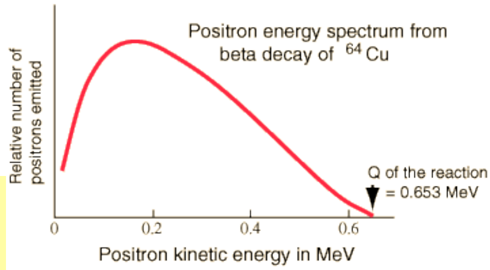
Beta Decay

Weak Nuclear Decay

β+ decay (A,Z) → (A,Z−1) + e^+ + ν_e.

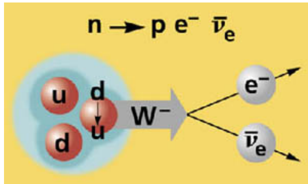
β− decay (A,Z) → (A,Z+1) + e^- + $\bar{\nu}_e$.

Continuous energy spectrum of $e^\pm$ ⇒
at least two decay products.
This led Pauli to postulate the neutrino.



Nuclear Interpretation

n→p e^- $\bar{\nu}_e$

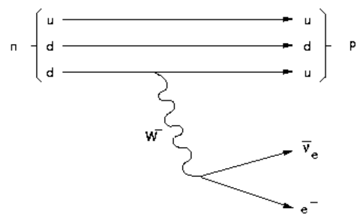


Modern quark level picture

Weak charged current mediated by exchange of virtual W± boson

d→u W

↳ e^- $\bar{\nu}_e$



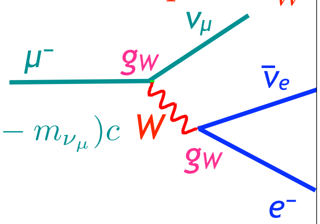
Muon Decay

How does a muon μ^- decay?

- Must decay into lighter particles: $e^\pm$, γ, ν.
  In particular, all hadrons are heavier than m_μ.

L_e, L_μ, L_τ conservation ⇒ only decay is $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$$\mathcal{M} \propto \frac{g_W^2}{q^2 - m_W^2}$$



Maximum momentum transferred by the W boson is $q = (m_\mu - m_{\nu_\mu})c$

$$\sigma \propto |\mathcal{M}|^2 \propto \frac{g_W^4}{(q^2 - m_W^2)^2} \to \frac{g_W^4}{m_W^4} \propto G_F^2$$

Width (or decay rate) $\Gamma_\mu = \hbar/\tau_\mu \propto \sigma$ measures how quickly the decay happens:

$$\Gamma_\mu \propto G_F^2$$

- Γ has dimensions of energy, [E];
- G_F^2 has dimensions $[E]^{-4}$

To balance dimensions, use m_μ (only other scale in the problem)

$$\Gamma_\mu = K\, G_F^2\, m_\mu^5 \quad (K\text{: dimensionless constant})$$

full calculation gives: $\Gamma_\mu = G_F^2\, m_\mu^5/(192\,\pi^3)$

Experimental measurements

- τ_μ = 2.19703 × 10^{-6} s
- m_μ = 105.658369×10^5 MeV/c^2

used to extract G_F (and g_W)

⇒ **G_F =1.16637(1) × 10^{-5} GeV^{-2}**

4^{th} year project:
measure τ_μ and m_μ

Lepton Universality

Tau Lepton Decay

- m_τ = 1.777 GeV/c^2 > m_μ, m_π, m_ρ, ...
- Several weak decay modes possible

Decay Mode	Branching Fraction $\mathrm{BF} = \frac{\Gamma(\tau^- \to X)}{\Gamma_\tau}$
$\tau^- \to e^- \bar{\nu}_e \nu_\tau$	BF=17.8%
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$	BF=17.4%
$\tau^- \to$ hadrons+ν_τ	BF=64.7%

