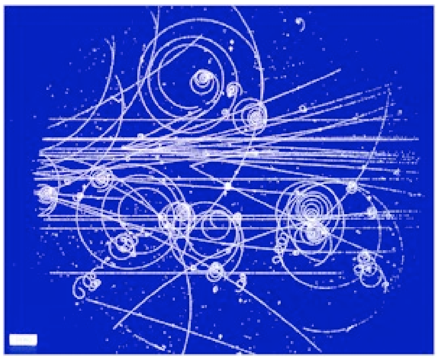
Nuclear and Particle Physics Junior Honours:

Particle Physics

Lecture 2: Practical Particle Physics

February 12th 2007

How do we study the particles and the forces?



- Natural Units
- Particle properties: lifetime and width
- Relativistic kinematics
- Scattering: cross sections, $\sqrt{s}$
- Fermi's golden rule
- Conservation laws

Summary of Last Lecture

The Standard Model of Particle Physics

An elegant theory that describes accurately (almost) all measurements in particle physics

Matter

- fermions
- 3 generations of quarks & leptons

Quarks and Leptons			Charge, e
ν_e e	ν_μ μ	ν_τ τ	0 −1
u d	c s	t b	+2/3 −1/3

- Antimatter
- Quarks form into hadrons - **mesons** and **baryons**

Forces

- mediated by the exchange of gauge bosons

Interaction	Gauge Bosons	Charge, e
Strong	gluons	0
Electro-magnetic	Photon	0
Weak	W, Z	0, ±1
Gravity	graviton	0

Introduction: Measurements in Particle Physics

- What properties of particles can we measure?
- How do we study the interactions, or the forces, between them?

Static Particle Properties

- Mass, m, Charge, q
- Magnetic moment
- Spin and Parity, J^{π}

Force	Typical Lifetimes	Typical Cross Sections
Strong	10^{-20} - 10^{-23} s	10 mb
Electromag	10^{-20} - 10^{-16} s	10^{-2} mb
Weak	10^{-13} - 10^{3} s	10^{-13} mb

Particle Decays

- Particle lifetime, τ, and width, Γ
- Allowed and forbidden decays → conservation laws

Particle Scattering

Two types: Elastic scattering *e.g.* $e^-p \to e^-p$; inelastic scattering *e.g.* $e^+e^- \to \mu^+\mu^-$

- Total cross section, σ.
- Differential cross section, $d\sigma/d\Omega$

Natural Units I

kg m s

SI units: [M] [L] [T]

- For everyday physics SI units are a natural choice: $M_{(\text{JH student})} \sim 80$kg.
- Not so good for particle physics: $M_{\text{proton}} \sim 10^{-27}$kg
- We choose to work in a different basis - **Natural Units**. based on the language of particle physics: quantum mechanics and relativity.
- The basis of natural units are:
  - ★ unit of action in QM: $\hbar$ (Js)
  - ★ velocity of light: c (ms^{-1})
  - ★ Unit of energy: GeV = 10^9 eV = 1.60×10^{-10} J

Energy GeV	Time $(\text{GeV}/\hbar)^{-1}$
Momentum GeV/c	Length $(\text{GeV}/\hbar c)^{-1}$
Mass GeV/c^2	Area $(\text{GeV}/\hbar c)^{-2}$

Natural Units II

Simplify even further by choosing $c = \hbar = 1$!

All quantities are expressed in powers of GeV

Energy GeV	Time GeV^{-1}
Momentum GeV	Length GeV^{-1}
Mass GeV	Area GeV^{-2}

Convert to SI units by reintroducing missing factors of $\hbar$ and c

- Example: Area = 1 GeV^{-2}

$$[L]^2 = [E]^{-2}[\hbar]^n[c]^m = [E]^{-2}\,[E]^n[T]^n\,[L]^m[T]^{-m} \qquad n = 2, m = 2$$

Area (in SI units) = 1 $\text{GeV}^{-2} \times \hbar^2 c^2 = 3.89 \times 10^{-32}\ \text{m}^2$ = 0.389 mb

