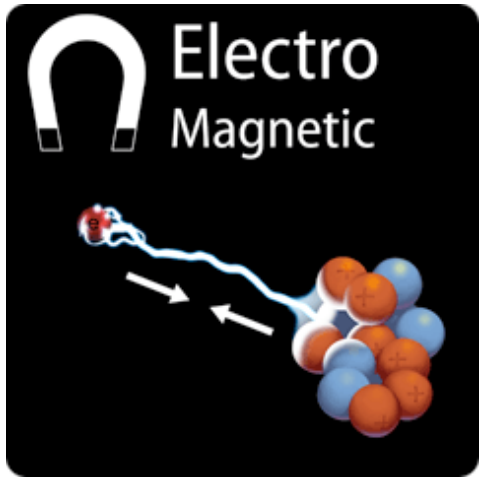
Nuclear and Particle Physics Junior Honours:

Particle Physics

Lecture 3: Quantum Electro-Dynamics & Feynman Diagrams

February 15th 2007



- Antimatter
- Virtual Particles

- Quantum description of electromagnetism
- QED vertex

- Feynman Diagram and Feynman Rules
- Examples

Schrödinger and Klein Gordon

- Quantum mechanics describes momentum and energy in terms of operators:

$$\hat{E} = i\hbar\frac{\partial}{\partial t} \qquad \hat{\vec{p}} = -i\hbar\vec{\nabla}$$

- $E=p^2/2m$ gives time-dependent ***Schödinger***: $$-\frac{\hbar}{2m}\nabla^2\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)$$

- Solution with a definite energy, E: $$\Psi_E(\vec{r},t) = \psi_E(\vec{r})\exp\{-iEt/\hbar\}$$

- However for particles near the speed of light $E^2=p^2c^2+m^2c^4$ ⇒

$$-\hbar\frac{\partial^2}{\partial t^2}\Psi(\vec{r},t) = -\hbar^2c^2\nabla^2\Psi(\vec{r},t) + m^2c^4\Psi(\vec{r},t)$$

Klein Gordon Equation

- Solutions with a definite energy, $E_p=+(p^2c^2+m^2c^4)^{1/2}$, and momentum, p:

$$\Psi(\vec{r},t) = N\exp\{i(\vec{p}\cdot\vec{r} - E_pt)/\hbar\}$$

- Also solutions with a energy, $E_p=-(p^2c^2+m^2c^4)^{1/2}$, and momentum, $-p$:

$$\Psi^*(\vec{r},t) = N^*\exp\{i(-\vec{p}\cdot\vec{r} + E_pt)/\hbar\}$$

- Negative energy solutions are a direct result of $E^2=p^2c^2+m^2c^4$.
- We interpret these as **anti-particles**

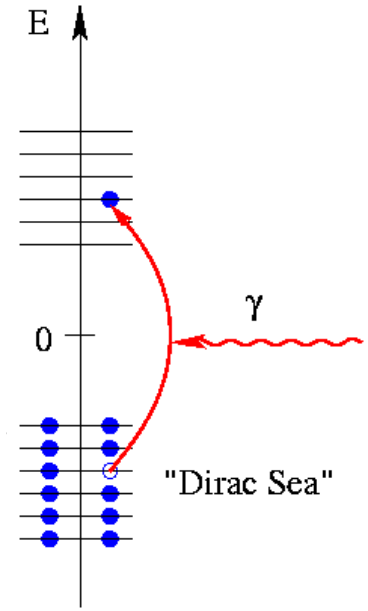
Klein Gordon equation is non-examinable

Antimatter

Klein-Gordon equation predicts negative energy solutions.

