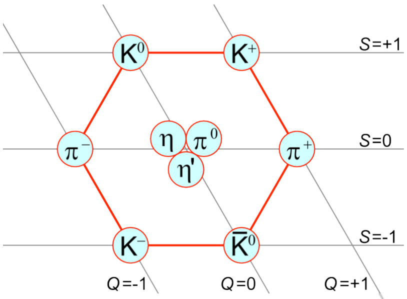
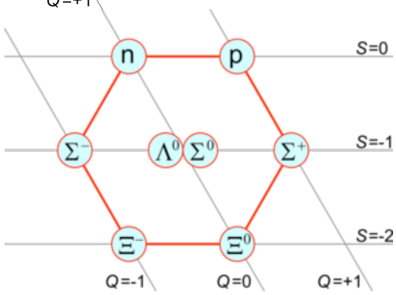
Nuclear and Particle Physics Junior Honours:

Particle Physics

Lecture 6. Hadrons: Mesons & Baryons

February 26th 2007





- Isospin again
  - Isospin conservation
- Spin, parity
- Mesons
- Baryons
- Hadron Masses

Much Ado about Isospin

Isospin was introduced as a quantum number before it was known that hadrons are composed of quarks.

Now we know it describes the number of up and down quarks in hadrons.

- Total isospin, I
- third component of isospin, I_Z

$$I_Z = \frac{1}{2}\left[N(\mathrm{u}) - N(\mathrm{d}) + N(\bar{\mathrm{d}}) - N(\bar{\mathrm{u}})\right]$$

Quark	I	I_Z
u	1/2	+1/2
d	1/2	−1/2
ū	1/2	−1/2
d̄	1/2	+1/2

Two hadrons with the same isospin, I, exhibit a symmetry: they have roughly equal mass and the strong force between the constituent quarks is equal.

Example: Pions: π^+, π^0 and π^- have m($\pi^\pm$)=139.6 MeV/c^2, m(π^0)=135.0 MeV/c^2.

- π^+ is ud̄: I_Z = ½+½=1
- π^0 is uū or dd̄: I_Z = ½+(−½)=0
- π^- is dū: I_Z = (−½)+(−½)=−1

Total isospin is the highest value of the I_Z. π^+, π^0, π^- all have I= 1

Isospin Conservation

Total isospin *I* is conserved in strong interactions: use to understand strong interaction cross sections and decays.

Example: *Δ*-resonances, created in pion-nucleon scattering:

- $\pi^+ p \to \Delta^{++} \to \pi^+ p \qquad \pi^- p \to \Delta^0 \to \pi^- p \qquad \pi^- p \to \Delta^0 \to \pi^0 n$

Isospin addition of the final and initial hadrons:

- $\pi^+ p \quad |1,+1\rangle \oplus |\tfrac{1}{2},+\tfrac{1}{2}\rangle \;=\; |\tfrac{3}{2},+\tfrac{3}{2}\rangle$
- $\pi^- p \quad |1,-1\rangle \oplus |\tfrac{1}{2},+\tfrac{1}{2}\rangle \;=\; \sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle - \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle$
- $\pi^0 n \quad |1,\ 0\rangle \oplus |\tfrac{1}{2},-\tfrac{1}{2}\rangle \;=\; \sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2},-\tfrac{1}{2}\rangle + \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2},-\tfrac{1}{2}\rangle$

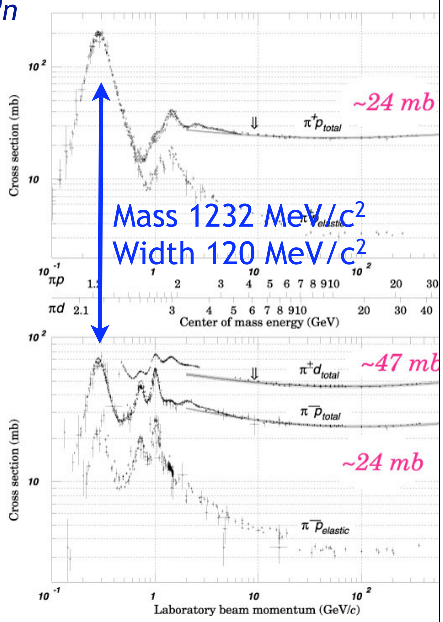
Matrix element: components for *I*=3/2 and *I*=1/2:

$$\mathcal{M}(\pi^+ p \to \Delta^{++} \to \pi^+ p) = \mathcal{M}_{3/2}$$
$$\mathcal{M}(\pi^- p \to \Delta^0 \to \pi^- p) = \tfrac{1}{3}\mathcal{M}_{3/2} + \tfrac{2}{3}\mathcal{M}_{1/2}$$
$$\mathcal{M}(\pi^- p \to \Delta^0 \to \pi^0 n) = \tfrac{\sqrt{2}}{3}\mathcal{M}_{3/2} - \tfrac{\sqrt{2}}{3}\mathcal{M}_{1/2}$$

