Discrete Groups and Flavour Physics *

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The rare and the beautiful -- Wylerfescht

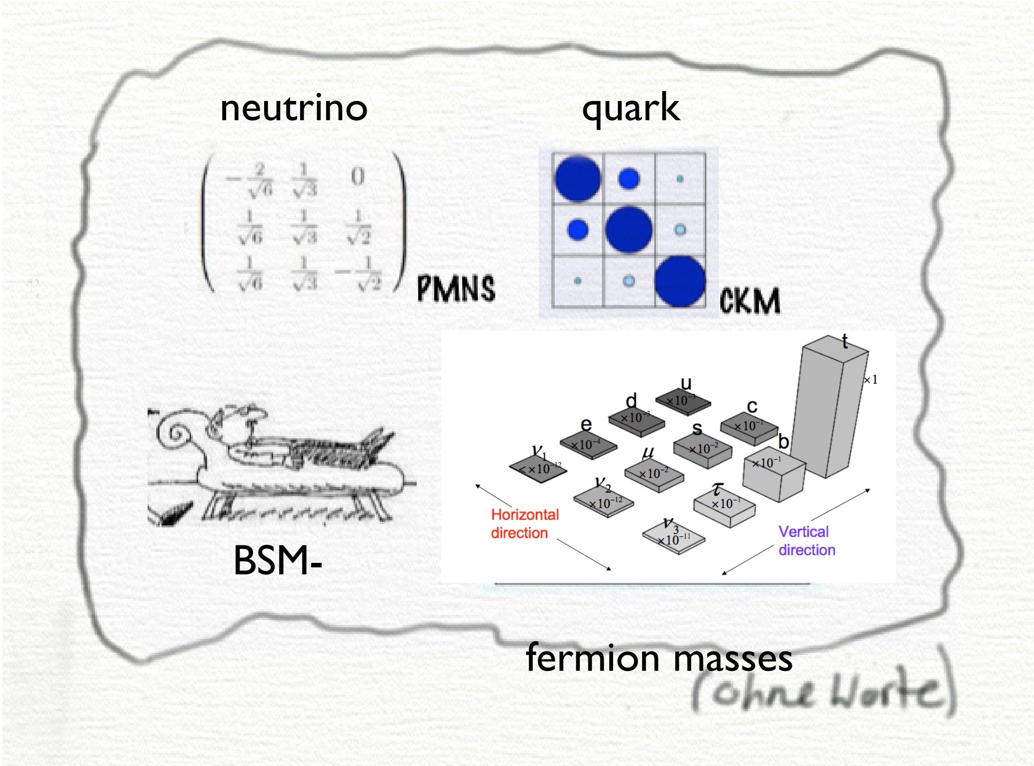
Overview

- Discrete groups -- discrete SU(3) subgroups
- A_4 an example -- 79' model building
  02'.. tri-bi-mixing lepton sector

conceptual cut -- discrete groups link

- Discrete Minimal Flavour Violation

Why discrete groups? ... Theory of flavour?



Are these patterns random?

no ← → yes

- Can (discrete/continuous) **family-symmetries** explain them?
  Who gave us the Yukawa matrices?

- Is the mechanism linked with the TeV-scale?
  (SM suggests that in a way: why $m_t \sim \Lambda_{EW}$?)

Def: discrete group = group with countable many elements

two facts (for general orientation):

- Any discrete group can be embedded into permutation group S_n (analogue manifold & R^n) *
- Order & irreducible representations (irreps): $|D| = \sum |\mathrm{irrep}(D)|^2$ ⇒ finite many of them

Which discrete groups are subgroups of SU(3)? (3 because of 3 famillies)

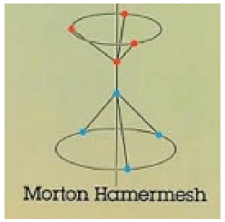
* 19th century used to be the definition (as opposed to abstract definition)

Discrete subgroups of SU(3)

- Classified in a classic book — Miller, Dickson, Blichfeld '1916
  Analyzed further eightfold way — Fairbairn, Fulton, Klink '64
  Further analyzed (lattice ...) — Bovier, Luling, Wyler '81 *
  Rescrutinized tri-bi-hype — Luhn, Nasri Ramand., '06-08