Dirac Interpretation:

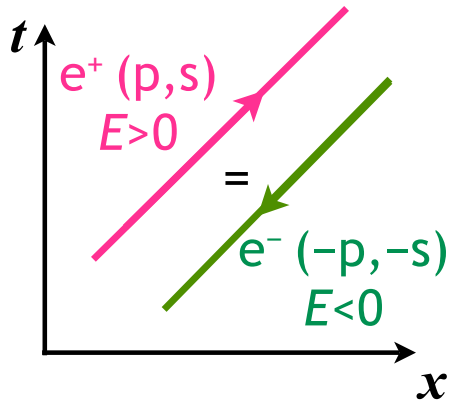
- The vacuum is composed of $E<0$ energy levels. Each level is filled with two electrons of opposite spin: the Dirac sea.
- A 'hole' in the sea of −ve charge and $E<0$ appears as a state with +ve charge and $E>0$.
- This idea lead Dirac to predict the **positron**, discovered in 1931.



Feynman-Stueckelberg

- negative energy particles moving backwards in space and time correspond to
- positive energy antiparticles moving forward in space and time

$$\exp\left\{-i\left((-E)(-t)-(-\vec{p})\cdot(-\vec{r})\right)/\hbar\right\}$$
$$= \exp\left\{-i\left(Et-\vec{p}\cdot\vec{r}\right)/\hbar\right\}$$



Quantum Electrodynamics (QED)

QED is the quantum theory of electromagnetic interactions.

Classical electromagnetism:

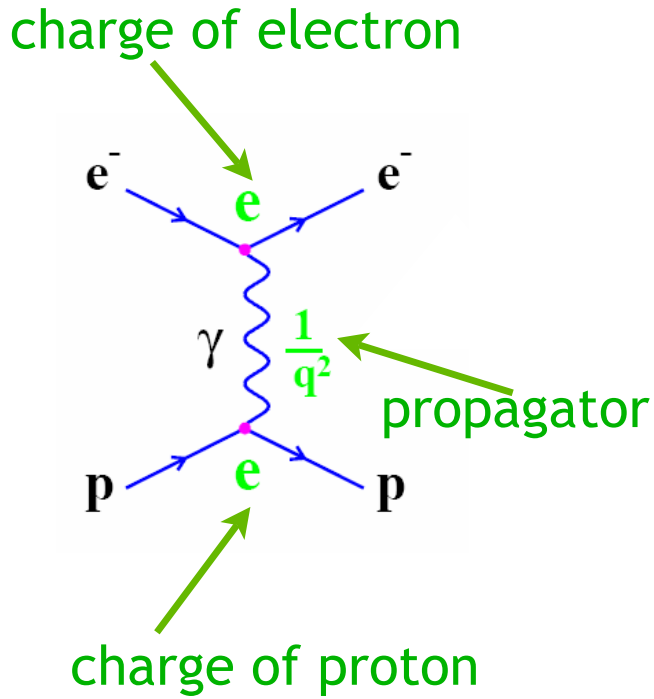
- Force between charged particle arise from the electric field

$$\vec{E} = \frac{q\,\hat{r}}{4\pi\epsilon_0 r^2}$$

- act instantaneously at a distance

Quantum Picture:

- Force between charged particle described by exchange of virtual "field quanta" - photons.
- Strength of interaction is related to charge of particle.



Matrix Element

Full derivation in 2nd order perturbation theory.

Gives propagator term: $1/(q^2 - m_\gamma^2)$ for exchange boson

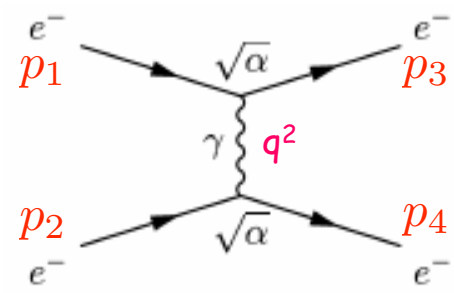
$$\mathcal{M}_{fi} = \frac{e^2}{(q^2 - m_\gamma^2)}$$

Virtual Particles

Force between two charged particles described by exchange of **virtual photons**.