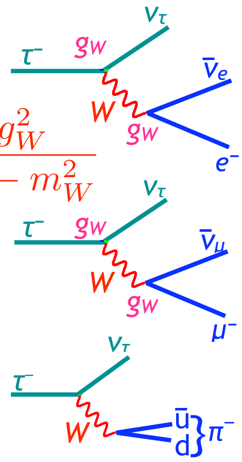
Decays $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ and $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ have same matrix element as $\mu^- \to e^- \bar{\nu}_e \nu_\tau$

$$\Gamma(\tau^- \to e^- \,\bar{\nu}_e \,\nu_\tau) = 0.178\,\Gamma_\tau \propto G_F^2\, m_\tau^5$$

$$\frac{\Gamma(\tau^- \to e^- \bar{\nu}_e \,\nu_\tau)}{\Gamma(\mu^- \to e^- \bar{\nu}_e \,\nu_\tau)} = \frac{0.178\,\Gamma_\tau}{\Gamma_\mu} = \frac{0.178\,\tau_\mu}{\tau_\tau} = \frac{m_\tau^5}{m_\mu^5}$$

$$\Rightarrow \;\; \tau_\tau = \mathrm{BF}(\tau \to e^- \nu_e \nu_\tau)\,\tau_\mu \frac{m_\mu^5}{m_\tau^5} = 2.91 \times 10^{-13}\ \mathrm{s}$$

$$\mathcal{M} \propto \frac{g_W^2}{q^2 - m_W^2}$$



Compare to measured τ_τ = (2.906 ± 0.011) × 10^{-13} s

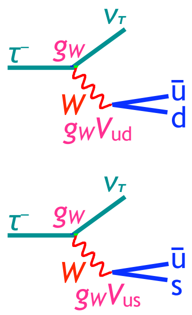
Coupling of to W-boson to all leptons is equal = g_W

LEPTON UNIVERSALITY

Weak Interactions of Quarks

How do quarks interact with the *W* boson?

- Quarks decays are enhanced by number of quark colours, N_C=3.
- Tau decay: W^- boson in can decay into ū+d or ū+s
  - All other quarks (and hadrons) are too heavy



Coupling at Wud $We\nu$ Wus vertices not equal

$g_W V_{ud}$ g_W $g_W V_{us}$

Quark coupling suppressed by a flavour-dependent factor V:

V_{ud}=0.974	V_{us}=0.227	V_{ub}=0.004
V_{cd}=0.230	V_{cs}=0.972	V_{cb}=0.042
V_{td}=0.008	V_{ts}=0.041	V_{tb}=0.999

(values from experimental measurements)

Couplings within a generations are the largest.

Main quark flavour changes are:

d ↔ u s ↔ c b ↔ t

Any change (Q=+2/3 e quark) ↔ (Q=−1/3 e quark) is allowed

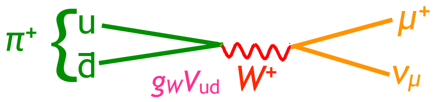
Weak Hadron Decays

As the strong and electromagnetic forces conserve *S*, *C*, *B*

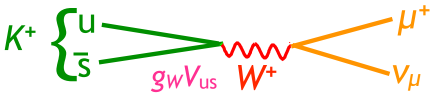
Lightest hadrons with non-zero *S*, *C*, *B* quantum numbers **must** decay by weak force!

Consider the interactions of the constituent quarks.

- *e.g.* $\pi^+ \to \mu^+ \nu_\mu$ $K^+ \to \mu^+ \nu_\mu$



$$\mathcal{M}(\pi^+ \to \mu^+ \nu_\mu) = \frac{g_W^2\, V_{\mathrm{ud}}}{q^2 - m_W^2}$$



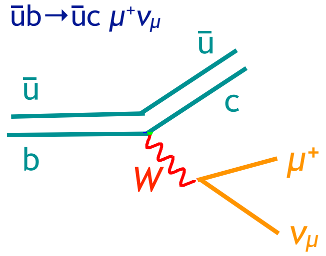
$$\mathcal{M}(K^+ \to \mu^+ \nu_\mu) = \frac{g_W^2\, V_{\mathrm{us}}}{q^2 - m_W^2}$$

$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = \frac{V_{\mathrm{sd}}^2}{V_{\mathrm{ud}}^2} = 0.055$$

Confirmed experimentally!