Cross sections ∝ $\mathcal{M}^2$

- $\sigma(\pi^+ p \to \Delta^{++} \to \pi^+ p) \approx$ 200 mb 9 x
- $\sigma(\pi^- p \to \Delta^0 \to \text{all}) \approx$ 71 mb 3 x
- $\sigma(\pi^- p \to \Delta^0 \to \pi^- p) \approx$ 24 mb 1 x

Consistent with *Δ*-resonances having *I*=3/2



Mesons

Mesons: bound state of a quark and an anti-quark. They have:

- Zero net colour charge. $|\psi\rangle = \frac{1}{\sqrt{3}}|r\bar{r} + g\bar{g} + b\bar{b}\rangle$
- Zero net baryon number. $\mathcal{B}$=+1/3 + (−1/3) = 0

Parity of a meson:
$\pi(q\bar{q}) = \pi(q)\pi(\bar{q})(-1)^L = (+1)(-1)(-1)^L = -1^{L+1}$

Psuedo-scalar mesons: $J^\pi=0^-$ Ground state of $q\bar{q}$ combination

- Angular momentum, ***L*=0**
- Spin of quark and antiquark anti-aligned ↑↓ or ↓↑ ***S*=0**
- Total angular momentum ***J*=*L*+*S*=0**

Vector Mesons: $J^\pi=1^-$ First excited state of $q\bar{q}$ combination.

- Angular momentum, ***L*=0**
- Spin of quark and antiquark aligned ↑↑ or ↓↓ ***S*=1**
- Total angular momentum ***J*=*L*+*S*=1**

Mesons are bosons, they have integer spin: 0, 1ℏ, 2ℏ, ...

Light Mesons - Quark Model

Mesons with u, d, s quarks only.

Three quantum numbers are used to distinguish states:

- Strangeness, *S*; Isospin, *I*, I_Z; Charge, *Q*.

$$Q = I_Z + \frac{1}{2}(S + \mathcal{B})$$

- Also hypercharge Y=S+$\mathcal{B}$

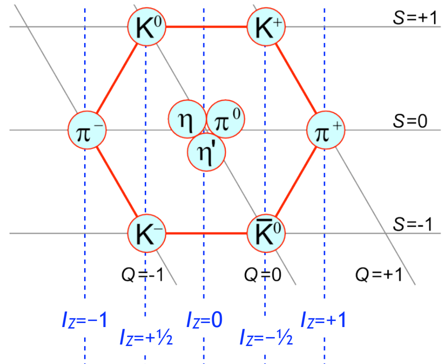
Quark flavour combinations:

- $u\bar{d}$, $u\bar{s}$, $d\bar{u}$, $d\bar{s}$, $s\bar{u}$, $s\bar{d}$ non-zero net flavour
- $u\bar{u}$, $d\bar{d}$, $s\bar{s}$ zero net flavour

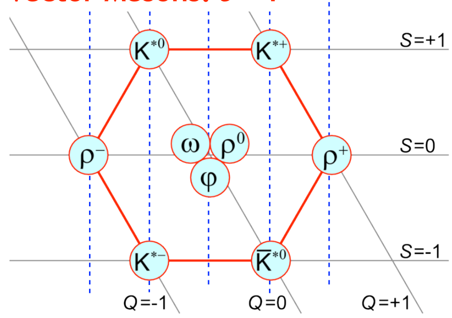
identical quantum numbers

- Physical mesons are linear superposition of these states: *e.g.* $\pi^0 = \frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$

Psudeo-scalar mesons: J^π=0⁻



Vector Mesons: J^π=1⁻



Baryons

Spin statistics theorem: see Quantum Lecture 16

- Baryons are bound state of three quarks. $\mathcal{B}$=+1/3 + 1/3 + 1/3 = 1
- **Spin statistics theorem**: Systems of identical fermions, have wavefunctions which are anti-symmetric under interchange of any pair of particle labels.
- The total baryon wavefunction is:

Ψ(total) = Ψ(space) Ψ(spin) Ψ(quark flavour) Ψ(quark colour)

- Only states with angular momentum *L*=0 are observed.

Parity of a baryon with L=0
$\pi(q_1q_2q_3) = \pi(q_1)\pi(q_2)\pi(q_3) = (+1)(+1)(+1) = +1$

- Ψ(space) is symmetric
- Spins aligned: ↑↑↑ or ↓↓↓ S=+3/2 ψ(spin) is symmetric
- Not all spins aligned: ↑↑↓ ↑↓↑ ↓↑↑ S=+1/2

Total angular momentum ***J=L+S=S***

Baryons are fermions, they have half-integer spin: 1/2ℏ, 3/2ℏ, ...

