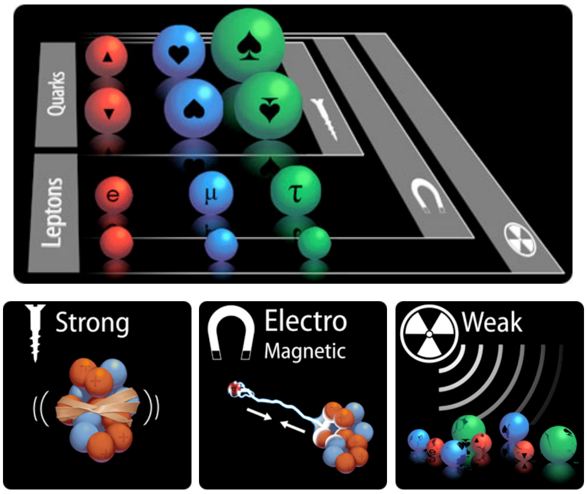
## Nuclear and Particle Physics Junior Honours:

# Particle Physics

## Review Lecture

March 19th 2007



- Natural Units
- Relativistic Dynamics
- Anti-matter
- Quarks, Leptons & Hadrons
- Feynman Diagrams and Feynman Rules
- Decays
- QED, QCD, Weak
- What you don’t need to know

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# The Standard Model

The Standard Model describes more-or-less everything we currently know about particle physics: the matter **particles** and the three of the four forces which describe their **interactions**.

**Matter:** aka the fermions

| Leptons | | | Charge, e |
|---|---|---|---|
| $\nu_e$<br>$e^-$ | $\nu_\mu$<br>$\mu^-$ | $\nu_\tau$<br>$\tau^-$ | 0<br>−1 |
| Quarks | | | |
| u<br>d | c<br>s | t<br>b | +2/3<br>−1/3 |

**Forces**

- Interactions are propagated by the exchange of bosons

| Interaction | Bosons | Q, e |
|---|---|---|
| Strong | gluons, g | 0 |
| Electro-magnetic | photon, γ | 0 |
| Weak | $W^\pm$, $Z^0$ | 0, ±1 |
| Gravity | ? | ? |

Two processes/quantities used to examine interactions:

- Decay lifetimes
- Scattering cross sections

# Relativistic Dynamics

Relativistic Dynamics is used to describe **decays** and **scattering**.

Four momentum:

$$\underline{\underline{p}} = (E/c, p_x, p_y, p_z) = (E/c, \vec{p})$$

If we square four-momentum:

$$\underline{\underline{p}}^2 = \frac{E^2}{c^2} - \vec{p}\cdot\vec{p} = m^2 c^2$$ we get the mass squared!

- In a **decay** the four-momentum is conserved *e.g.* $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$$\underline{\underline{p}}_\mu = \underline{\underline{p}}_e + \underline{\underline{p}}_{\bar{\nu}_e} + \underline{\underline{p}}_{\nu_\mu}$$ Square both sides: $$m_\mu^2 c^2 = (\underline{\underline{p}}_e + \underline{\underline{p}}_{\bar{\nu}_e} + \underline{\underline{p}}_{\nu_\mu})^2$$

- In a **scattering** the four-momentum is conserved *e.g.* $e^+e^- \to \mu^+\mu^-$

$$\underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-} = \underline{\underline{p}}_{\mu^+} + \underline{\underline{p}}_{\mu^-}$$

- The square of the initial four momentum is called ***s***. The energy in the Centre of Mass frame is **$\sqrt{s}$,** this is the energy available to make new particles, *e.g.*

$$s = (\underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-})^2$$

In both decay and scattering: the boson transfers the momentum from the final to the initial state!