- *e.g.* $B^- \to D^0 \mu^+ \nu_\mu$

  $\bar{u}b \to \bar{u}c\, \mu^+ \nu_\mu$



Electroweak Theory

Glashow, Weinberg & Salam: Noble Prize in Physics 1979

"for their contributions to the theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current"

The weak and electromagnetic interactions are manifestations of a underlying force: **the electroweak force.**

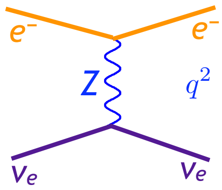
- Couplings of the γ, *W*, *Z* boson are related: $e = g_W \sin\theta_W = g_W' \cos\theta_W$
- Mass of the *W* and *Z* bosons are related: $m_Z^2 = m_W^2 / \cos^2\theta_W$

Just three fundamental parameters *e.g.* α_{EM}, G_F, $\sin\theta_W$ required to describe:

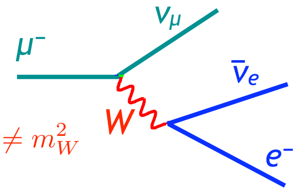
- couplings of W, Z and γ to quarks and leptons
- boson masses
- self-interactions of the {*W*,*Z*,γ}

W and *Z* bosons

Virtual $W^{\pm}$ and Z^0 bosons responsible for weak decays and scattering of neutrinos
e.g. $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ $\quad \nu_e e^- \to \nu_e e^-$

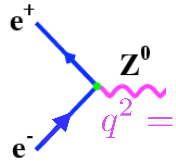


$$q^2 = \frac{E_Z^2}{c^2} - \vec{p}_Z \cdot \vec{p}_Z \neq m_Z^2$$



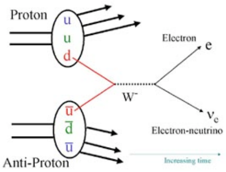
$$q^2 = \frac{E_W^2}{c^2} - \vec{p}_W \cdot \vec{p}_W \neq m_W^2$$

Real $W^{\pm}$ and Z^0 bosons can be produced in high energy collisions: e^+e^- or $p\bar{p}$



$q^2 = m_Z^2$

$e^+e^- \to Z^0$
$e^+e^- \to W^+W^-$



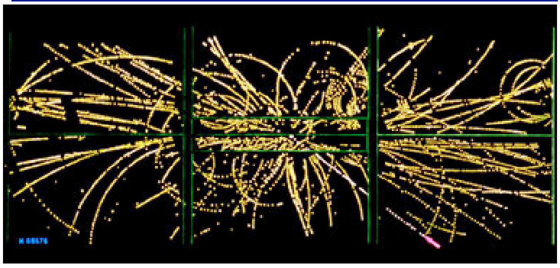
$u\bar{u} \to Z^0$, $d\bar{d} \to Z^0$
$\bar{u}d \to W^-$, $\bar{d}u \to W^+$

Detect clean signature of leptons decays: $W \to e\nu$, $W \to \mu\nu$, $Z \to ee$, $Z \to \mu\mu$

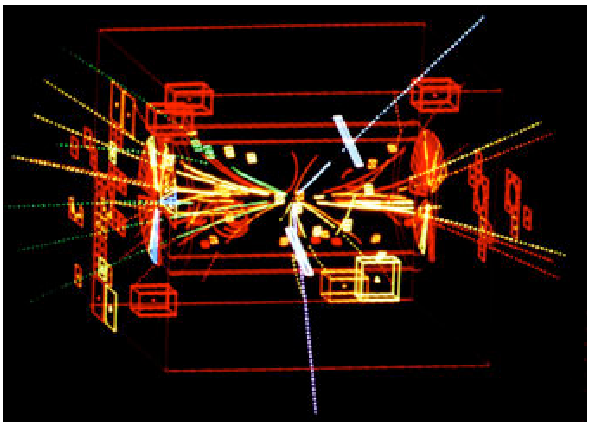
Discovery of *W* and *Z* bosons at CERN in 1983 at the $Sp\bar{p}S$ collider.

$E_p = E_{\bar{p}} = 270$ GeV

$p\bar{p} \to W^- \to e^- \bar{\nu}_e$ event at UA1 experiment



Nobel Prize for Physics 1984



To Carlo Rubbia and Simon van der Meer, from CERN

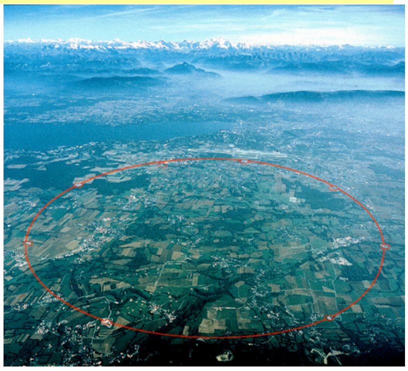
"For their decisive contributions to large projects, which led to the discovery of the field particles *W* and *Z*, communicators of the weak interaction."