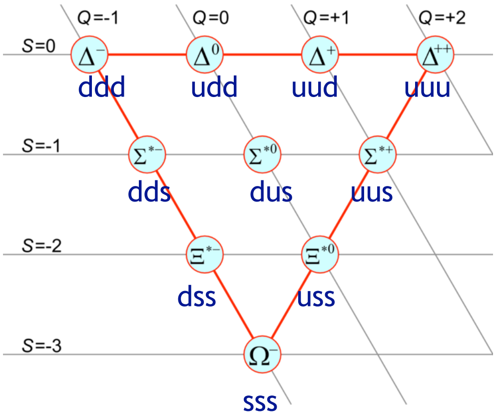
Baryon Decuplet

Baryons with J^π=3/2$^+$

- **Ψ(space)** is symmetric
- **ψ(spin)** is symmetric ↑↑↑ or ↓↓↓
- **Ψ(quark flavour)** is symmetric *e.g.* uuu, (uud+udu+duu)/√3
- **Ψ(quark colour)** must be anti-symmetric:

$$\Psi(\mathrm{colour}) = (rgb - rbg + gbr - grb + brg - bgr)/\sqrt{6}$$

The quark model predicted the unobserved Ω^- baryon and suggest colour degree of freedom for quarks.



Delta M~1232 MeV

Sigma* M~1385 MeV

Cascade* M~1232 MeV

Omega M~1672 MeV

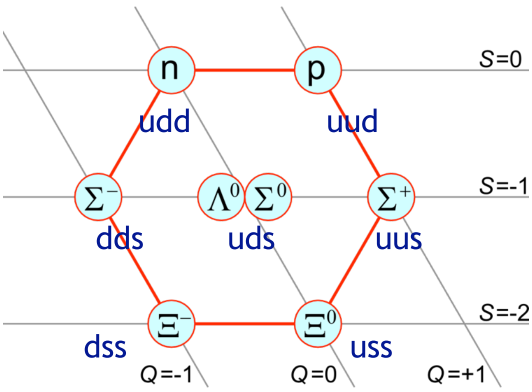
Baryon Octet

Baryons with J^π=1/2$^+$

- **Ψ(space)** is symmetric
- **ψ(spin)**
- **Ψ(quark flavour)**

} symmetric: (2u↑u↑d↓+2d↓u↑u↑+2u↑d↓u↑−u↓d↑u↑−u↑u↓d↑−u↓u↑d↑−d↑u↓u↑−u↑d↑u↓−d↑u↑u↓)/√(18)

- **Ψ(quark colour)** must be anti-symmetric.



nucleons M~940 MeV

Sigma M~1193 MeV

Lambda M~1116 MeV

Cascade M~1318 MeV

- These are the lightest baryons, stable or long-lived
- Anti-baryons also form an octet and decuplet

Hadron Masses

Hard to measure the quark mass, as we can never isolate a quark.

- At high energy, $q^2 > 1\ GeV^2$ the masses are very light:
  - $m_u < m_d \sim 5\ MeV/c^2$, $m_s \sim 100\ MeV/c^2$
- **Constituent mass** is relevant for quark model $q^2 < 1\ GeV^2$
  - $m_u = m_d \sim 300\ MeV/c^2$, $m_s \sim 500\ MeV/c^2$

- Mass is due to mass of quarks and spin couplings:

$$m(q\bar{q}) = m_1 + m_2 + A\frac{\vec{S}_1 \cdot \vec{S}_2}{m_1 m_2}$$

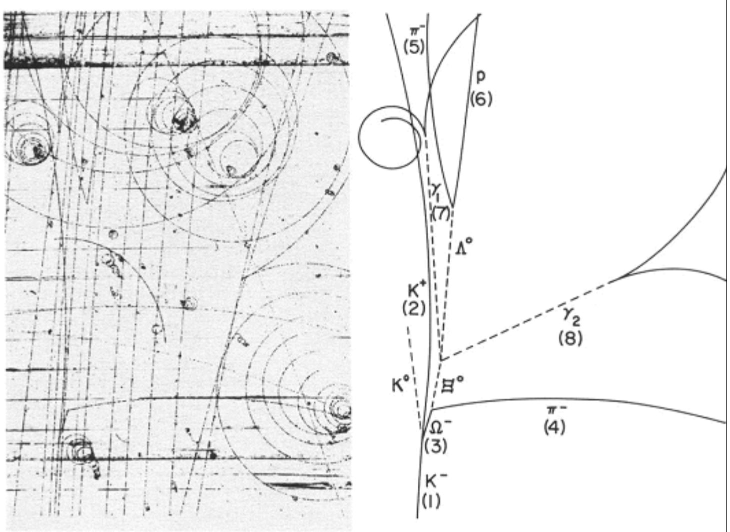
$$\begin{aligned} \vec{S}_1 \cdot \vec{S}_2 &= 1/2\,(\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2) \\ &= 1/2\,[S(S+1) - S_1(S_1+1) - S_2(S_2+1)] \\ &= \begin{cases} +1/4 & \text{for } S = 1 \\ -3/4 & \text{for } S = 0 \end{cases} \end{aligned}$$