- <u>Virtual Particles</u>
  - do not have same rest mass as physical particles: $m_X^2 \neq E_X^2 - \vec{p}_X^2$
  - We say *X* is "off-mass shell"
- **Heisenberg Uncertainty Principle**: can borrow energy to create particle if energy ($\Delta E = mc^2$) repaid within time (Δt), where $\Delta E \Delta t \approx \hbar$
- In electromagnetic interactions mass of photon propagator is non-zero.

Example: electron-electron scattering



- Momentum-squared transferred by the photon is: $q^2 = (p_3 - p_1)^2 = (p_2 - p_4)^2$
- The photon is virtual as $q^2 \neq m_\gamma^2$

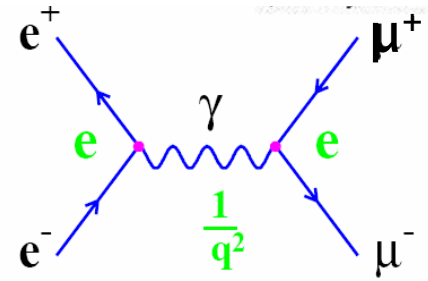
- Only intermediate photons may be virtual. Final state ones must be real!

Feynman Diagrams

"A Feynman diagram is a pictorial representation of a process corresponding to a particular transition amplitude"

Richard Feynman receiving the 1967 Noble prize in physics for his invention of this technique.

- The **probability amplitudes** for all processes - scattering, decay, absorption, emission can be described by Feynman Diagrams.
- Feynman diagrams are a very useful and **powerful** tool in particle physics. We will use them a lot in this course. We use them a lot in our research!



- <u>Conventions</u>
  - Time flows from left to right (occasionally upwards)
    - Fermions are solid lines with arrows
    - Bosons are wavy (or dashed) lines

- <u>Feynman Rules</u>: allow quantitative results for the matrix elements at different orders in perturbation theory.

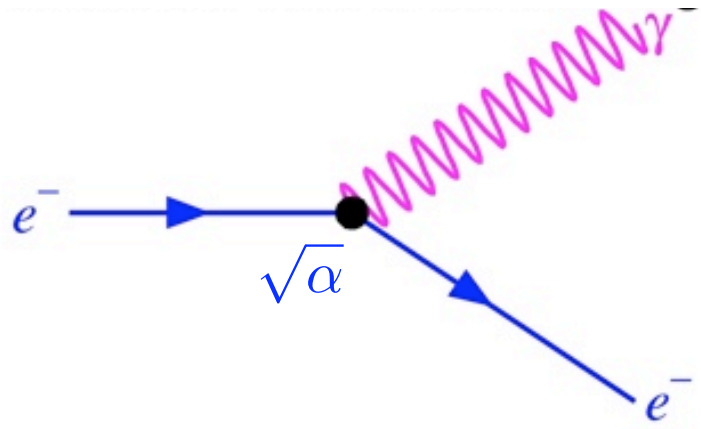
Electromagnetic Vertex

Basic electromagnetic process:

- Initial state fermion
- Absorption or emission of a photon
- Final state fermion

Examples: $e^- \to e^- \gamma$, $e^- \gamma \to e^-$

All electromagnetic interactions are described by the **vertex** and a **photon propagator**



Coupling strength

Matrix element is proportional to the fermion charge: $\mathcal{M}_{fi} \propto e$

The rate of the interaction is proportional to $|\mathcal{M}_{fi}|^2 \propto e^2$