Trihedral like: $\Delta(3n^2)$ / $\Delta(6n^2)$

- $Z_n \times Z_n \rtimes Z_3$ / S_3
- largest irreps 3/6-dim
- Analogue Dihedral group (chemistry) $Z_n \rtimes Z_2$

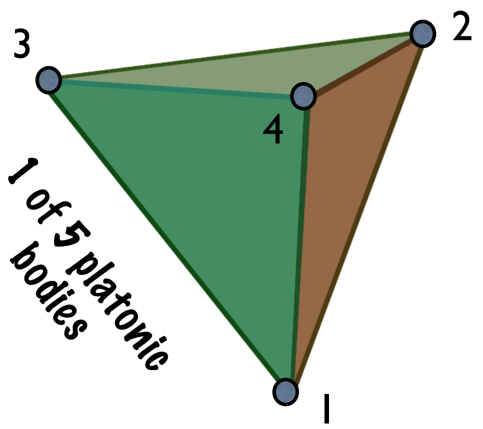


Crystallographic groups , Σ

- finite many of them
- maximal subgroups
  $\Sigma(168) \sim PSL(2,7)$
  $\Sigma(216\varphi)$ hessian group
  $\Sigma(360\varphi)$
  (φ=1,3 related center SU(3))
- used in lattice $SU(3)_{color}$ discretizations '80

* some confusion (D)-groups, reemphasized Ludl'09, argue (D)-groups embedded in $\Delta(6n^2)$ Fischbacher RZ'09

Tetrahedral group $\cong A_4$



- 4 corners, fix one e.g. 4 120°-roation: (4)(234) & (4)(243) = 8
  3 opposite edges with 180°-roation: (12)(34) .... = 3
  identity 1

  elements 12

  $\Rightarrow T \cong \mathbf{A_4}$ (*even four permutation*) = $\Delta(12)$

- **Algebraic def:** $S^2=1,\ T^3=1,\ (ST)^3=1$

- **Irreps:** $|A_4| = 4!/2 = 12 = |\mathbf{1}|^2 + |\mathbf{1'}|^2 + |\overline{\mathbf{1}}'|^2 + |\mathbf{3}|^2$

- Example Kronecker product: $\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{1'} + \overline{\mathbf{1}}' + \mathbf{3}_s + \mathbf{3}_a$

  denote: $\mathbf{3} \sim (x_1, x_2, x_3)$ & $\mathbf{3} \sim (y_1, y_2, y_3)$ $\mathbf{1'} \sim (\omega^2 x_1 y_1, \omega x_2 y_2, x_3 y_3)$ $\omega=\exp(2\pi i/3)$

  ; $T: \mathbf{1'} \rightarrow \omega^2\ \mathbf{1'}$

A_4 in model-building

Model building 70'
Tri-bi-maximal mixing '00

Connection? ... they were just first in line ....

A_4 quark sector in the '70

Wyler'79

- Not much known about 3rd generation (basically m_b) -- $\mathbf{\Theta_C \sim (m_d/m_s)^{1/2}}$ Cabibbo universality

a,b,c index family-symmetry
I^n invariant (constant tensor)

- Assume: $\mathcal{L}_{\rm Yuk} \sim \sum_n (I^n)_{abc} \bar{Q}_L^a D_R^b H^c$

- Need at least two invariants (o/w $m_d/m_u = m_s/m_c$) -- $\mathbf{3}^* \times \mathbf{3} = 2\,\mathbf{X}$ ... (not simply reducible)

- **A_4** candidate with low order ($\mathbf{3} \times \mathbf{3} = 2\,\mathbf{3} + ..$) ⇒ 2 invariants ⇒ 2 Yukawas (instead of 3)

- Work out (m_d, m_s, m_b) $\langle H \rangle = (v, v, v_3)$ with $v \ll v_3$ (⇒ Higgs potential add. family singlet)
  third mass adjusting the 2 Yukawas

Results:

- Cabibbo universality ✓
- $|V_{ub}|/|V_{cb}| \sim 10$ (reversed hierarchy)
- 2 Yukawa & 3 masses → relation: $m_d\, m_s / m_b^2 = m_u\, m_c / m_t^2 \Rightarrow m_t \approx 15\text{GeV}$

(not known '79)

Tri-bi-maximal mixing & A_4 or rather S_4

Data suggests (not exclude):

- Neutrino sector: 'know' mixing -- masses less known

$$U_{TB} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & +1/\sqrt{2} \end{pmatrix}$$

Harrison, Perskins, Scott '99'02

- Go into basis leptons are diagonal

$$\begin{aligned} M_\nu &= M^\nu_{\mathrm{TB}} \equiv U_{\mathrm{TB}}\,\mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^T_{\mathrm{TB}} \\ M_l &= \mathrm{diag}(m_e, m_\mu, m_\tau) \end{aligned}$$

- $AM_\nu A^\dagger = M_\nu$ for $A \in \mathbf{Z}_2 \times \mathbf{Z}_2$ generated S, U
- $AM_l A^\dagger = M_l$ for $A \in \mathbf{Z}_3$ generated by T

- The three generators S,T,U define **S_4** (and not A_4 ... but was a good start)
  Suggests (original) family symmetry S_4 (or any group containing S_4 e.g. PSL(2,7) ...)

- Model building **flavon** φ_T with T-invariant VEV, Frogatt-Nielsen etc ..

$$\mathcal{L}^L_{\mathrm{YUK}} \sim \Psi(\phi_T + \phi_0)\Psi^c$$

Minimal Flavour Violation

Is there something (very) special about the Yukawa matrices?

Minimal Flavour Violation

- **Yukawa** = 0 continuous global symmetry: $G_F = U(3)^5 = G_q \times G_l$, $G_q = U(3)_Q \times U(3)_{UR} \times U(3)_{DR}$
  Yukawa $\neq 0$ breaks down to: $G_q = U(3)_q^3 \to U(1)_{Baryon}$

- Let **Yukawa** formally transform as $Y_D \sim (3^*,1,3)_{Gq}$ & $Y_U \sim (3^*,3,1)_{Gq}$

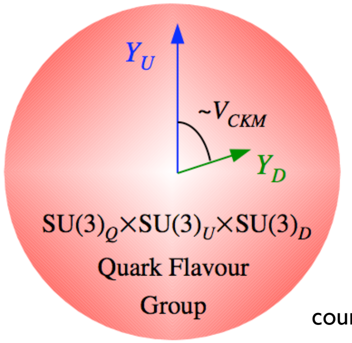
➡ G_F symmetry restored

MFV: effective field theory invariant under global G_F
(criterion of naturalness applied coefficient O(1))

D'Ambrosio, Giudice, Isidori & Strumia '02

- **Yukawa**'s promoted to spurions
  $<Y_{U/D}> \neq 0$ VEV breaks G_q

N.B. interpretation of symmetry breaking
other options a) explicit(soft) breaking
b) anomalous breaking



courtesy G.Isidori

CKM due to miss-allignement of Yukawa's

* Add. assumption: CP-invariance — AGIS'02
No new Lorentz structure — Buras, Gambino, Gorgahn Jager, Silvestrini ...'00

A few remarks on MFV

(partly skip)

- Restricts the number of operators: *(denote: D = (d,s,b))*

Dynamics without dynamics

$$O^{\Delta F=1'} = (\bar{D}_L Y_U Y_U^\dagger Y_D \, \sigma \cdot F D_R) \qquad b \to s\gamma \text{ -type}$$
$$O^{\Delta F=1} = (\bar{D}_L Y_U Y_U^\dagger D_L) \cdot \bar{D}_L D_L \qquad B \to K\pi\pi$$
$$O^{\Delta F=2} = (\bar{D}_L Y_U Y_U^\dagger D_L)^2 \qquad B_d \to \bar{B}_d \text{ -type}$$

- ⇒ **correlations**: b → s, b→d, s→d transitions e.g. $\Delta M_d / \Delta M_s$ as in SM

testable hypothesis!