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# Natural Units

- Natural units: set **$c = \hbar = 1$**
  - All quantities can be expressed as a power of energy.
- Mass, momentum and energy measured in the same units: MeV or GeV

- Two important quantities for Lorentz transformations:

$$\beta = v/c \qquad\qquad \gamma(v) = 1/\sqrt{1-\beta^2}$$

**Natural Units**

Lorentz boosts: $\gamma = E/m \quad \gamma\beta = |\vec{p}|/m \quad \beta = |\vec{p}|/E$

Four momentum: $\underline{\underline{p}} = (E,\, p_x,\, p_y,\, p_z)$

**Invariant mass** $\underline{\underline{p}}^2 = E^2 - \vec{p}^{\,2} = m^2$

# Matter

**Six quarks and six leptons**

Matter is grouped into three **generations**.
Each generation consists of:

- 1 lepton with Q=−1e
- 1 neutral lepton Q=0 ($\nu$)
- 1 quark with Q=+2/3e
- 1 quark with Q=−1/3e

Each generation is successively heavier.

| 1st | 2nd | 3rd | Q |
|---|---|---|---|
| $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ | 0 |
| $e-$ | $\mu-$ | $\tau-$ | −1e |
| u | c | t | +2/3e |
| d | s | b | −1/3e |

**Quantum Numbers**

- **Leptons** $L_e$, $L_\mu$, $L_\tau$

| | $L_e$ | $L_\mu$ | $L_\tau$ |
|---|---|---|---|
| $\nu_e$, $e-$ | +1 | 0 | 0 |
| $\nu_\mu$, $\mu-$ | 0 | +1 | 0 |
| $\nu_\tau$, $\tau-$ | 0 | 0 | +1 |

- **Quarks:**
  - Isospin, $I_Z=\frac{1}{2}[N(u)-N(d)+N(\bar{d})-N(\bar{u})]$
  - Baryon number, $\mathcal{B}$=1/3 for quarks, $\mathcal{B}$=−1/3 for anti-quarks
  - Strangeness: $S$, Charm: $C$, Bottomness: $B$, Topness: $T$ - number of s, c, b, t
    $S=N(\bar{s})-N(s)$ $C=N(c)-N(\bar{c})$ $B=N(\bar{b})-N(b)$ $T=N(t)-N(\bar{t})$
  - Every quark carries a colour charge: red, blue or green

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# Anti-matter

Every matter particle has an anti-matter partner.

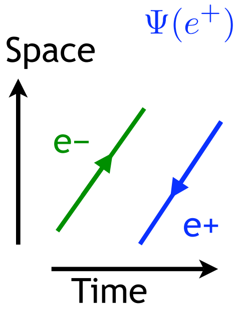
$$E^2=\vec{p}^{\,2}c^2+m^2c^4 \Rightarrow E=\pm\sqrt{\vec{p}^{\,2}c^2+m^2c^4}$$

- Particle is the positive energy state
- Anti-particle is negative energy state

$$\Psi(e^-)=A\,e^{i[Et-\vec{p}\cdot\vec{x}]/\hbar}$$

$$\Psi(e^+)=A\,e^{i[(-E)(-t)-(-\vec{p})\cdot(-\vec{x})]/\hbar}$$

| Leptons | | | Charge |
|---|---|---|---|
| $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ | 0 |
| $e-$ | $\mu-$ | $\tau-$ | −1 |
| Quarks | | | |
| u | c | t | +2/3 |
| d | s | b | −1/3 |



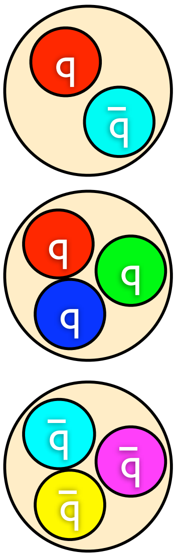
**Feynman's interpretation:** negative energy particle with charge $Q$ moving backward in space & time appears as positive energy particle with charge $-Q$ moving forward in space & time.

| Anti-leptons | | | Charge |
|---|---|---|---|
| $\bar{\nu}_e$ | $\bar{\nu}_\mu$ | $\bar{\nu}_\tau$ | 0 |
| $e+$ | $\mu+$ | $\tau+$ | +1 |
| Anti-quarks | | | |
| $\bar{u}$ | $\bar{c}$ | $\bar{t}$ | −2/3 |
| $\bar{d}$ | $\bar{s}$ | $\bar{b}$ | +1/3 |

Anti-matter particle has:

- Opposite electric charge, opposite colour charge
- Same mass & lifetime
- Opposite $\mathcal{B}$, $S$, $C$, $B$, $T$, $I_Z$, $L_e$, $L_\mu$ & $L_\tau$