Use the fine structure constant, α

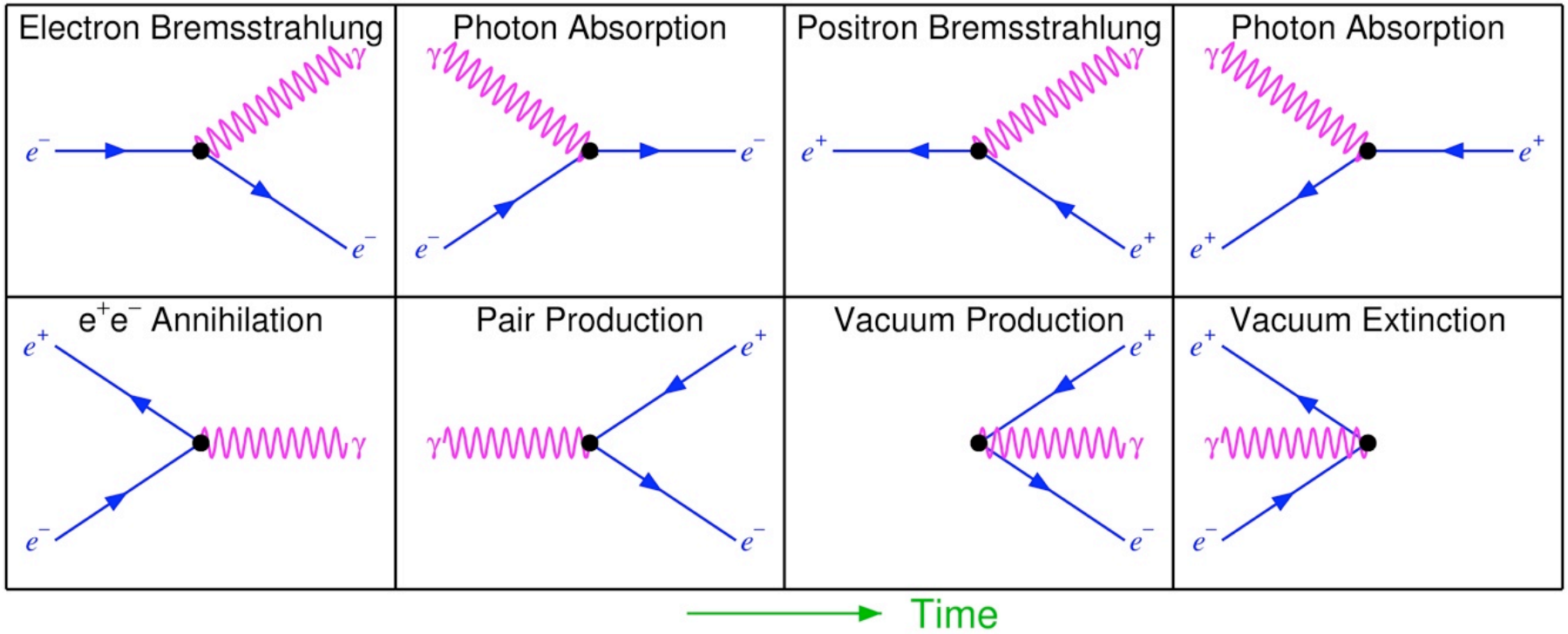
$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

The strength of the coupling at the vertex is $\sqrt{\alpha}$

QED Conservation Laws

- Momentum, energy and charge is conserved at all vertices
- Fermion flavour (e, μ, τ, u, d ...) is conserved:
  e.g. $u \to u\gamma$ allowed, $c \to u\gamma$ forbidden
- Parity, π, is conserved.

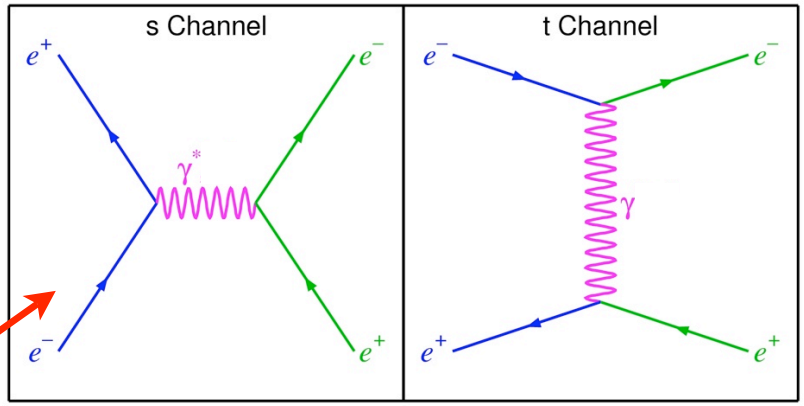
Basic QED Processes



- The vertex can be twisted: describing many processes. NB: electron going backwards in time is equivalent to a positron.
- None of above processes is real! Violate energy-momentum conservation:

$$p_\gamma^2 = (p_{e_1} - p_{e_2})^2 \neq m_\gamma^2$$

- Join two together to get a real processes



Feynman Rules for QED

Each line and vertex in a Feynman diagram corresponds to a term in the matrix element.

