**Beta Decay**

**Weak Nuclear Decay**

- $B^+$ decay $(A,Z) \rightarrow (A,Z-1) + e^+ + \nu_e$.
- $B^-$ decay $(A,Z) \rightarrow (A,Z+1) + e^- + \bar{\nu}_e$.

Continuous energy spectrum of $e^\pm$ \(\Rightarrow\) at least two decay products. This led Pauli to postulate the neutrino.

**Nuclear Interpretation**

\[ n \rightarrow p \ e^- \bar{\nu}_e \]

**Modern quark level picture**

Weak charged current mediated by exchange of virtual $W^\pm$ boson

\[ d \rightarrow u \ W \]

\[ \rightarrow e^- \bar{\nu}_e \]

---

**Muon Decay**

How does a muon $\mu^-$ decay?

- Must decay into lighter particles: $e^\pm$, $\gamma$, $\nu$.
  - In particular, all hadrons are heavier than $m_\mu$.
- $L_e$, $L_\mu$, $L_\tau$ conservation \(\Rightarrow\) only decay is $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$

Maximum momentum transferred by the $W$ boson is

\[ q = (m_\mu - m_{\nu_\mu})c \]

\[ \sigma \propto |\mathcal{M}|^2 \propto \frac{g_W^4}{(q^2 - m_W^2)^2} \rightarrow \frac{g_W^4}{m_W^4} \propto G_F^2 \]

Width (or decay rate) $\Gamma_\mu = \hbar/\tau_\mu \propto \sigma$ measures how quickly the decay happens:

\[ \Gamma_\mu \propto G_F^2 \]

- $\Gamma$ has dimensions of energy, [E];
- $G_F^2$ has dimensions [E]$^{-4}$

To balance dimensions, use $m_\mu$ (only other scale in the problem)

\[ \Gamma_\mu = K \ G_F^2 \ m_\mu^5 \quad (K: \text{dimensionless constant}) \]

full calculation gives:

\[ \Gamma_\mu = G_F^2 \ m_\mu^5/(192 \pi^3) \]

**Experimental measurements**

- $\tau_\mu = 2.19703 \times 10^{-6}$ s
- $m_\mu = 105.658369 \times 10^5$ MeV/c$^2$

used to extract $G_F$ (and $g_W$)

\[ \Rightarrow G_F = 1.16637(1) \times 10^{-5} \text{GeV}^{-2} \]

4th year project: measure $\tau_\mu$ and $m_\mu$
Lepton Universality

Tau Lepton Decay
• \( m_\tau = 1.777 \text{ GeV}/c^2 > m_\mu, m_\pi, m_\rho, \ldots \)
• Several weak decay modes possible

Decays \( \tau^- \to e^- \bar{\nu}_e \nu_\tau \) and \( \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau \) have same matrix element as \( \mu^- \to e^- \bar{\nu}_e \nu_\mu \):

\[
\frac{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)} = \frac{0.178 \Gamma_\tau}{0.178 \Gamma_\mu} = \frac{m_\tau}{m_\mu} = 1.777 \text{ GeV}/c^2 \times \frac{m_\tau}{m_\mu} = 2.91 \times 10^{-13} \text{ s}
\]

Compare to measured \( \tau_\tau = (2.906 \pm 0.011) \times 10^{-13} \text{ s} \)

Coupling of to \( W \)-boson to all leptons is equal = \( g_W \)

LEPTON UNIVERSALITY

Weak Interactions of Quarks

How do quarks interact with the \( W \) boson?
• Quarks decays are enhanced by number of quark colours, \( N_c=3 \).
• Tau decay: \( W^- \) boson in can decay into \( \bar{d} \) or \( \bar{s} \)
• All other quarks (and hadrons) are too heavy

Coupling at \( Wd,韦v,韦s \) vertices not equal

\[
g_WV_{ud} \quad g_W \quad g_WV_{us}
\]

Quark coupling suppressed by a flavour-dependent factor \( V \):

<table>
<thead>
<tr>
<th>( V_{ud} )</th>
<th>( V_{ub} )</th>
<th>( V_{us} )</th>
<th>( V_{cb} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.974</td>
<td>0.227</td>
<td>0.004</td>
<td></td>
</tr>
<tr>
<td>0.230</td>
<td>0.972</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td>0.008</td>
<td>0.041</td>
<td>0.999</td>
<td></td>
</tr>
</tbody>
</table>

Couplings within a generations are the largest.

Main quark flavour changes are:

\( d \leftrightarrow u \quad s \leftrightarrow c \quad b \leftrightarrow t \)

Any change \( (Q=+2/3 \text{ e quark}) \leftrightarrow (Q=-1/3 \text{ e quark}) \) is allowed
Weak Hadron Decays

As the strong and electromagnetic forces conserve $S$, $C$, $B$
Lightest hadrons with non-zero $S$, $C$, $B$ quantum numbers must decay by weak force!
Consider the interactions of the constituent quarks.

