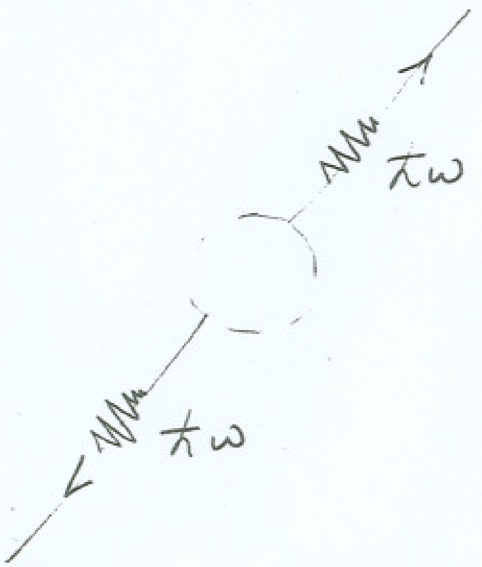
$e^+ + e^- \longrightarrow e^+ + e^-$ $l=0$ state. $S=0$.

($S=1$ state possible $\sim \frac{1}{1000}$ $S=0$ state. <u>and</u> $S=1$ state decays to 3γ's).

So

Initial state $L=0$, $S=0$.

Then decays to 2γ's.



$$2\hbar\omega = 2m_0c^2$$
$$\hbar\omega = m_0c^2.$$

Photon has $S=1$ $\therefore$ $S_3 = 1, 0, -1$?

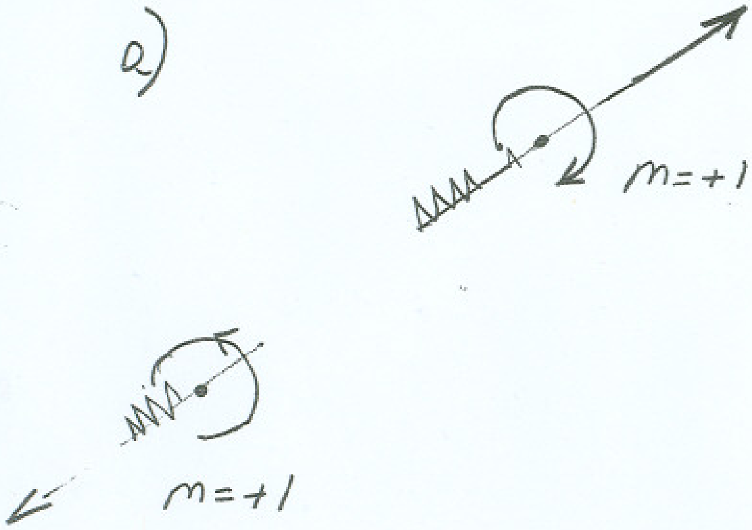
<u>not so</u> for particles of zero rest mass (or $v=c$) can only have $m = \pm j$ along $\underline{v}$.

So photon can only have $m = \pm 1$, $m=0$ not possible.

For decay into two photons we have to have 0 spin in final state.

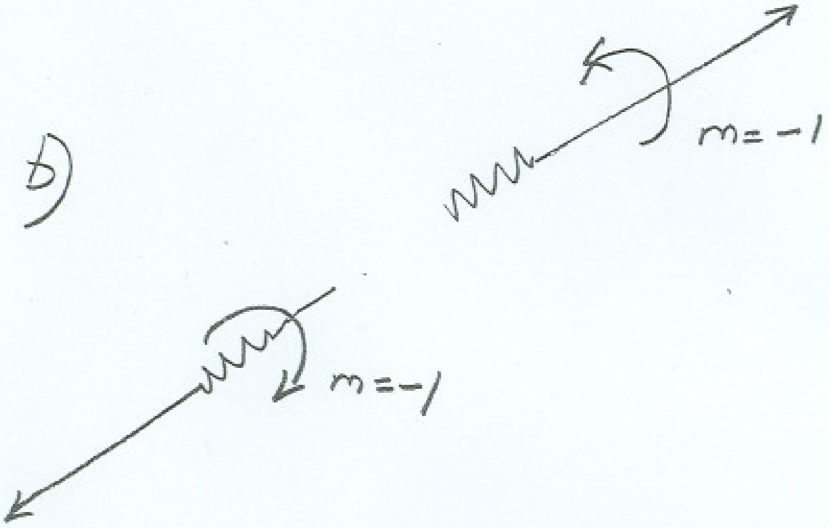
So following are possibles:

a)



m=+1 are right handed photons (circular polarised).

b)



m=-1 are left handed photons.

if $|R\rangle$ indicates the state of a right handed photon and $|L\rangle$ a lefthanded.

then

a) state $= |R_1\rangle|R_2\rangle$

b) state $= |L_1\rangle|L_2\rangle$.