# Hadrons



**Free quarks are never observed.**

Quarks are always found in bound colour-neutral states:

- Mesons: a quark and an anti-quark
- Baryons: three quarks
- Anti-baryons: three anti-quarks

**Colour confinement**

- The quarks are confined to hadrons due to QCD
- gluon self-interactions... & coupling constant $\alpha_S$ increases as quarks become further apart

**Interactions**

- Consider the interactions of the individual quarks

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# Forces & Interactions

**Three forces to consider: strong (QCD), electromagnetic (QED) & weak.**

- Weak force has two parts: charged current and neutral current

**Forces are propagated by the exchange of bosons.**

- Bosons exchange four momentum, $\underline{\underline{q}}$ , between the initial and final state

Strength of interaction is acts on some properties of the particle, *e.g.* electromagnetic force is couples to electric charges of interacting particles

| | | |
|---|---|---|
| **Strong** | exchange of gluons | couples to colour charge |
| **Electromagnetic** | exchange of photons | couples to electric charge |
| **Weak Neutral Current** | exchange of $Z^0$ boson | couples to all fermions with strength $g'_W$ |
| **Weak Charged Current** | exchange of $W^{\pm}$ boson | couples to all fermions with strength $g_W$ |

The exchanged bosons are often **virtual** (as opposed to real).

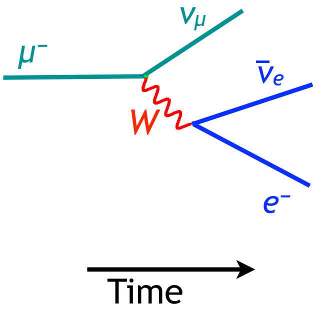
Virtual: square of four momentum is not mass squared: $\underline{\underline{q}}^2 = E^2 - \vec{p}\cdot\vec{p} \neq m^2_{\text{boson}}$

Allowed by HUP; we can never directly detect virtual bosons: only their effects.

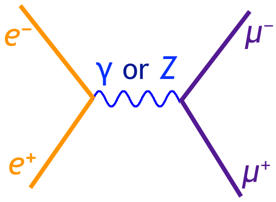
# Feynman Diagrams

Feynman diagrams are used to **illustrate** and **calculate** decays and scattering.

- e.g. muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$



- e.g. $e^+e^- \to \mu^+\mu^-$ scattering



Use the Feynman Rules to calculate the matrix element, $\mathcal{M}$, from diagram

- For decay the width of the decay, Γ, is proportional to $\mathcal{M}^2$
- For scattering the cross section, σ, is proportional to $\mathcal{M}^2$

Use four momentum conservation to calculate boson four momentum, $\underline{\underline{q}}$

- Muon decay

$$\underline{\underline{q}} = \underline{\underline{p}}_\mu - \underline{\underline{p}}_{\nu_\mu} = \underline{\underline{p}}_e + \underline{\underline{p}}_{\nu_e}$$

- $e^+e^- \to \mu^+\mu^-$ scattering

$$\underline{\underline{q}} = \underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-} = \underline{\underline{p}}_{\mu^+} + \underline{\underline{p}}_{\mu^-}$$

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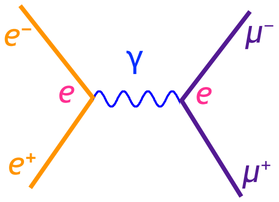
# Feynman Rules

The matrix element, $\mathcal{M}$, is the amplitude, per unit time, for a given process to happen.

We calculate $\mathcal{M}$ from:
- the vertex couplings at the vertex
- the boson propagator term



## The Feynman Rules

- Write down the **coupling at the each vertex**:
  - *charge of the fermion* (for EM), $g_s$ (for QCD), $g'_W$ (for $Z$)
  - $g_W$ (for $W$-$\ell$-$\nu$ vertex), $g_W V_{qq'}$ (for $W$-$q$-$q'$ vertex),
- Work out the four-momentum transferred by the boson, $\underline{\underline{q}}$
- Write down the **propagator term** for each boson: $1/(\underline{\underline{q}}^2 - m^2_{\text{boson}})$

$\mathcal{M}$ is proportional to vertex couplings and propagator terms *e.g.*

$$\mathcal{M}(e^+e^- \to \mu^+\mu^-) = e^2/\underline{\underline{q}}^2$$

- If process involves hadrons consider interactions of the constituent quarks

# Decays

We use **decays** and **scattering cross section** to understand interactions.