Other common units:

- Masses and energies measured in MeV
- cross section measured in barn, b ≡ 10^{-28} m^2
- lengths in fm = 10^{-15} m
- electric charge in units of e

Two useful relations: $\hbar c = 197\ \text{MeV fm}$ $\qquad \hbar = 6.582 \times 10^{-22}\ \text{MeV s}$

Particle Lifetime

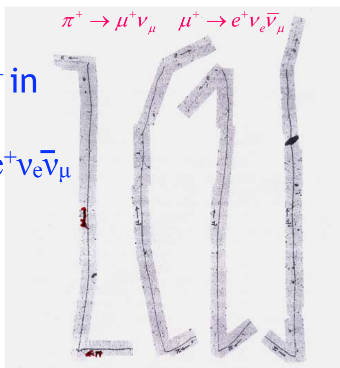
- The signature of many particle interactions is a decay.
- Most particles decay. Decay is characterised by the **lifetime** of the decay, the time taken for the sample to reduce to 1/e of original sample.

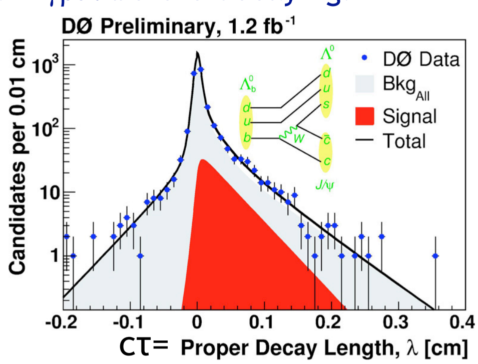
$$\frac{dN}{dt} = -\frac{t}{\tau} = -\frac{\Gamma}{\hbar}t \qquad \Rightarrow \qquad N(t) = N_0 \exp(-t/\tau) = N_0 \exp(-\Gamma t/\hbar)$$

Γ = width (next page)

- In the lab, time is dilated. Particles travel $L = \gamma\beta c\tau$ before decaying.

Discovery of π^+ in cosmic rays:
$\pi^+ \rightarrow \mu^+\nu_\mu$, $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$



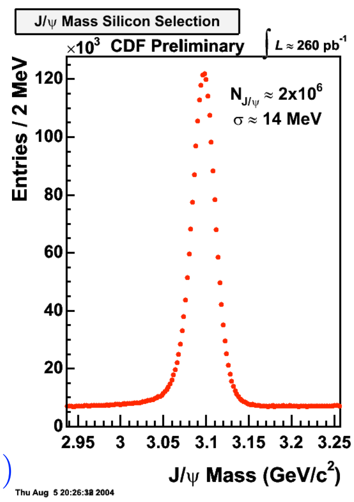


$\tau(\Lambda_b) = (1.30 \pm 0.15) \times 10^{-12}$ s

Particle Resonance

unmeasurable!

- Lifetimes of particles can be very short, *e.g.* lifetime of Δ^{++} (uuu) is 5×10^{-24} s, cτ=1.5 fm.
- Heisenburg Uncertainty Principle: energy and time are related: $\Delta E \Delta t \approx \hbar$
- **Natural width of a particle** $\Gamma \equiv \hbar/\tau = 1/\tau$
- For a short-lived particles, a 'resonance' appears in the mass spectrum. *e.g.* J/ψ meson ($c\bar{c}$) decays to two muons: $J/\psi \to \mu^+\mu^-$



$$\begin{aligned} M^2(J/\psi) &= (p_{\mu^+} + p_{\mu^-})^2 \\ &= p_{\mu^+}^2 + p_{\mu^-}^2 + 2p_{\mu^+}p_{\mu^-} \\ &= m_\mu^2 + m_\mu^2 + (E_{\mu^+}E_{\mu^-} - \vec{p}_{\mu^+}\cdot\vec{p}_{\mu^-}) \end{aligned}$$

- The mass of J/ψ not fixed, has an intrinsic uncertainty!
- The **total width** of a particle is the sum of widths for all possible final states.