- **e.g.** $\pi^+ \rightarrow \mu^+ \nu_\mu$, $K^+ \rightarrow \mu^+ \nu_\mu$

\[
\mathcal{M}(\pi^+ \rightarrow \mu^+ \nu_\mu) = \frac{g_W^2 V_{ud}}{q^2 - m_W^2}
\]

\[
\mathcal{M}(K^+ \rightarrow \mu^+ \nu_\mu) = \frac{g_W^2 V_{us}}{q^2 - m_W^2}
\]

\[
\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = \frac{V_{sd}^2}{V_{ud}^2} = 0.055
\]

**Confirmed experimentally!**

---

Electroweak Theory

**Glashow, Weinberg & Salam: Noble Prize in Physics 1979**

"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"

The weak and electromagnetic interactions are manifestations of a underlying force: the **electroweak force**.

- Couplings of the $\gamma$, $W$, $Z$ boson are related:
  \[ e = g_W \sin \theta_W = g'_W \cos \theta_W \]
- Mass of the $W$ and $Z$ bosons are related:
  \[ m_Z^2 = m_W^2 / \cos^2 \theta_W \]

Just three fundamental parameters **e.g.** $\alpha_{EM}$, $G_F$, $\sin \theta_W$ required to describe:

- couplings of $W$, $Z$ and $\gamma$ to quarks and leptons
- boson masses
- self-interactions of the $\{W,Z,\gamma\}$
W and Z bosons

Virtual W± and Z0 bosons responsible for weak decays and scattering of neutrinos e.g. \( \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \) \( \nu_e e^- \rightarrow \nu_e e^- \)

\[
q^2 = \frac{E_W^2}{c^2} - \vec{p}_W \cdot \vec{p}_W \neq m_W^2
\]

Real W± and Z0 bosons can be produced in high energy collisions: \( e^+ e^- \) or \( p\bar{p} \)

\[
e^+ e^- \rightarrow Z^0
\]

\[
e^+ e^- \rightarrow W^+ W^-
\]

\[
\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \]


Discovery of W and Z bosons at CERN in 1983 at the SpS collider.

\( E_p = E_R = 270 \text{ GeV} \)

\( p\bar{p} \rightarrow W^- \rightarrow e^- \bar{\nu}_e \) event at UA1 experiment

Nobel Prize for Physics 1984

To Carlo Rubbia and Simon van der Meer, from CERN

“For their decisive contributions to large projects, which led to the discovery of the field particles W and Z, communicators of the weak interaction.”
**W and Z boson tests at LEP**

LEP - the Large Electron Positron Collider at CERN  
The world's highest energy e+e− collider:  
- 27 km circumference.  
- Ran from 1989 to 2000  
- Energy of mass energy, \( \sqrt{s} = 89 \) to 206 GeV  
- Four experiments: Aleph, Delphi, L3, Opal

**Z-boson production**  
- Resonant production at \( \sqrt{s} = m_Z \)  
- ~4 million e+e− \( \rightarrow Z^0 \) events

**W-boson production**  
- Resonant production at \( \sqrt{s} \geq 2m_W \)  
- ~8000 e+e− \( \rightarrow Z^0 \rightarrow W^+W^- \) events

**LEP Measurements**  
- Properties of the W and Z bosons: mass, width, couplings  
- QCD measurements, weak decays  
- Precision tests of the Standard Model

---

**Z0 resonance**

- Need to consider both Z-boson and photon diagrams and interference.  
- Near \( \sqrt{s} = m_Z \), mainly Z-boson exchange:  
  \[
  \sigma(e^+e^- \rightarrow Z \rightarrow f\bar{f}) \propto \frac{g_W^4}{(q^2 - m_Z^2)^2}
  \]
  
- Need to take into account width of the boson \( \Gamma = \frac{\hbar}{\tau_Z} \), \( \tau_Z \sim 10^{-25} \) s  
  \[
  \sigma(E = q^2) = \sigma_{\text{max}} \frac{\Gamma_Z^2}{4} \left(\frac{q^2 - m_Z^2}{\Gamma_Z^2}\right)
  \]

- Measurements at LEP:  
  - \( m_Z = 91.1876 \pm 0.0021 \) GeV/c²  
  - \( \Gamma_Z = 2.49529 \pm 0.0023 \) GeV
Number of Neutrinos

Total width of the Z-boson (Γ_Z) is sum of all final state widths:
Γ_Z = Γ(Z → q̅q) + Γ(Z → e^+e^-) + Γ(Z → μ^+μ^-) + Γ(Z → τ^+τ^-) + N_ν Γ(Z → νν̅)

Lepton universality: partial widths to leptons are equal
Γ(Z → e^+e^-) = Γ(Z → μ^+μ^-) = Γ(Z → τ^+τ^-) = 83.984 ± 0.086 MeV

Z→ν̅ν leave no signal in the detector. However ... measured width depends on the number of neutrinos flavours the Z decays into.