- Is there still room for large effects:? Yes in certain channels
  e.g. B → ll -- enhancement due to large tanβ = v_u / v_d

- SUSY & MFV make proton long lived! No need for R-parity!

Nikoladitis & Smith '07

- No model of MFV *(... seems as hard as creating a **Theory of Flavour**)*

- MFV is also a language -- you can compare your BSM-flavour physics to MFV

If we take it beyond that ....

Consequences of spurious Goldstone bosons

- If G_q-symmetry SSB by Yukawa's $\langle Y_{U/D} \rangle \neq 0$ $\Rightarrow$ 3x8 + 2 = 26 (massless) Goldstone bosons
- Mid 70's 80's study breaking of continuous family symmetries -- dubbed Goldstone modes **familons**
- Physics should be the same:

$$\mathcal{L}^{\rm eff} \simeq \frac{1}{\Lambda_F} (\partial_\mu \phi_F)\,(\bar{s}\gamma_\mu d_L) + \ldots$$

Φ_F weakly coupled (not detected) ---- $K^+ \to \pi^+ \nu\nu$ vs $K^+ \to \pi^+ \Phi_F$

$\Rightarrow \Lambda_F > 10^8$ TeV (infer from e.g. Feng et al '97)

- If $\Lambda_F \sim \Lambda_{MFV}$ flavour difficuly to detect

caveat:

If flavour violation in soft-breaking terms e.g. SUSY-GUT then $\Lambda_F \sim \Lambda_{GUT}$ .. $\Lambda_{MFV} \sim \Lambda_{SUSY}$

ways out

- **Discrete symmetry**
  (no Goldstone modes)

  this work ..

- **Gauge the symmetry**
  (new massive gauge bosons)

  Albrecht, Feldmann, Mannel 'announced'

There are (plenty) discrete subgroups very good -- end of the day?

By going to a discrete symmetry .....

1. Get rid of familons (goldstone bosons) 
2. Reduce symmetry ⇒ new flavour structure (dangerous?) 

Is there room for a TeV-scale dMFV-scenario ?

Formulation: discrete Minimal Flavour Violation (dMFV)

Fischbacher RZ 2008 PRD79

1. $G_q \to D_q$ = **$D3_Q$ x $D3_{UR}$ x $D3_{DR}$** $D3 \subset SU(3)$, not discuss U(1)'s cold be Z_n
2. Specify the 3D **irrep** of D3
3. (possibly) Yukawa expansion $Y \to \kappa Y \quad \kappa \leq 1$ *

- Model independent approach: ⇒ study of **invariants** (~ effective operators)

$$\mathcal{L}^{\text{eff}} \sim \sum_n \frac{c_n}{\Lambda^{\dim(\mathcal{I}_n)-4}} \mathcal{I}_n(u, d, Y_U, Y_D) + h.c.$$

- Cutting a long story short: new invariants = non-MFV transitions

1. New invariants (typically) ⇒ anarchic flavour transitions
2. Moreover: **Yukawas** (modulo CKM) diagonalized via G_q
   If we break it down to D_q (⇒ more observable mixing angels!)

* $\kappa \cong 1$ non-linear MFV (σ-model ... Feldmann, Mannel '08 , Kagan et al '09) $\kappa \ll 1$ linear MFV

Invariants

Let **A**, **B**,**C**, ... be irreps of some group then **A** x **B** x **C** x ... = **n** **I** + ...

n: Number of invariants = Number times **I** appears in *(follows from* ***V*** *x* ***V**** *= I* ***I*** *+ .....* ⇔ ***V*** *irrep)*

- Denote tensor n **3** and m **3*** indices by $T^{(m,n)}$ $D \equiv (d, s, b)_L \in T^{(1,0)}$, $\Delta \equiv Y_U^\dagger Y_U \in T^{(1,1)}$ *

dMFV | **MFV**

$$\Delta F = 1' \qquad \mathcal{I}_n^{(2,2)} = (\mathcal{I}_n)^{ab}{}_{rs} \; (\bar{D}^r \Delta_a{}^s D_b) \qquad (\bar{D}_L \Delta_U Y_D \, \sigma \cdot F D_R)$$

$$\Delta F = 2 \qquad \mathcal{I}_n^{(4,4)} = (\mathcal{I}_n)^{abcd}{}_{rstu} \; (\bar{D}^r \Delta_a{}^s D_b) \; (\bar{D}^t \Delta_c{}^u D_d) \qquad (\bar{D}_L \Delta_U D_L)^2$$

Q: Are there SU(3) subgroups with no new a: $I^{(4,4)}$ b: $I^{(2,2)}$-invariants?

