Interpretation: First example

s-channel unitarity (1-loop)

\[ \tilde{D}(p^2) = \frac{-1}{p^2 - M^2} + \frac{-1}{p^2 - M^2} \Sigma(p^2) \frac{-1}{p^2 - M^2} + \ldots = \frac{-1}{p^2 - M^2 + \Sigma(p^2)} \]

\[ 2 \text{Im} \tilde{D}(p^2) = \sim (-)^{\text{prop}} \text{Im}[-\Sigma(p^2)] = (-)^2 M \Gamma > 0 \]

unitarity? causality?

sum all bubbles (use narrow width approximation)

Im[\tilde{D}(p^2)] \sim (-)^{\text{prop}} \text{Im}[-\Sigma(p^2)] = (-)^2 M \Gamma > 0

(-1)^2 = 1

negative residue and width cancel!
The Lee-Wick Standard Model (LWSM)

Construction principle in HD formalism

\[
\delta \mathcal{L}_{\text{scalar}} = -\frac{1}{M_\phi^2} \left| \hat{D}^2 \phi \right|^2 \\
\delta \mathcal{L}_{\chi-\text{fermion}} = +\frac{1}{M_\psi^2} \bar{\psi} i \hat{D} \hat{\gamma} \hat{\gamma} \psi \\
\delta \mathcal{L}_{\text{gauge}} = +\frac{1}{M_A^2} \text{Tr} (\hat{D} \hat{F})^2_{\mu}
\]

- gauge invariance (minimal coupling) enforces derivatives in \( \mathcal{L}_{\text{int}} \)
- \( \mathcal{L}_{\text{gauge}} \) could be used as PV regulator for non-abelian gauge theories

Grinstein, O’Connel, Wise (0704.1845)
Absence of quadratic divergences -- HD formalism

- Top and Higgs loop: no new "∂" simply log convergent
- Gauge boson loop: new "∂" ⇒ closer look

\[ d_{div} = 6 - 2L - E_S - E_V - 2E_g = 2 \]

\[ \mathcal{L}^{int} = h^\dagger D^4 h \sim h^\dagger (\partial^2 + \partial \cdot A + A \cdot \partial + A^2)^2 h \]

- In Lorentz gauge (\( \partial \cdot A = 0 \)) & partial-∫ ⇒ log-convergent

Grinstein, O’Connel, Wise (0704.1845)
Flavour sector ... CKM -- AF formalism

massive fermion = 2×chiral femion
⇒ three LW fermion generations !!

\begin{align*}
Q_L & \leftrightarrow (\tilde{Q}_L, \tilde{Q}_R') \\
q_R & \leftrightarrow (\tilde{q}_L', \tilde{q}_R)
\end{align*}

\[
\begin{pmatrix}
m_t & -m_t & 0 \\
-m_t & m_t & -M_t \\
0 & -M_T & 0
\end{pmatrix}
\begin{pmatrix}
m_t & -m_t & 0 \\
-m_t & m_t & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

top mass matrix
top Yukawa matrix

are not simultaneously diagonal

non-unitary correction to CKM matrix (generalize unitarity)

e.g. \(|V_{tx}| \rightarrow |V_{tx}|(1 + \frac{1}{2} \left(\frac{m_t}{m_{\tilde{t}}}\right)^2) > 1\) possible (x=t)

Krauss, Underwood, RZ '07
Electroweak (precision) constraints

Energy

low \quad M_Z \quad above \quad LHC (M_W..) \quad ILC?

- low: $\nu$-e, $\nu$-N scattering, atomic parity
- Z-pole: LEP1/SLC asymmetries FB, polarization initial/final state scattering
- above: LEP2 asymmetries/cross section (contact interactions)

Generally: trade EW parameters ($g_1, g_2, \nu$) for three best measured parameters

$m_Z = 91.1976(21)$ GeV, $\alpha^{-1}(m_Z) = 127.918(18)$, $G_F = 1.16637(1) \times 10^{-5}$ GeV$^{-2}$

Quantity $X$: $X = X(G_{F}^{SM}, ... | M_{LW}, ...) = X(G_{SM}, M_{LW}, ...)$

1. Lagrangian (“theory”) \quad 2. $G_F = G_{F}^{SM}(1 + \delta_{LW})$ (“experiment”)
Work out effective Lagrangians (step 1)

\[ \mathcal{L}_Z^{\text{eff}} = - \left( \rho_f \sqrt{2} G_F \right)^{1/2} 2 m_Z J_Z \cdot Z, \quad J_Z^\mu \equiv (J_3 - s_\ast^2 (m_Z) J_Q)^\mu \]

\[ \mathcal{L}_{\text{Low}}^{\text{eff}} = - \frac{4 G_F}{\sqrt{2}} \left( J_+ \cdot J_+ + \rho_\ast (0) J_{nc}^2 \right) + C_Q J_Q^2 \quad J_{nc}^\mu \equiv (J_3 - s_\ast^2 (0) J_Q)^\mu \]

\[ \mathcal{L}_{\text{QED}}^{\text{eff}} = - \frac{1}{16 \pi \alpha} F^2 + .. \]