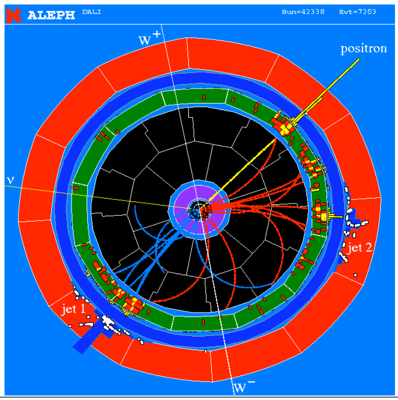
W and *Z* boson tests at LEP

LEP - the Large Electron Positron Collider at CERN

The world's highest energy e+e– collider:

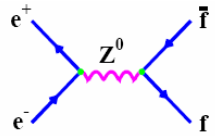
- 27 km circumference.
- Ran from 1989 to 2000
- Energy of mass energy, $\sqrt{s} = 89 \text{ to } 206 \text{ GeV}$
- Four experiments: Aleph, Delphi, L3, Opal





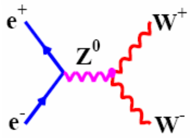
***Z*-boson production**

- Resonant production at $\sqrt{s}=m_Z$
- ~4 million $e^+e^- \to Z^0$ events



***W*-boson production**

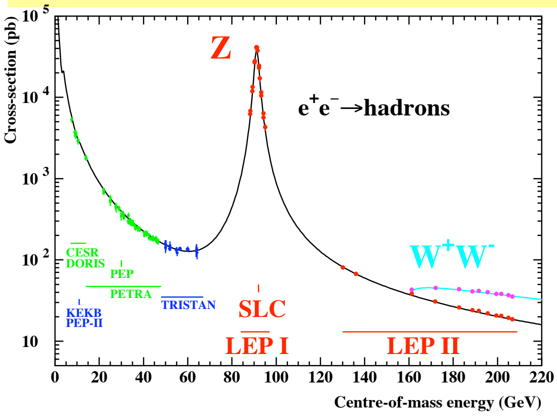
- Resonant production at $\sqrt{s} \geq 2m_W$
- ~8000 $e^+e^- \to Z \to W^+W^-$ events

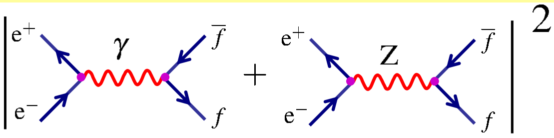


LEP Measurements

- Properties of the W and Z bosons: mass, width, couplings
- QCD measurements, weak decays
- Precision tests of the Standard Model

Z^0 resonance

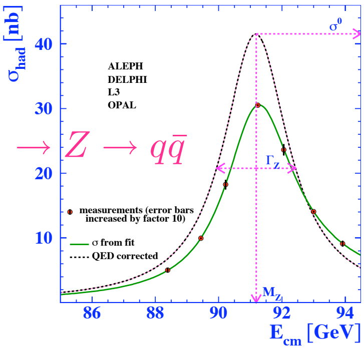




- Need to consider both *Z*-boson and photon diagrams and interference.
- Near $\sqrt{s}=m_Z$, mainly *Z*-boson exchange:

$$\sigma(e^+e^- \to Z \to f\bar{f}) \propto \frac{{g'_W}^4}{(q^2 - m_Z^2)^2}$$

$$e^+e^- \to Z \to q\bar{q}$$



- Need to take into account width of the boson $\Gamma = \hbar/\tau_Z$ $\tau_Z \sim 10^{-25}$ s

$$\sigma(E = q^2) = \sigma_{\max} \frac{\Gamma_Z^2/4}{(q^2 - m_Z)^2 + \Gamma_Z^2/4}$$

- Measurements at LEP:
  - $m_Z = 91.1876 \pm 0.0021$ GeV/c^2
  - $\Gamma_Z = 2.49529 \pm 0.0023$ GeV