Now we must also conserve parity but

$\Psi =$  $(s=0 \quad L=0)$

state have negative parity because

$P\Psi$ = parity of Ψ = (parity of e^+)(parity of e^-)(parity of relative motion)

$= -ve$.

because parity of antiparticle = - parity of particle

But

$$P|R_1\rangle|R_2\rangle = +|L_1\rangle|L_2\rangle.$$

and

$$P|L_1\rangle|L_2\rangle = +|R_1\rangle|R_2\rangle$$

So. $|R_1\rangle|R_2\rangle$, $|L_1\rangle|L_2\rangle$ do not have definite parity

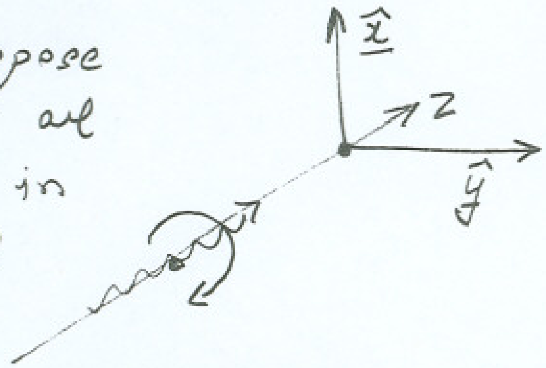
But if $\Psi = |R_1\rangle|R_2\rangle - |L_1\rangle|L_2\rangle$

then $P\Psi = |L_1\rangle|L_2\rangle - |R_1\rangle|R_2\rangle = -\left(|R_1\rangle|R_2\rangle - |L_1\rangle|L_2\rangle\right)$

$\therefore$ parity of $\Psi = |R_1\rangle|R_2\rangle - |L_1\rangle|L_2\rangle$ is $-$ve and therefore this represents correct wavefunction.

Now the polariser is sensitive to linear polarisation not circular. $|R_1\rangle$ and $|L\rangle$ represents circular polarised light $\therefore$ we have to transform Ψ to linearly polarised representation.

Suppose $\hat{x}, \hat{y}$ are fixed in space



Suppose $|x\rangle$ = state of photon, going along z, and has <u>electric</u> polarisation in $\hat{x}$ direction.

$$\text{Then } |R\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle + i|y\rangle\right)$$

$$\text{and } |L\rangle = \frac{1}{\sqrt{2}}\left(|x\rangle - i|y\rangle\right)$$

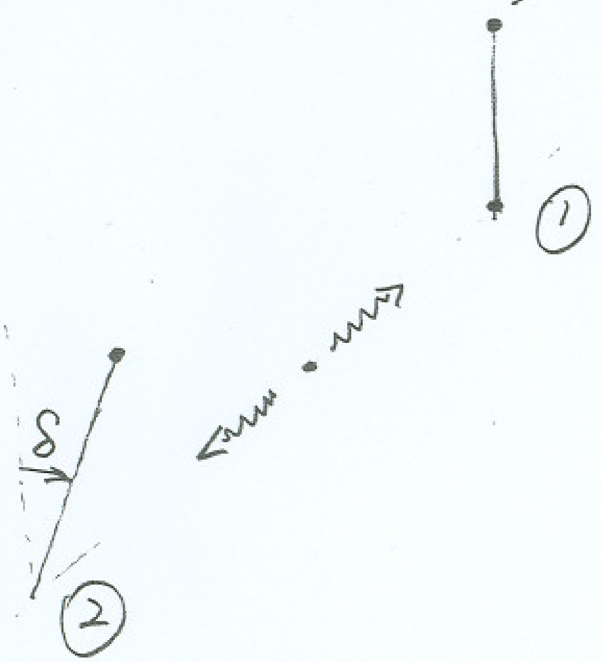
So now.