⭐ Out of \((G_F, m_Z, \alpha)\) only \(m_Z\) affected

⭐ Low energy not affected (formally) -- ” \(\partial^4 / M_{\text{LW}}^2 \Rightarrow 0\)”

Match with experiment (step 2)

\[ s_W^2 c_W^2 = \frac{\pi \alpha (M_Z)}{\sqrt{2} G_F M_Z^2} \frac{(1 + \delta_G)(1 + \delta_Z)}{1 + \delta_\alpha} \]
perform the $\chi^2$-fit ..... 

traces of the "little hierarchy" problem?
Oblique formalism (S,T,U)

Kennedy & Lynn, Altarelli & Barbieri, Peskin & Takeuchi ∼90’

⭐ If “new physics” in (W,Z) self energy (e.g. 4th family, technicolor)

⭐ If $\varepsilon = (M_W / M_{\text{New}})^2 << 1 \Rightarrow$ oblique approximation

$$\Pi_{ab}(\epsilon) = \Pi_{ab}(0) + \Pi'_{ab}(0)\epsilon + O(\epsilon^2)$$  

$ab \in \{BB, B3, 33, 11\}$

⭐ $\Rightarrow 8 - 3(G_F, m_Z, \alpha) - 2_{U(1)} = 3$ parameters S, (T, U)

e.g.  

$$\rho_*(0) = 1 + \alpha T ; \quad T = \frac{1}{M_W^2 \alpha}(\Pi_{33}(0) - \Pi_{11}(0))$$
Non-universal models ......

★ additional weak gauge bosons \( W, \tilde{Z} \) -- \( \mathcal{J} \)-out

\[
\int -\text{out} \quad \tilde{W}, \tilde{Z}
\]

\[ W_{\text{out}} = \alpha W + \beta \tilde{W} \]

\[ W_{\text{out}} \perp W_{\text{in}} \quad \mathcal{L}_{\text{eff}} \sim W_{\text{in}} \cdot J_{\text{lep}}^\text{SM} \]

⇒ theory “universal” form -- “universalized”

★ prize to pay: one higher order ⇒ 3 + 4 = 7 parameters \((S,X) (T,U,V) (W,Y)\)

\[
\Pi_{ab}(\epsilon) = \Pi_{ab}(0) + \Pi'_{ab}(0)\epsilon + \frac{1}{2}\Pi''_{ab}(0)\epsilon^2 + \mathcal{O}(\epsilon^3)
\]

Strumia’99 .......... Barbieri,Pomarol,Rattazi,Strumia’04

captures contact terms of LEP2
LWSM (AF) formalism

- Belongs to this class of models
- “Universalize” AF formulation $\Rightarrow$ HD formulation!!

\[
\Pi(q^2)_{33} = q^2 - m_W^2 - \frac{q^4}{M^2} ; \quad W = \frac{M_W^2}{2} \Pi'''_{33}(0) = -\frac{m_W^2}{M^2}
\]

**All leading order terms**

\[
S=0, \quad T=0, \quad W = - (m_W/M_2)^2, \quad Y = - (m_W/M_1)^2
\]

- Expand (previous) tree-level exact results in $\varepsilon$ -- they do agree!
- N.B. carefully avoided “custodial symmetry”; ambiguous $g_1 \neq 0$!
- Similar models x-dim. LW $\sim$ KK; but $W,Y \sim (m_W/M_{KK})^2 > 0$ no ghost :)

LHC Pheno I -- $gg \rightarrow h_0 \rightarrow \gamma\gamma$

- Low Higgs mass discovery channel sensitive to heavy quarks i.e. top

$$A \sim (1 \pm O(1/m_{LW}^p))$$

1. ghost reduction  
2. p=2 ; fast quadratic decoupling $(m_W/m_{LW})^2$ -- could have been linear ....

- scaling derived  
- hyperbolic rotation $(V_{tx}) +$

- 1. sign 2. scaling
LHC Pheno II -- $qq \rightarrow \tilde{Z}(\tilde{W}) \rightarrow ll(\nu)X$

Can LW gauge bosons be “uniquely” identified?

- wrong sign
  - 1. residue
  - 2. width

2. width $\sim 2$ GeV small -- at least $\Gamma_{LW}/M_W$ suppressed + hadronic env.