- Initial and final fermions are described by a wave function: $\psi_e(p^\mu)$
- Initial and final photons described by a photon wave function: $\psi_\gamma(p^\mu)$
- Internal virtual photons described by photon propagator $1/(q^2 - m_\gamma^2) = 1/q^2$
- At every vertex a coupling constant $\sqrt{\alpha}$ (and the vertex operator $\hat{O}_V$)
- Also internal fermions: descried by fermion propagator

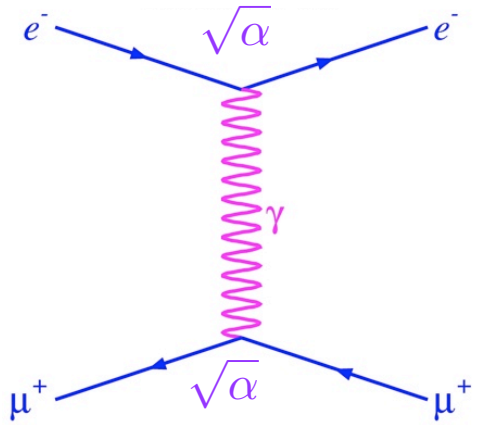
(Some technical differences between the way we treat fermions and bosons)

Example electron - elastic muon scattering: eμ→eμ

Matrix element:

- muon wave function $\sim \langle \psi_e(p_1)|\, e\, \hat{O}_V\, |\psi_e(p_3)\rangle$
- photon propagator $\sim 1/q^2$
- electron wave function $\sim \langle \psi_\mu(p_2)|\, e\, \hat{O}_V\, |\psi_\mu(p_4)\rangle$

$$q^2 = (p_1 - p_3)^2 = (p_4 - p_2)^2$$



$$\langle \psi_e(p_1)|\, e\, \hat{O}_V\, |\psi_e(p_3)\rangle \frac{1}{q^2} \langle \psi_\mu(p_2)|\, e\, \hat{O}_V\, |\psi_\mu(p_4)\rangle \propto \frac{e^2}{q^2}$$

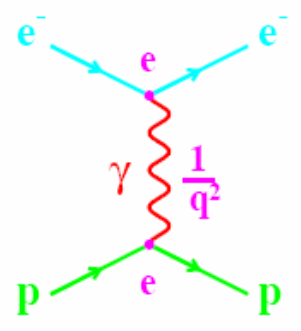
$$\text{rate} \propto |\mathcal{M}_{fi}|^2 \propto \frac{\alpha^2}{q^4}$$

QED Scattering Examples

- **Elastic electron-proton scattering: $e^-p \to e^-p$**

$$\mathcal{M} \propto \frac{e^2}{q^2} = \frac{4\pi\alpha}{q^2}$$

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{M}|^2 \propto \frac{e^4}{q^4} = \frac{16\pi^2\alpha^2}{q^4}$$



- Momentum transferred to photon from e^-:

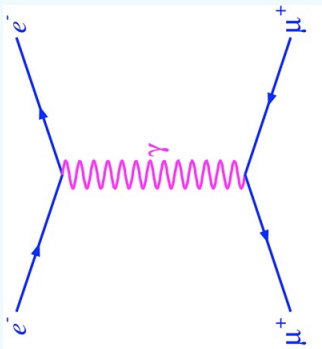
$$\begin{aligned} q^2 &= (p_f - p_i)^2 = p_f^2 + p_i^2 - 2p_f p_i \\ &= 2m_e^2 - 2(E_f E_i - |\vec{p}_f||\vec{p}_i|\cos\theta) \\ &\approx -4E_f E_i \sin^2(\theta/2) \end{aligned}$$