$$\Psi = \frac{1}{2}\Big(\big(|x_1\rangle + i|y_1\rangle\big)\big(|x_2\rangle + i|y_2\rangle\big) - \big(|x_1\rangle - i|y_1\rangle\big)\big(|x_2\rangle - i|y_2\rangle\big)\Big)$$

$$= \frac{i}{2}\big(|y_1\rangle|x_2\rangle + |x_1\rangle|y_2\rangle\big) \times 2$$

$$= i\big(|y_1\rangle|x_2\rangle + |x_1\rangle|y_2\rangle\big).$$

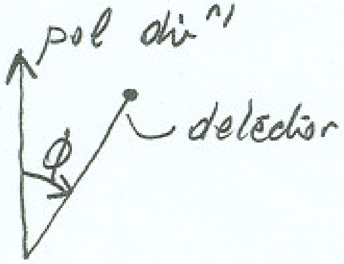
We know how our apparatus responds to linear polarisation so now we can calculate its response. To simplify suppose we keep one detector fixed (say vertical) and move the other detector, the other detector is at an angle δ to vertical.



Now if we examine Ψ we see it contains two components for detector 1 $|x_1\rangle$, $|y_1\rangle$

What is the response of 1 to $|x_1\rangle$.

__Well__ - general formula for scattering



$$\frac{d\sigma}{d\Omega} \propto a + b\cos^2\phi$$

(assume $\theta \sim 90^\circ$)

But a photon in $|x_1\rangle$ state has polarisation vertical, but on p.5 the detector is vertical also. $\therefore$ $\phi = 0$. so for this state

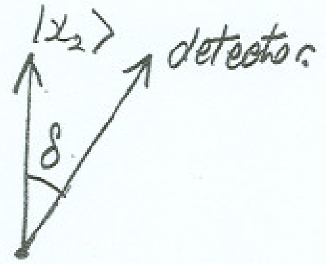
$$\frac{d\sigma}{d\Omega} \propto a + b\cos^2(\phi = 0) = a + b.$$

Similarly a photon in $|y_1\rangle$ state has polarisation horizontal, so for this state.

$$\frac{d\sigma}{d\Omega} \propto a + b\cos^2(\phi = \pi/2) = a.$$

Now lets look at the second photon approaching analyser (5) at δ to vertical.

look at $|x_2\rangle$



$$\frac{d\sigma}{d\Omega} \propto (a + b\cos^2\delta)$$

for $|y_2\rangle$
$$\frac{d\sigma}{d\Omega} \propto (a + b\sin^2\delta).$$

now $\langle x_1|\Psi\rangle = i\,|y_2\rangle$

So this scattering component will have intensity

$$(a+b)(a+b\sin^2\delta)$$

and $\langle y_1|\Psi\rangle = i\,|x_2\rangle$

scattering component

$$(a)(a+b\cos^2\delta).$$

Total intensity

$$= (a+b)(a+b\sin^2\delta) + a(a+b\cos^2\delta)$$

$$= a^2+ab+a^2 + b(a+b)\sin^2\delta + ab\cos^2\delta$$

$$= a^2+ab+a^2+ b(a+b) + (ab-(a+b)b)\cos^2\delta$$

$$= (2a^2+2ab+b^2 - b^2\cos^2\delta).$$

But $a = k/k_0 + \frac{k_0}{k} = 2+0.5 = 2.5$

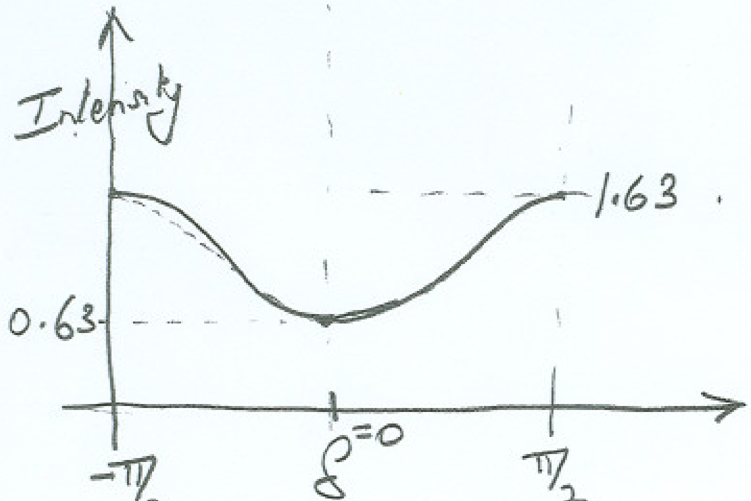
$b = -2.$

$\therefore$ Intensity $= 2(2.5)^2 - 10 + 4 - 4\cos^2\delta$

$$= 12.5 - 6 - 4\cos^2\delta$$

$$= 6.5 - 4\cos^2\delta = (1.63 - \cos^2\delta)4$$

So:



Ratio $\frac{I(\theta=\pi/2)}{I(\theta=0)} = 2.59.$

Local wavefunction:

$f_1(x)\, f_2(x)$

Assume now $\Psi = |\psi_1(\theta)\rangle |\psi_2(\theta + \pi/2)\rangle$

separate spacial parts.

i.e. this is a local wavefunction which has photon 1 with polarisation (θ) and photon 2 $(\theta + \pi/2)$.

Then detector response

$$= (a + b\cos^2\theta)(a + b\cos^2(\theta + \pi/2 - \delta)).$$

$\therefore$ average over $\theta \times 2\pi$

$$= \int \Big[a^2 + b^2\cos^2\theta\cos^2(\theta + \pi/2 - \delta)$$

$$+ ab\cos^2\theta + ab\cos^2(\theta + \pi/2 - \delta)\Big] d\theta$$

$$= \Big[2\pi a^2 + b^2\int \cos^2\theta \sin^2(\theta - \delta)\, d\theta$$

$$+ ab\int \cos^2\theta\, d\theta + ab\int \sin^2(\theta - \delta)\, d\theta\Big]$$

$$= \Big[2\pi a^2 + b^2\frac{\pi}{4}(1 + 2\sin^2\delta) + ab\pi + ab\pi\Big]$$

$$\text{Average} = \frac{1}{2\pi}\Big[2\pi a^2 + \frac{b^2\pi}{4} + 2ab\pi + \frac{b^2\pi}{2}\sin^2\delta\Big]$$

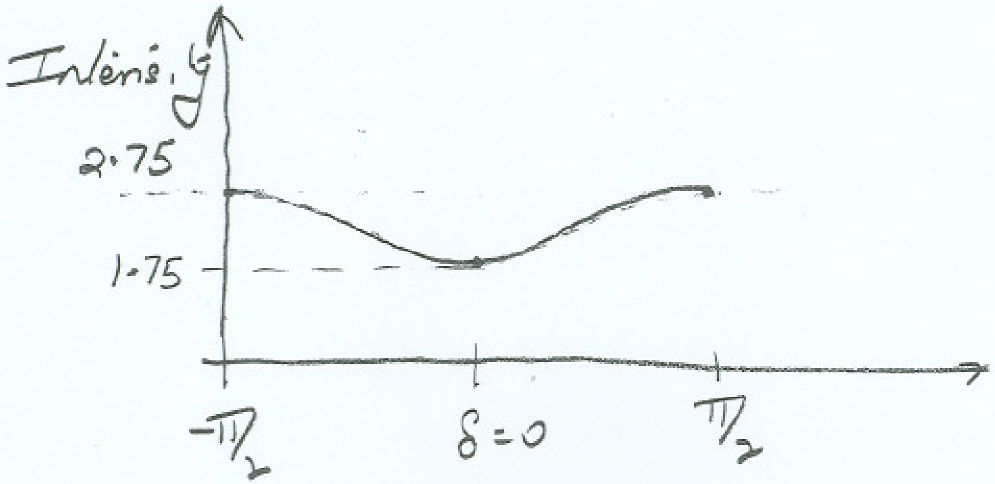
$$= \Big[\Big(a^2 + \frac{b^2}{8} + ab\Big) + \frac{b^2}{4}\sin^2\delta\Big]$$

if $a = 2\cdot5 \quad b = -2.$

$$a^2 + \frac{b^2}{8} + ab = (2\cdot5)^2 + \frac{4}{8} - 5 = 6\cdot25 + \tfrac{1}{2} - 5$$
$$= 1\cdot75$$

$$\frac{b^2}{4} = 1.$$

$\therefore$ Have. $1\cdot75 + \sin^2\delta = 2\cdot75 - \cos^2\delta.$



$$\frac{I(\pi/2)}{I(0)} = \underline{\underline{1\cdot57}}$$

So we can see this local wave function gives a shallower dip.