*assuming D_{UR} indices can be contracted

.... results on invariants:

- a: **no** (no discrete SU(3) subgroups no new $\boldsymbol{I}^{(4,4)}$-invariants!)
- b: **yes** (are discrete SU(3) subgroups no new $\boldsymbol{I}^{(2,2)}$-invariants!)

group	order	pairs $(\mathbf{3}, \bar{\mathbf{3}})$	$\mathcal{I}^{(2,2)}$	$\mathcal{I}^{(3,3)}$	$\mathcal{I}^{(4,4)}$
SU(3)	∞	1	2	6	23
$\Sigma(360\varphi)$	1080	2	2	6	28
$\Sigma(216\varphi)$	648	3	2	7	40
$\Sigma(168)$	168	1	2	7	44
$\Sigma(72\varphi)$	216	4	2	11	92

subgroup

- Show: absence 27-dim irrep ⇒ new $\boldsymbol{I}^{(4,4)}$-invariants (c.f. backup-slide)

  absence 8-dim irrep ⇒ new $\boldsymbol{I}^{(2,2)}$--invariants

Q: Does this mean there's no TeV-scale dMFV?

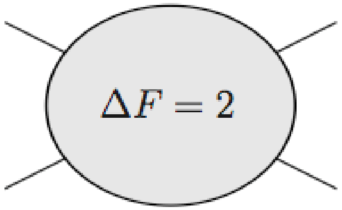
Have to refine notion of model independence.

The $\Delta F=2$ generation mechanism has to be reflected upon.

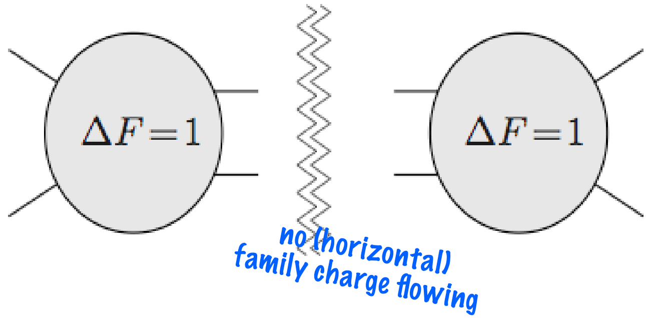
"Family (ir)reducible"

factorization

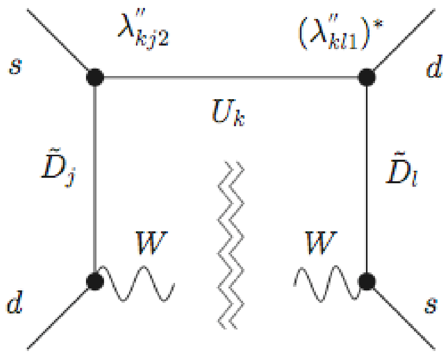
"family irreducible" $I^{(4,4)}$



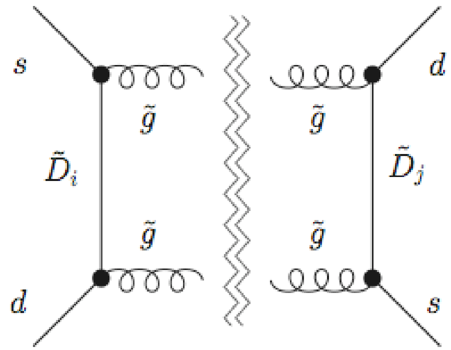
"family reducible" $I^{(4,4)} \to I^{(2,2)} \times I^{(2,2)}$