Consistent with just three neutrinos!
⇒ Only three generations of matter ?!

W-boson

At LEP three diagrams contribute to W^+W^- production:

Predictions of electroweak model

Measurements from LEP confirms existence of Z^0 W^+ W^- vertex

Today the Tevatron also investigates W-bosons p̅p→W

From measurements at LEP & Tevatron:
• m_W = 80.413 ± 0.048 GeV/c^2
• Γ_W = 2.141 ± 0.041 GeV
The Higgs Boson

The Higgs boson: missing piece of the Standard Model.
- Interactions of the fermions with the Higgs boson explains the mass of fermions.
- Interactions of the W and Z bosons with the Higgs boson explains the mass of the W and Z bosons.
- Coupling at Higgs vertex is proportional to particle mass.

The Higgs Boson

• Interactions of the fermions with the Higgs boson - described by Fermi-bosons interactions
  - fermion flavour change
  - W-boson vertex: fermion flavour change
    - $e^-\leftrightarrow\nu_e, \mu^-\leftrightarrow\nu_\mu, \tau^-\leftrightarrow\nu_\tau$
    - (Q=+2/3 e quark) $\leftrightarrow$ (Q=-1/3 e quark)
    - quark coupling: $g_W V_{qf}$
  - Z-boson vertex: no flavour change
    - lepton coupling: $g_W$
    - propagator: $1/(q^2-m_Z^2)$

Weak Interaction Summary

<table>
<thead>
<tr>
<th>Weak force acts on all quarks and leptons.</th>
<th>Two massive bosons propagate the weak interaction: W and Z.</th>
<th>Lepton Universality: interactions of all leptons are identical.</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-boson vertex: fermion flavour change</td>
<td>$e^-\leftrightarrow\nu_e, \mu^-\leftrightarrow\nu_\mu, \tau^-\leftrightarrow\nu_\tau$</td>
<td>$Z$-boson vertex: no flavour change</td>
</tr>
<tr>
<td>$W$-boson interactions described by Fermi constant: $G_F \propto \frac{g_W^2}{m_W^2}$</td>
<td>Electromagnetic &amp; weak are manifestations of a single unified electroweak interaction. (with just 3 parameters)</td>
<td><strong>Standard Model</strong> describes electroweak and QCD. Beautifully verified by experiment, apart from missing Higgs boson.</td>
</tr>
</tbody>
</table>

Particle widths, $\Gamma = \hbar/\tau \propto \sigma$
- Total width is sum of all final states widths:
  - e.g. $\Gamma_\tau = \Gamma(\tau\rightarrow\mu\nu\nu) + \Gamma(\tau\rightarrow\nu\nu) + \Gamma(\tau\rightarrow\text{hadrons}+\nu)$
Standard Model Interactions

<table>
<thead>
<tr>
<th>QED</th>
<th>QCD</th>
<th>Weak Neutral Current</th>
<th>Weak Charged Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>quantum theory of EM interactions</td>
<td>quantum theory of strong interactions</td>
<td>quantum theory of weak interactions</td>
<td></td>
</tr>
<tr>
<td>mediated by exchange of virtual photons</td>
<td>mediated by exchange of gluons</td>
<td>mediated by exchange of Z bosons</td>
<td>mediated by exchange of W bosons</td>
</tr>
<tr>
<td>acts on all charged particles</td>
<td>acts on quarks only</td>
<td>acts on all quarks and leptons</td>
<td></td>
</tr>
<tr>
<td>couples to electric charge</td>
<td>couples to colour charge</td>
<td>does not change quark or lepton flavour</td>
<td>changes quark and lepton flavours</td>
</tr>
<tr>
<td>coupling strength $\propto e \propto \frac{1}{Q}$</td>
<td>coupling strength $\propto g_s \propto \frac{1}{Q} s$</td>
<td>coupling strength $\propto g_W' \propto \frac{1}{Q^2}$</td>
<td>coupling strength $\propto g_W \propto \frac{1}{Q^2 w}$</td>
</tr>
<tr>
<td>propagator: $1/q^2$</td>
<td>propagator: $1/q^2$</td>
<td>propagator: $1/(q^2-M^2)$</td>
<td>propagator: $1/(q^2-M^2 w^2)$</td>
</tr>
</tbody>
</table>

The massive bosons: $W, Z \&$ Higgs