1. measure cross section rather than amplitude!

$$\sigma \sim (\lambda_{qqZ}^2 \lambda_{llZ}^2 \cdot \lambda_{qqZ} \lambda_{llZ})$$

obvious formal cure: same initial final state vertices!

gauge bosons to two jets (dijets) -- challenging background

Bhabba scattering $e^+e^- \rightarrow e^+e^-$ for ILC

Answer: yes!
Idea behind unitarity

negative norm states & unitarity are in straight contradiction

have seen tree-level (improved pert. th) works ... but how about loops? .

basic-picture: S-matrix block-diagonal on particle/ghost-space

\[ S = \begin{pmatrix} S & 0 \\ 0 & X \end{pmatrix} \]  

\[ S^\dagger S = 1 \]

\[ e \neq 0, \text{ the states } \{|e^+ e^-\rangle_{m_{\tilde{A}}}, |\tilde{A}\rangle\} \text{ mix for } m_{\tilde{A}} > 2m_e \]

\[ \mathcal{H}^{int} = \begin{pmatrix} 0 & i\Gamma/2 \\ i\Gamma/2 & \Gamma \end{pmatrix}, \quad \Gamma = \frac{1}{3} \alpha m_{\tilde{A}} \]

\[ \Rightarrow |\pm\rangle \sim (|\tilde{A}\rangle \pm |e^+ e^-\rangle_{m_{\tilde{A}}}) \text{ with } E_c = m_{\tilde{A}} \pm i\Gamma/2 \]

negative norm states have complex energies
Viewpoint of analyticity
(rather non-analytic points:)

positive norm: \[ M_c^2 = M_{LW}^2 - iM_{LW}\Gamma \]

negative norm:

\[ i\Delta(q^2) = \frac{1}{q^2 - M_c^2} + \frac{1}{q^2 - M_c^{*2}} + \int_{4m^2} ds \rho(s) \frac{q^2 - M_c}{s - q^2 - i0} \quad q^2 \approx M_c^2 \gg \Gamma^2 \quad \frac{1}{q^2 - M_c^2} \]
Ready for Loops
(rather non-analytic points:)

\[ i \Delta(q^2) = \frac{1}{q^2 - M_c^2} + \frac{1}{q^2 - M_c^*^2} + \int_{4m^2} \frac{ds \rho(s)}{s - q^2 - i0} \underset{q^2 \approx M^2 \gg \Gamma^2}{\rightarrow} \frac{1}{q^2 - M_c^2} \]

\[ \int_p \Delta(p^2) \Delta((p + q)^2) \]

Examples: Lee-Wick poles in \( C_{p0} \)

\[ p_0 = \pm \sqrt{p^2 + M_c^2} \]

\[ p_0 = \pm \sqrt{p^2 + M_c^*^2} \]

Status: no examples known in literature where this does not work!
(A)Causality

(In proofs of analyticity causality is ingredient ...may expect trouble)

- Classical level: higher derivatives .. need future BC to eliminate unstable mode:
- Quantum level: acausality is microscopic phenomenon

Distance of backward propagation prop. to $1/\Gamma_{\text{LW}}$ .. microscopic

Alvarez et al '09 proposal to measure with primary & secondary vertex ... lee-wick electron cand. since it is lightest.
Epilogue I (formal)

Unitarity to all order in perturbation theory?

Non-perturbative definition via canonical formalism does not yield CLOW prescription

Boulware, Gross ‘84

Is LWFT Lorentz invariant?
No with LW prescription -- Yes? with CLOW prescription

Is LWFT effective description of sthg beyond field theory?
(borne out in desperation of the hierarchy problem)

Proposal path-∫ quantum fields restricted test fcts space yields CLOW prescription

Tonder, Dorca ‘06

Not discussed high T physics e.o.s. w=1 (p = w ρ) ......
Epilogue II (pheno)

🌟 2007 GOW extended LWFT to SM: chiral fermions -- non-abelian gt
counting of d.o.f. not trivial when \( M_{SM} = 0 \) (\( M_{LW} \neq 0 \) by construction)

🌟 not suprisingly LWSM has curious phenomenology
e.g. possible \( |V_{tb}| >1 \)
negative residue “cought” in: \( xx \rightarrow Z_{LW} \rightarrow xx ; xx=(qq,ll) \)

🌟 EWPO constrains ~ 2-3TeV for weak LW gauge bosons
again distinguished other models e.g. x-dim

🌟 Issues left out:
LWFT as lattice regulator (avoids discretization effects cut-off) used 1990
LWFT running of beta-fct, LWFT & massive vector boson scattering

Merci pour votre attention!