(Total width = transition amplitude, $T_{i\to f}$, can be calculated using Fermi's golden rule.)

Review: Relativistic Dynamics

Please review dynamics and relativity lectures 13-15.

- Two important quantities for Lorentz transformations:

$$\beta = v/c \qquad \gamma(v) = 1/\sqrt{1-\beta^2}$$

- Four-momentum of a particle: $p^\mu = (E/c,\, p_x,\, p_y,\, p_z)$
- Energy of a particle $E^2 = \vec{p}^2c^2 + m^2c^4 \quad E = \gamma mc^2$
- Scalar product of 4-momentum: $p^2 = p^\mu \cdot p_\mu = (E/c)^2 - \vec{p}^2 = m^2c^2$
- Particles with *m*=0 travel at the speed of light

Mistake in handouts

Natural Units

Lorentz boosts: $\gamma = E/m \quad \gamma\beta = |\vec{p}|/m \quad \beta = |\vec{p}|/E$

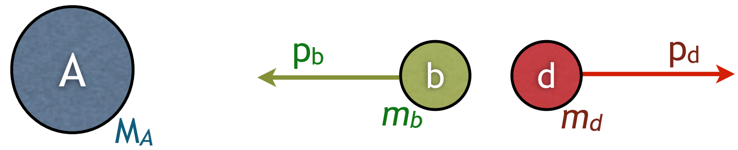
Four momentum: $p^\mu = (E,\, p_x,\, p_y,\, p_z)$

Invariant mass $p^2 = E^2 - \vec{p}^2 = m^2$

Review: Particle Decay

Decay of an unstable particle at rest:

A→*b d*



$$p_A^\mu = (M_A, 0) \qquad p_b^\mu = (E_b, \vec{p}_b) \qquad p_d^\mu = (E_d, \vec{p}_d)$$

Before After

- Four-momentum conservation:

$$p_A^\mu = p_b^\mu + p_d^\mu \qquad \Rightarrow \qquad p_b^\mu = p_A^\mu - p_d^\mu$$

$$p_b^2 = p_A^2 + p_d^2 - 2p_A \cdot p_d \; = M_A^2 + m_d^2 + 2M_A\, E_d \; = m_b^2$$

$$\Rightarrow E_d = \frac{M_A^2 + m_d^2 - m_b^2}{2M_A} \qquad \vec{p}_b = -\vec{p}_d$$

For moving particles, apply appropriate Lorentz boost.

- Example: $\pi^+ \to \mu^+ \nu_\mu$ work in rest frame of pion. $m_\nu \approx 0$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = 109.8 \text{ MeV} \qquad |\vec{p}_\nu| = |\vec{p}_\mu| = \sqrt{E_\mu^2 - m_\mu^2} = 29.8 \text{ MeV}/c$$

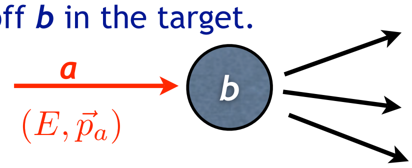
Collisions and Scattering

Consider a collision between two particles: ***a*** and ***b***

- Elastic collision: ***a*** and ***b*** scatter off each other ***a b*** → ***a b***
- Inelastic collision: new particles are created ***a b*** → ***c d …***

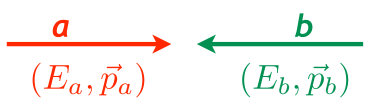
Two main types of particle physics experiment:

- **Fixed Target Experiments**: A beam of ***a*** are accelerated into a target at rest. ***a*** scatters off ***b*** in the target.



LAB Frame

- **Collider experiments** beams of ***a*** and ***b*** are brought into collision
  - ***a*** and ***b*** collide, usually $\vec{p}_a = -\vec{p}_b$

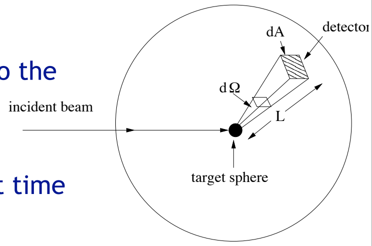


CM Frame

Fixed Target Scattering

Please review dynamics and relativity lecture 2.

a b → c d ...