Number of Neutrinos

Total width of the Z-boson (Γ_Z) is sum of all final state widths:

$$\Gamma_Z = \Gamma(Z \to q\bar{q}) + \Gamma(Z \to e^+e^-) + \Gamma(Z \to \mu^+\mu^-) + \Gamma(Z \to \tau^+\tau^-) + N_\nu \Gamma(Z \to \nu\bar{\nu})$$

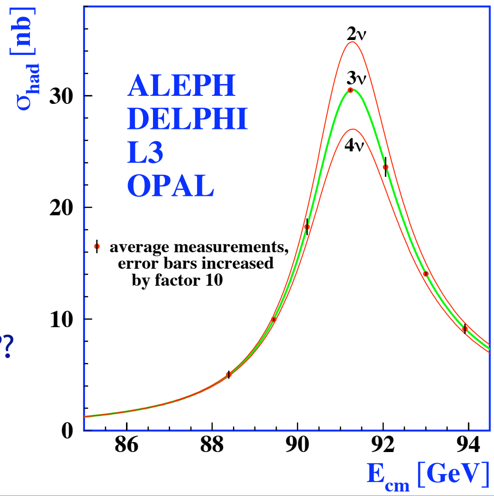
Lepton universality: partial widths to leptons are equal

$$\Gamma(Z \to e^+e^-) = \Gamma(Z \to \mu^+\mu^-) = \Gamma(Z \to \tau^+\tau^-) = 83.984 \pm 0.086 \text{ MeV}$$

$Z \to \nu\,\bar{\nu}$ leave no signal in the detector.
However ... measured width depends on the number of neutrinos flavours the Z decays into.

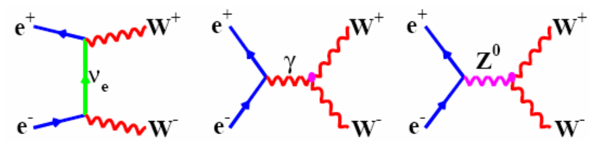
Consistent with just three neutrinos!

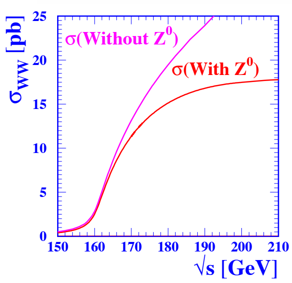
⇒ Only three generations of matter ??



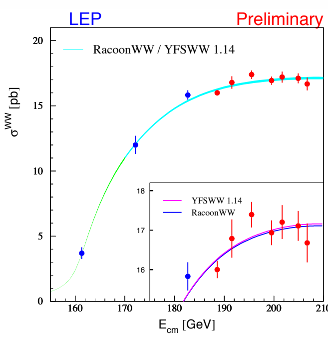
W-boson

At LEP three diagrams contribute to W^+W^- production:



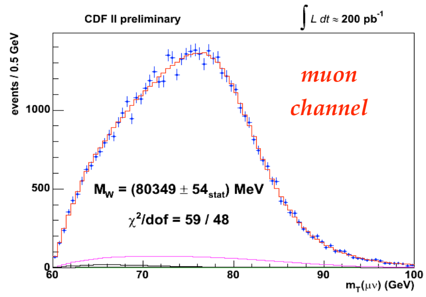


predictions of electroweak model



Measurements from LEP confirms existence of $Z^0\,W^+\,W^-$ vertex

Today the Tevatron also investigates W-bosons
$p\bar{p} \to W$



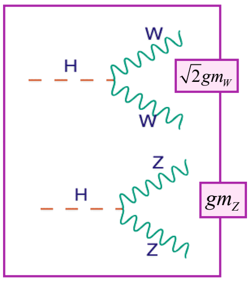
From measurements at LEP & Tevatron:

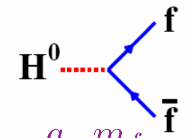
- m_W = 80.413 ± 0.048 GeV/c²
- Γ_W = 2.141 ± 0.041 GeV

The Higgs Boson

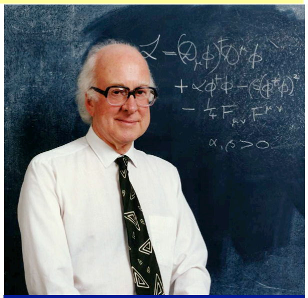
The Higgs boson: missing piece of the Standard Model.