  - $m_u = m_d = 310\ MeV/c^2$
  - $m_s = 483\ MeV/c^2$
  - $A = (2m_u)^2 \times 160\ MeV$
    - **Excellent agreement!** ➜

	Mass [MeV/c²]	
Mesons	Prediction	Experiment
π	140	138
K	484	496
ρ	780	770
ω	780	782
K*	896	894
ϕ	1032	1019

Discovery of the Ω⁻

$$\begin{aligned} K^- + p \to \Omega^- + K^- + K^0 \\ \mapsto \Xi^0 + \pi^+ \\ \mapsto \Lambda^0 + \pi^0 \\ \mapsto \gamma + \gamma \\ \mapsto e^+e^- \\ \mapsto e^+e^- \\ \mapsto \pi^- p \end{aligned}$$



The quark model predicted a Ω⁻ baryon composed of sss: the missing member of the baryon decuplet $J^{\pi}=3/2^+$.

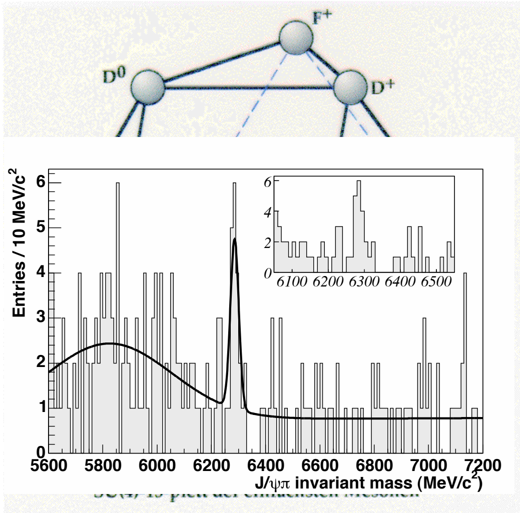
Discovered in 1964 in Brookhaven national lab: K⁻ beam onto hydrogen target: bubble chamber detector.

Heavier Mesons and Baryons

We can also use the quark model to predict hadrons with charm and bottom quarks.

Need to use more quantum numbers:

- Charge, Q (or isospin, I)
- Strangeness, S
- Charm, C and/or bottom-ness, B
- Hypercharge Y = $\mathcal{B}$+S+C+B+T



Charmed Mesons and Baryons

- $J^\pi=0^-$: $D^0 = c\bar{u}$, $D^+=c\bar{d}$, $D_S^+=c\bar{s}$
- $J^\pi=1^-$: $D^{*0} = c\bar{u}$, $D^{*+}=c\bar{d}$, $D_S^{*+}=c\bar{s}$
- $J^\pi=½^+$: $D^0 = c\bar{u}$, $D^+=c\bar{d}$, $D_S^+=c\bar{s}$

Bottom Hadrons

- $J^\pi=0^-$: $B^+ = u\bar{b}$, $B^0=d\bar{b}$, $B_S^+=s\bar{b}$
- $J^\pi=1^-$: $B^{*+} = u\bar{b}$, $D^{*+}=d\bar{b}$, $D_S^{*+}=s\bar{b}$
- $J^\pi=½^+$: Λ_b^0 = bud

Most recently discovered meson $B_C^+=\bar{b}c$

The top quark does not form hadrons!

Summary

J^π = (Total angular mom=L+S)^{parity}

Quark model explains hadrons in terms of quarks:

- Ground state of hadrons have zero angular orbital momentum L=0.
- mesons are formed of quark-anti-quark: **bosons**
  - **Pseudo-scalar mesons: $J^\pi=0^-$** **Vector Mesons: $J^\pi=1^-$**
- baryons are formed of three quarks: **fermions**
  - **Decuplet: $J^\pi=3/2^+$** **Octet: $J^\pi=1/2^+$**
- Hadrons carry the same quantum numbers as their constituent quarks

Hadrons with the same isospin (I) and strangeness (S) have roughly the same mass - due to the isospin symmetry between up and down quarks.

- Mass explained using constituent quark masses and the spin couplings.

Two useful relations: hypercharge, $Y = \mathcal{B}+S+C+B+T$; charge, $Q=I_Z+Y/2$

Baryons wave function must be anti-symmetric. Leads to the prediction of the colour quantum number.