- beam of n_a particles per unit area at velocity v_a
- Incident flux, F_a - number of particles crossing area normal to the beam direction per unit time $F_a = n_a\, v_a$
- n_b: number of target particles
- We **measure** dN scattered particles in solid angle dΩ per unit time
- Integrated event rate (per unit time) $N = \int \frac{dN}{d\Omega} d\Omega$

- How do we relate this to the underlying physics we want to study?
- **Differential cross section:** probability to observe a scattered particle in dΩ

$$\frac{d\sigma}{d\Omega} = \frac{\text{Scattered flux/Unit of solid angle}}{\text{Incident flux/Unit of surface}} \qquad \sigma = \int\int \frac{d\sigma}{d\Omega} d\Omega$$

- **The total cross section, σ**, effective area of the scattering, normalised to incident flux.
- Define the **luminosity**, $\mathcal{L} = n_a v_a n_b$

$$\frac{dN}{d\Omega} = n_a v_a n_b \frac{d\sigma}{d\Omega} = \mathcal{L}\frac{d\sigma}{d\Omega} \quad \Rightarrow \quad N = \mathcal{L}\sigma$$

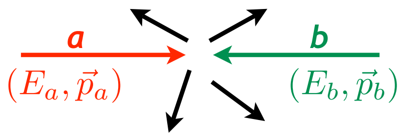
Cross sections measured in barn, 1b = 10^{-28} m^{2}.
Generally: nb, fb

Event rate = luminosity × cross section

Luminosity: inverse area/unit time: measured in $10^{30\ldots34}$ cm^{-2}s^{-1}

Collider Scattering

a b → c d ....



- Define the invariant quantity, ***s***:

$$\begin{aligned} s &= (p_a^\mu + p_b^\mu)\,(p_{a\,\mu} + p_{b\,\mu}) \\ &= p_a^2 + p_b^2 + 2p_a\,p_b \\ &= m_a^2 + m_b^2 + 2(E_aE_b - |\vec{p}_a||\vec{p}_b|\cos\theta) \end{aligned}$$

- √s is the energy in centre of momentum frame $E_{\mathrm{CoM}} = \sqrt{s}$
- This is the total energy of the collision available to crate new particles!
- Normally E>>m, and θ=180˚ $E_{\mathrm{CoM}} \approx \sqrt{4E_aE_b}$

- Example: Large Electron Positron Collider at CERN collided electrons and positrons with E_e=45.6 GeV from 1989-1995.

  E_{CM} = 2E_e = 91.2 GeV.
- Cross section to create Z -bosons at √s≈91.2 GeV: σ (e^+e^-→Z→hadrons) ≈ 26 nb
- How many Z→hadrons events were created for the total integrated luminosity of ∫ℒ dt = 162 pb^{-1}?

Event rate = luminosity × cross section

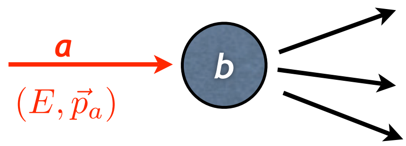
⇒ Number of events = time-integrated luminosity × cross section

- Number of events = ∫ℒ dt × σ = 26,000 × 162 = 4,536,000

$\sqrt{s}$ at Collider and Fixed Target

Fixed Target Collision

$$\begin{aligned} s &= m_a^2 + m_b^2 + 2(E_aE_b - |\vec{p}_a||\vec{p}_b|\cos\theta) \\ &= m_a^2 + m_b^2 + 2E_am_b \end{aligned}$$