- Interactions of the fermions with the Higgs boson explains the mass of fermions.
- Interactions of the *W* and *Z* bosons with the Higgs boson explains the mass of the *W* and *Z* bosons.
- Coupling at Higgs vertex is proportional to particle mass.



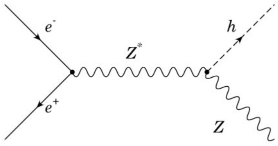


$$\frac{g_w\, m_f}{2\, m_W}$$



Peter Higgs
emeritus professor in the School of Physics

LEP searched for the Higgs $e^+e^- \to Z \to ZH$ with no success! $m_{Higgs} > 114.4$ GeV/c^2.



Finding the Higgs would validate the Standard Model:
One of the main reasons for building the LHC collider!

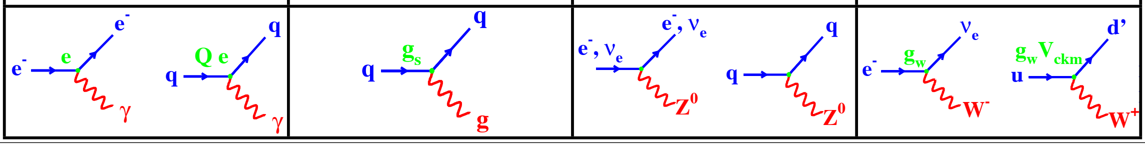
Weak Interaction Summary

<table>
<tr><td>Weak force acts on all quarks and leptons.</td><td>Two massive bosons propagate the weak interaction: W and Z.</td><td>Lepton Universality: interactions of all leptons are identical.</td></tr>
<tr><td colspan="2">W-boson vertex: fermion flavour change
$e^- \leftrightarrow \nu_e$ $\mu^- \leftrightarrow \nu_\mu$ $\tau^- \leftrightarrow \nu_\tau$
(Q=+2/3 e quark) ↔ (Q=−1/3 e quark)
quark coupling: $g_W V_{qq'}$ lepton coupling: g_W
propagator term: $1/(q^2 - m_W^2)$</td><td>Z-boson vertex:
no flavour change

coupling: g'_W
propagator: $1/(q^2 - m_Z^2)$</td></tr>
<tr><td rowspan="2">At low energies ($q^2 << m_W^2$) W-bosons interactions described by Fermi constant:
$G_F \propto \frac{g_w^2}{m_W^2}$</td><td>Electromagnetic & weak are manifestations of a single unified electroweak interaction.
(with just 3 parameters)</td><td>Standard Model describes electroweak and QCD. Beautifully verified by experiment, apart from missing Higgs boson.</td></tr>
<tr><td colspan="2">Particle widths, $\Gamma = \hbar/\tau \propto \sigma$ $\mathrm{BF} = \Gamma(\tau^- \to X)/\Gamma_\tau$
Total width is sum of all final states widths:
e.g. $\Gamma_\tau = \Gamma(\tau \to \mu\nu\nu) + \Gamma(\tau \to e\nu\nu) + \Gamma(\tau \to \text{hadrons} + \nu)$</td></tr>
</table>

Standard Model Interactions

QED	QCD	Weak Neutral Current	Weak Charged Current
quantum theory of EM interactions	quantum theory of strong interactions	quantum theory of weak interactions	
mediated by exchange of virtual photons	mediated by exchange of gluons	mediated by exchange of *Z* bosons	mediated by exchange of *W* bosons
acts on all charged particles	acts on quarks only	acts on **all** quarks and leptons	
couples to electric charge	couples to colour charge	does not change quark or lepton flavour	changes quark and leptons flavours
coupling strength ∝ $e \propto \sqrt{\alpha}$	coupling strength ∝ $g_S \propto \sqrt{\alpha_S}$	coupling strength ∝ g'_W	coupling strength ∝ $g_W \propto \sqrt{\alpha_W}$
propagator: $1/q^2$	propagator: $1/q^2$	propagator: $1/(q^2 - M_Z^2)$	propagator: $1/(q^2 - M_W^2)$



The massive bosons: *W*, *Z* & Higgs