- **Rutherford scattering**: e^- Au$\to e^-$ Au, can neglect recoil of the gold atoms: $E=E_i=E_f$

$$\frac{d\sigma}{d\Omega} \propto \frac{Z^2\pi^2\alpha^2}{E^4\sin^4(\theta/2)}$$

LAB Frame

- **Inelastic $e^-e^+\to\mu^+\mu^-$**



- Momentum transferred by photon:

$$q^2 = (p_{e^+} + p_{e^-})^2 = s$$

- Use Fermi's golden rule, including density of final states:

$$\sigma = \frac{16\pi E^2}{3}|\mathcal{M}|^2 = \frac{4\pi\alpha^2}{3s}$$

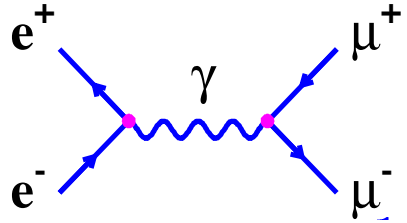
CoM Frame

Higher Orders

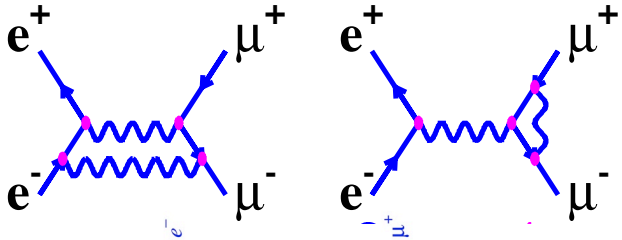
- QED is formulated from time dependent **perturbation theory**.
- Perturbation series: break up the problem into a piece we can solve exactly plus a small correction.
- *e.g.* for $e^+e^- \to \mu^+\mu^-$ scattering.
  - Many more diagrams have to be considered for a accurate prediction of **σ**.
- As α is small the lowest order in the expansion dominates, and the series quickly converges!

Lowest Order

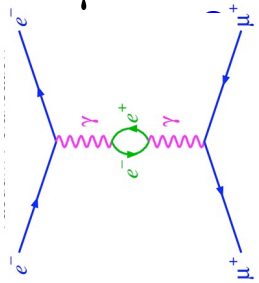
$|\mathcal{M}|^2 \propto \alpha^2$



2nd Order



$|\mathcal{M}|^2 \propto \alpha^4$



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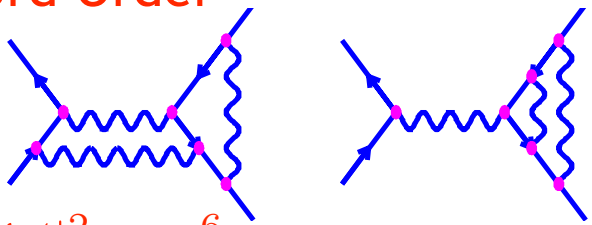
3rd Order



+ ...

$|\mathcal{M}|^2 \propto \alpha^6$

Summary

Relativistic quantum mechanics predicts negative energy particles: **antiparticles.** Two interpretations:

- a positive energy particle travelling backwards in time.
- a 'hole' in a vacuum filled with negative energy states.

Quantum Electro Dynamics (QED) is the quantum mechanical description of the electromagnetic force.

Electromagnetic force propagated by "off-mass shell" photon: $q^2 \neq m_\gamma^2$

Feynman diagrams can be used to **illustrate** and **calculate** a QED matrix elements.

Feynman rules are used to calculate the matrix element.

Cross section is proportional to the matrix element squared.

All QED interactions are described by a **fermion-fermion-photon vertex**:

- Strength of the vertex is the charge of the fermion.
- Fermion flavour and energy-momentum are conserved at vertex.

The **photon propagator** ~$1/q^2$ where q is the 4-momentum transferred by the photon.