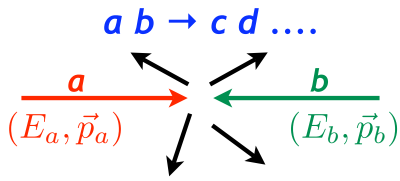
- For $E_a >> m_a, m_b$ $\quad s = 2E_am_b$

$$E_{\mathrm{CoM}} = \sqrt{2E_am_b}$$

- *e.g.* At NA48 450 GeV protons hit protons in target:

$$E_{\mathrm{CoM}} \approx \sqrt{2\times 450\times 1} = 30\ \mathrm{GeV}$$

Collider Experiment



$$s = m_a^2 + m_b^2 + 2(E_aE_b - |\vec{p}_a||\vec{p}_b|\cos\theta)$$

- For $E_a = E_b >> m_a, m_b$, $\cos\theta = \pi$

$$s = 4E^2 \qquad E_{\mathrm{CoM}} = 2E$$

- *e.g.* The Sp$\bar{\mathrm{p}}$S collider at CERN in the ’80s collided p and $\bar{\mathrm{p}}$ with E≈270GeV

$$E_{\mathrm{CoM}} = 540\ \mathrm{GeV}$$

- In a fixed target experiment most of the proton’s energy is wasted providing momentum to the COM system rather than being available for the interaction.

Fermi’s Golden Rule

What do measured cross sections tell us about the properties of the interactions?

- The transition rate, the probability of the interaction per unit time, depends on the cross section and the flux:

$$T_{i\to f} = F\sigma$$

- **Fermi’s Golden Rule:** gives for a transition between two eigenstates of a system.
(from time dependent perturbation theory)

$$T_{i\to f} = \frac{2\pi}{\hbar}|\mathcal{M}_{fi}|^2\rho$$

density of final states

Matrix element, containing the fundamental physics

$$\mathcal{M}_{fi} = \left\langle \psi_f \middle| \hat{H} \middle| \psi_i \right\rangle$$

- *e.g.* scattering reaction ***a b*** → ***c d***
  - The initial state $|\psi_i\rangle$ is the two particles ***a*** and ***b*** (including 4-momenta).
  - The final state $|\psi_f\rangle$ is two particles ***c*** and ***d*** (including 4-momenta).
  - Density of final states: measures how many possible final states for ***a b*** scattering exist
- In the next lecture we will see how to calculate matrix elements.

Conservation Laws

Noether's Theorem: Every symmetry of nature has a conservation law associated with it, and vice-versa.

- **Energy & Momentum; Angular Momentum**
  conserved in all interactions
  Symmetry: translations in space and time; rotations in space
- **Charge conservation**
  conserved in all interactions
  Symmetry: gauge transformation - underlying symmetry in QM description of electromagnetism
- **Lepton Number and Quark Number symmetry**
  L_e, L_μ, L_τ number of quarks minus number of anti-quarks $N_q - N_{\bar{q}}$
  symmetry: mystery!
- **Quark Flavour, Isospin, Parity**
  conserved in strong and electromagnetic interactions
  violated in weak interactions
  Symmetry: unknown

Summary

Natural Units: set $\hbar$=c=1 • Measure energies in GeV • Every quantity is measured as a power of energy	**Particle lifetime** and **width** Most particles decay, described by: • lifetime, τ, time taken for sample to decrease to 1/e. • Width, $\Gamma=\hbar/\tau$
Invariant Mass $p^2 = E^2 - \vec{p}^2 = m^2$ For a decay ***A→ab*** $M_A^2 = (p_a^\mu + p_b^\mu)^2$	Collider and Fixed Target **Scattering** *e.g.* ***ab→cd...*** $s = (p_a^\mu + p_b^\mu)^2 \quad E_{\mathrm{CoM}} = \sqrt{s}$ More energy available at a collider to make new particles.
The strength of an interaction can be described by the cross section, σ. Measured in barn = 10^{-28} m^2. Event rate = luminosity × cross section Number of events = time-integrated luminosity × cross section	Cross sections (and widths) can be calculated using Fermi's golden rule. **Noether's Theorem**: Every symmetry of nature has a conservation law associated with it, and vice-versa.