Nuclear and Particle Physics Junior Honours:

Particle Physics

Lectures 8 & 9. The Weak Force

March 5th & 8th 2007

- Weak interactions
- Charged and neutral current
- Fermi Theory
- Beta Decay
- Muon Decay
- Lepton Universality
- Electroweak Theory & The Standard Model
- W and Z bosons
- Higgs

Introduction

The weak force is responsible for many processes:

- Decays of the muon and tau leptons
- Neutrino interactions (more to come...)
- Decays of the lightest mesons and baryons
- Radioactivity, nuclear fission and fusion

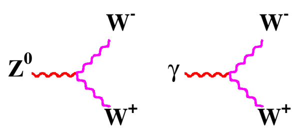
Characteristics of Weak Processes

- Long lifetimes 10^{-13} - 10^{3} s
- Small cross sections 10^{-13} mb

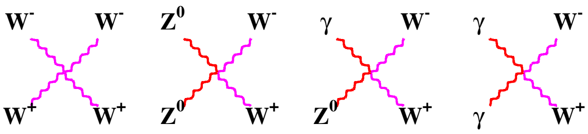
Weak Force is propagated by **massive** W^+, W^- and Z^0 bosons

- M_Z = 91.2 GeV/c^2; M_W = 80.4 GeV/c^2, both spin $1\hbar$
- Z^0 has no electric charge, $W^\pm$ has electric charge $\pm 1e$

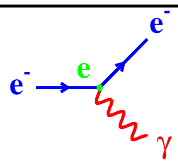
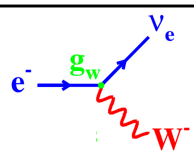
- *W* and *Z* bosons interactions different (related by symmetry of the weak interaction)



- *W* and *Z* can interact with each other
- *W* and γ interact (as *W* is charged)



Weak Vertices

QED	W-boson
mediated by the exchange of virtual photons	mediated by the exchange of W boson
acts on all charged particles	acts on all quark and leptons
coupling strength $\propto e \propto \sqrt{\alpha}$	coupling strength $\propto g_W \propto \sqrt{\alpha_W}$
propagator term: $1/(q^2 - m_\gamma^2) = 1/q^2$	propagator term: $1/(q^2 - m_W^2)$
For many processes: $\mathcal{M} \propto e^2/q^2$	For many processes: $\mathcal{M} \propto g_W^2/(q^2 - m_W^2)$
	

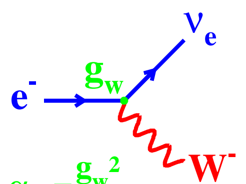
The matrix element, $\mathcal{M}$, is the amplitude of a process. The cross section, σ measures the probability - how quickly, or how often - a process happens, $\sigma \propto \mathcal{M}^2$

Weak Charged Current Interaction

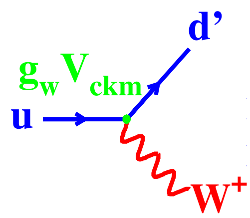
Interactions of the $W^\pm$ boson (*W*-boson is charged, hence the name)

- Weak charged current is propagated by exchange of virtual *W* bosons
- Charged current acts on **all** fermions - quarks and leptons
- An electron emitting an *W*-boson can't remain an electron - violates conservation of charge!
- Charged current **changes the flavour of the fermion**:
  - an electron turns into a electron neutrino
  - an up quark turns into a down quark

    and vice versa!
- Coupling strength at every vertex $\propto g_W$
- Propagator term describing the *W*-boson $\propto 1/(q^2 - m_W^2)$
  - q is the four-momentum transferred by the *W*-boson



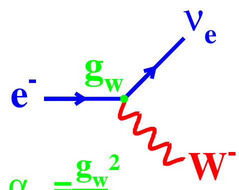
$$\alpha_w = \frac{g_w^2}{4\pi}$$



Allowed Flavour Changes

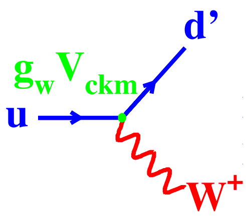
At a *W*-boson vertex:

- Lepton numbers: L_e, L_μ and L_τ, **must** be conserved:
  Allowed lepton flavour changes: $e^- \leftrightarrow \nu_e$ $\quad \mu^- \leftrightarrow \nu_\mu$ $\quad \tau^- \leftrightarrow \nu_\tau$