- Composite TC model, R-parity violating MSSM



- SM, R-parity conserving MSSM:



TeV-scale dMFV scenario

"family reducability" is sufficient property for "TeV-scale dMFV scenario" for D_q:

$$\Sigma(168)\,,\quad \Sigma(72\varphi)\,,\quad \Sigma(216\varphi) \quad \text{and} \quad \Sigma(360\varphi)$$

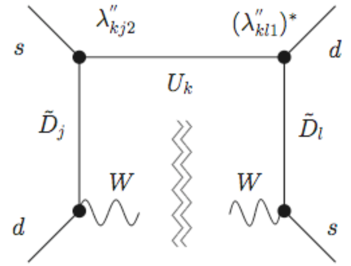
Dangerous invariants factorize: $\boldsymbol{I}^{(4,4)} \to \boldsymbol{I}^{(2,2)}\, \boldsymbol{I}^{(2,2)}$

- TeV-scale? Recall: $C_{SM}/C_{MFV} \geq (0.5\ \text{TeV}/m_W)^2$ -- Yukawa expansion: what κ bound $C_{MFV} \sim C_{dMFV}$?
    - most suitable candidate $\Sigma(360\varphi)$ only $\boldsymbol{I}^{(4,4)}$ new invariants
      MFV: $s \to d$ $O(\lambda^5)$ strong suppression, $\Delta S = 2$ real part $O(\lambda^{10})$
      dMFV: $s \to d$ $O(\lambda)$ (from examples .."worst case")

      $\Rightarrow$ **MFV : dMFV** $= \lambda^{10} : \lambda^6 \kappa^4$ equal $\kappa_{\Sigma(360\varphi)} \approx \lambda \approx 0.2$

- sufficient but not necessary! Consider R-parity violating MSSM
  Can convince yourself that
  not lead to "dangerous" non-fac. $\boldsymbol{I}^{(4,4)}$

(essentially) each vertex is D_q-invariant !



Epilogue

- Discrete groups are fun .... (and have potential)
- Would be good to work out in more generality how breaking patterns $G_{cont} \rightarrow G_{discrete} \rightarrow G'_{discrete}$ works out (systematically)
- TeV-scale dMFV scenario possible for crystal-like groups $\Sigma(360\varphi)$, $\Sigma(216\varphi)$, $\Sigma(72\varphi)$, $\Sigma(168)$
  (with moderate ($\kappa \sim O(0.2)$) possible (model-independent))
- *Happy Birthday* ..... greetings from Bob Shrock, Pasquale Di Bari , ...

Backup Slides

Necessarily new $\boldsymbol{I}^{(4,4)}$-invariants !!

- **What level?:** Kronecker product decompose any different than SU(3) !

Look at: $$\mathcal{I}_n^{(4,4)} = (\mathcal{I}_n)^{abcd}_{rstu}\,\left(\bar{D}^r \Delta_a{}^s D_b\right)\left(\bar{D}^t \Delta_c{}^u D_d\right)$$

- SU(3): $(\mathbf{3} \times \mathbf{3}^* \times \mathbf{3} \times \mathbf{3}^*)_S \times (\mathbf{3} \times \mathbf{3}^* \times \mathbf{3} \times \mathbf{3}^*)_S$ =

  $(\mathbf{8} \times \mathbf{8})_S \times (\mathbf{8} \times \mathbf{8})_S + \ldots =$

  $(\mathbf{1} + \mathbf{8} + \mathbf{27}) \times (\mathbf{1} + \mathbf{8} + \mathbf{27}) + \ldots$

$\Rightarrow$ if $D_Q \subset$ SU(3) has no **27** $\Rightarrow$ new $\boldsymbol{I}^{(4,4)}$ invariants (new $\Delta F = 2$ structure)

- 1. **Dihedral groups** $\Delta(3n^2)$, $\Delta(6n^2)$ max 3,6D irrep $\Rightarrow$ out

  2. **Crystallographic groups** .. look at character tables reveals there is none
  (N.B. $27^2 = 729$ almost saturates the largest group (3x360=1080) ...)