- A decay can only occur if Mass(initial) > Mass(final)
- The stronger the interaction, the quicker the particle will decay.

Measurable quantities:

- **lifetime: $\tau$** Dimensions: time.
- **total width: $\Gamma = \hbar/\tau$** Dimensions: energy.
- **Width of individual decay mode** *e.g.* $\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$

| Force | Typical Lifetimes |
|---|---|
| Strong | $10^{-20}$ - $10^{-23}$ s |
| EM | $10^{-20}$ - $10^{-16}$ s |
| Weak | $10^{-13}$ - $10^{3}$ s |

$$\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) \propto \left(\mathcal{M}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)\right)^2$$

- The total width is the sum of **all** the individual decay modes *e.g.*

$$\Gamma_\tau = \Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) + \Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) + \Gamma(\tau^- \to \nu_\tau + \text{hadrons})$$

- The **branching fraction** is the fraction of particles will decay into a particular final state *e.g.*

$$\text{BF}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = \frac{\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{\Gamma_\tau}$$

- The sum of all possible branching fractions adds up to 1.

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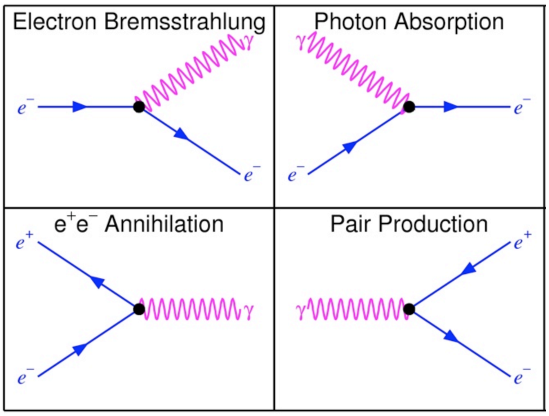
# Quantum Electrodynamics

**QED is quantum theory of electromagnetic interactions.**

- All charged particles interact via QED
- All interactions are described by fermion-fermion-photon (γ) vertex:
  - Fermion emits or absorbs a photon
  - γ→fermion anti-fermion or fermion anti-fermion→γ
- Fermion flavour does not change when it emits or absorbs a photon *e.g.* an e− remains an e−, b-quark remains a b-quark

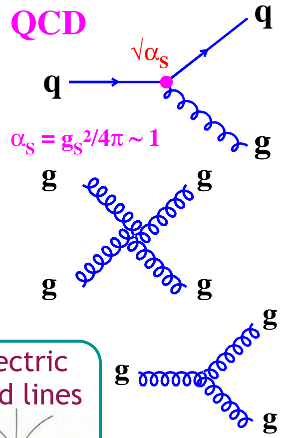


- Strength of vertex is proportional to charge of fermion
- Cross sections, decay width ∝ $\mathcal{M}^2$ write in powers of $\alpha$

$$\alpha = \frac{e^2}{4\pi\epsilon_0} \approx \frac{1}{137}$$

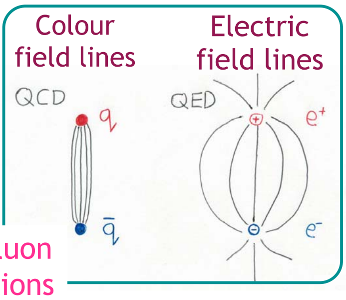
QED conserves:
$Q$, $I_Z$, $S$, $C$, $B$, $T$, $\mathcal{B}$, $L_e$, $L_\mu$, $L_\tau$

# Quantum Chromodynamics



**QCD is quantum theory of strong interactions.**

- Acts on colour charged *i.e.* only quarks and gluons interact via QCD
- quark-quark-gluon vertex:
  - A quark emits or absorbs a gluon
  - gluon→quark anti-quark or quark anti-quark→gluon
- Quark flavour does not change, but colour charge changes