$\alpha_w = \frac{g_w^2}{4\pi}$

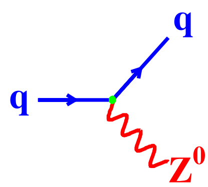
- Baryon number, $\mathcal{B}$, is conserved
- Strangeness, Charmness ... *S*, *C*, *B*, *T* are violated:
  Allowed quark flavour changes:
  (Q=+2/3 *e* quark) ↔ (Q=−1/3 *e* quark)
  (d s b) ↔ (u c t)
- Main quark flavour changes are within generations:
  d ↔ u s ↔ c b ↔ t

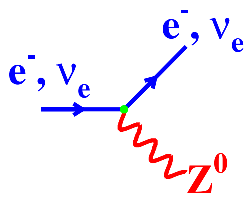


Neutral Current Weak Interactions

Interactions of the Z^0 boson

- Neutral current acts on **all** fermions - quarks and leptons
- Propagated by virtual *Z*-bosons
- Neutral current **conserves** flavour of the fermion
- Coupling strength ∝ g'_W
- Propagator describing the *Z*-boson ∝ $1/(q^2 - M_Z^2)$





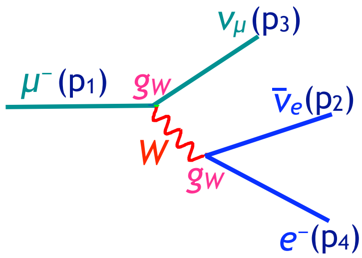
Anywhere a photon could be exchanged a Z^0 boson can be exchanged.
(Almost vice-versa, although Z^0 boson also has neutrino interactions too!)

Electromagnetic and weak neutral current interactions are linked!

Feynman Rules for Weak Interaction

How to calculate the matrix element, $\mathcal{M}$, for a decay or scattering process
$\mathcal{M}$ describes the amplitude for a process
$\sigma \propto |\mathcal{M}|^2$ is the cross section - how likely/quickly process will happen

***e.g.* decay of a muon** $\mu^- \to e^- \nu_\mu \bar{\nu}_e$

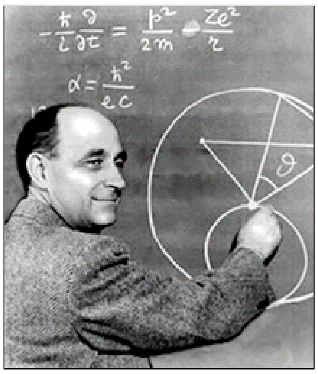


- Draw the Feynman diagram for the process
  - four momentum for each particle
- Check quantum numbers conservation at every vertex
  - For both W and Z: L_e, L_μ and L_τ, $\mathcal{B}$, Q
  - For Z only: no change of quark or lepton flavour
- Is energy and momentum conserved? mass(initial) > mass(final)
- Write down the **coupling at the each vertex**: g_W (for W) or g'_W (for Z)
- Work out the four-momentum transferred by the boson, $q = (p_3 - p_1) = (p_4 + p_2)$
- Write down the **propagator term** for each boson: $1/(q^2 - m_{boson}^2) = 1/(q^2 - m_W^2)$
- $\mathcal{M}$ is proportional to vertex couplings and propagator terms

$$\mathcal{M} \propto \frac{g_w^2}{(q^2 - m_W^2)}$$

Fermi Theory

Weak Interactions at Low Momentum Transfer



For muon decay, and many other weak processes:

$$\mathcal{M} \propto \frac{g_w^2}{(q^2 - m_W^2)}$$

At low momentum transfer, $q^2 << m_W^2$:

$$\mathcal{M} \to \propto \frac{g_w^2}{m_W^2}$$

Introduce **Fermi coupling constant:**

$$G_F \propto \frac{g_w^2}{m_W^2} \qquad G_F = \frac{\sqrt{2}\, g_w^2}{8\, m_W^2}$$

- Dimension $[E]^{-2}$
- From experimental measurements: $G_F = 1.16637 \times 10^{-5}$ GeV^{-2}

Range of interaction: massive exchange boson ↔ short range.

$$\Delta x \approx \frac{\hbar}{\Delta p} = \frac{\hbar}{m_W c} = 0.002 \text{ fm}$$

Measurements of $G_F \,\&\, M_W \;\Rightarrow\; g_w = 0.66 \;\Rightarrow\; \alpha_w = \frac{g_w^2}{4\pi} = \frac{1}{29} > \alpha_{EM} = \frac{1}{137}$

Weak interaction not intrinsically weak - appears weak due to large boson masses.