- As gluons also carry colour charge, the gluons interact with other gluons
- Potential between two quarks is:

$$V_{\mathrm{QCD}}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

quark-gluon interaction

gluon-gluon interactions

- Almost impossible to pull two quarks apart: **colour confinement**

QCD conserves:
$Q, I_Z, S, C, B, T, \mathcal{B}, L_e, L_\mu, L_\tau$

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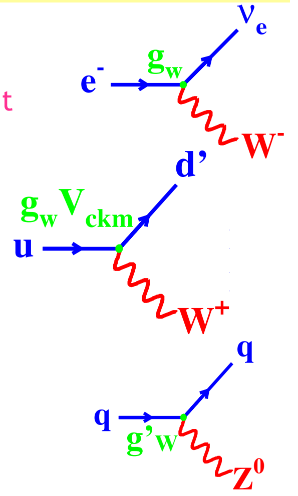
# Weak Interactions



Charged Current

Weak Force is propagated by massive $W^\pm$ & $Z^0$ bosons

Neutral Current

Weak force interacts on all quarks and leptons.

Charged current **changes the flavour of the fermion**:

- Allowed flavour changes: $\mathcal{B}$, $L_e$, $L_\mu$ and $L_\tau$ conserved

$e^- \leftrightarrow \nu_e$ $\mu^- \leftrightarrow \nu_\mu$ $\tau^- \leftrightarrow \nu_\tau$ $e^+ \leftrightarrow \bar{\nu}_e$ $\mu^+ \leftrightarrow \bar{\nu}_\mu$ $\tau^+ \leftrightarrow \bar{\nu}_\tau$

(Q=+2/3 quark) ↔ (Q=−1/3 quark)

(Q=−2/3 anti-quark) ↔ (Q=+1/3 anti-quark)

Strength of charged current:

- Leptons vertices, universal coupling: $g_W$
- Quark vertices, depends on quark flavour *e.g.* for Wud: $g_W V_{ud}$

Neutral current no **fermion flavour change**, coupling: $g'_W$

Handy hint: neutrinos are only
involved in weak interactions.

Weak force conserves:
$Q, \mathcal{B}, L_e, L_\mu, L_\tau$

# Weak Interactions at Low Energy

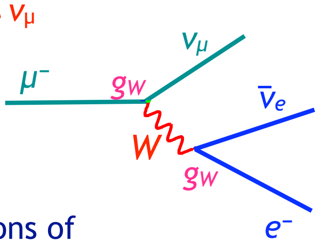
If four-momentum $\underline{\underline{q}}$ transferred by a *W*-boson is small, $\underline{\underline{q}} \ll M_W^2$ use Fermi constant, $G_F$, to describe process

$$G_F = \frac{\sqrt{2}\, g_w^2}{8\, m_W^2}$$

*e.g.* Fermi constant can be used to describe beta decay rates

*e.g.* muon decay has one allowed decay mode $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

$$\mathcal{M} \propto \frac{g_W^2}{\underline{\underline{q}}^2 - m_W^2} \longrightarrow \frac{g_W^2}{m_W^2} \propto G_F$$



Decay width: $\Gamma \propto \mathcal{M}^2 \propto G_F^2$

To balance the dimensions use something with dimensions of energy: use $m_\mu$ as lifetime will depend on $m_\mu$.

$$\Gamma_\mu = \Gamma(\mu \to e^- \bar{\nu}_e \nu_\mu) = K G_F^2\, m_\mu^5$$

K: dimensionless constant

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# What you don’t need to know…

**The masses of the particles**; they are **given on the constant sheet!** Except:

- neutrino mass is so small you can always ignore it $m_\nu \approx 0$!
- electron mass so small you can ignore it compared to other masses.
- *W* and *Z* bosons are much more massive than all lepton and hadron masses.

**The lifetimes of the particles**, they will be given if required. But remember typical lifetimes for the different forces.

**The quark content of the hadrons**. Except, handy to remember:

- proton is uud      anti-proton is: $\bar{u}\bar{u}\bar{d}$
- neutron is udd     anti-neutron is: $\bar{u}\bar{d}\bar{d}$

You can work out the charge of a particle from its symbol *e.g.* $Q(\Delta^{++}) = +2e$

- exceptions:
  - p and n don’t have superscript (but I hope you know the charge of these)
  - quarks have charge $+2/3e$, $-1/3e$