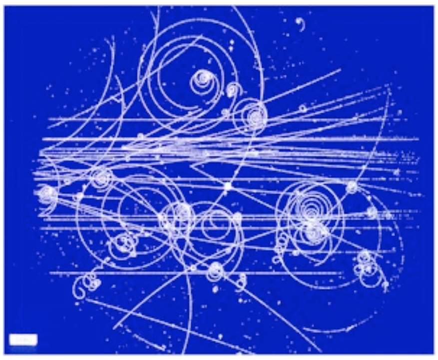
Physics 3:

Particle Physics

Lecture 2: Practical Particle Physics

February 14th 2008

How do we study the particles and the forces?
What properties of particles, decays & scatterings can we measure?



- Particle Properties
- Natural Units
- Relativistic kinematics
- Decay Properties
- Scattering Properties

Particle Properties

Static properties of the fundamental particles and the hadrons *e.g.*:

- Mass, m charge, Q
- spin, J parity, π

We assign additional quantum numbers to each particle:

- Lepton number, L: Number of leptons – number of anti-leptons
  - Electron number, L_e $L_e = N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e)$
  - Muon number, L_μ $L_\mu = N(\mu^-) - N(\mu^+) + N(\nu_\mu) - N(\bar{\nu}_\mu)$
  - Tau number, L_τ $L_\tau = N(\tau^-) - N(\tau^+) + N(\nu_\tau) - N(\bar{\nu}_\tau)$
- Quark number, N_q: Number of quarks – number of anti-quarks
- Baryon number, $\mathcal{B}$, $= 1/3\ N_q$

All of these quantum numbers (including Q) are conserved in all reactions: strong, electromagnetic and weak.

Natural Units I

kg m s

SI units: [M] [L] [T]

- For everyday physics SI units are a natural choice: **M(JH student)~80kg.**
- Not so good for particle physics: $M_{proton} \sim 10^{-27}$kg
- PP chooses a different basis - **Natural Units**, based on the language of particle physics: quantum mechanics and relativity.
- The basis of natural units are:
  - ★ unit of action in QM: $\hbar$ (Js)
  - ★ velocity of light: c (ms^{-1})
  - ★ Unit of energy: **GeV** $= 10^9$ eV $= 1.60 \times 10^{-10}$ J

Energy GeV	Time $(\mathrm{GeV}/\hbar)^{-1}$
Momentum GeV/c	Length $(\mathrm{GeV}/\hbar c)^{-1}$
Mass GeV/c^2	Area $(\mathrm{GeV}/\hbar c)^{-2}$

Natural Units II

Simplify even further by choosing $c = \hbar = 1$!

All quantities are expressed in powers of GeV

Energy GeV	Time GeV^{-1}
Momentum GeV	Length GeV^{-1}
Mass GeV	Area GeV^{-2}

Convert to SI units by reintroducing missing factors of $\hbar$ and c

- Example: Area = 1 GeV^{-2}

$$[L]^2 = [E]^{-2}[\hbar]^n[c]^m = [E]^{-2}\,[E]^n[T]^n\,[L]^m[T]^{-m} \quad n=2, m=2$$

Area (in SI units) = 1 $\mathrm{GeV}^{-2} \times \hbar^2 c^2 = 3.89 \times 10^{-32}$ m^2 = 0.389 mb

Other common units:

- Masses and energies measured in MeV
- cross section measured in barn, b ≡ 10^{-28} m^2
- lengths in fm = 10^{-15} m
- electric charge in units of e

Two useful relations: $\hbar c = 197$ MeV fm $\qquad \hbar = 6.582 \times 10^{-22}$ MeV s

Review: Relativistic Dynamics

Please review dynamics and relativity lectures 13-15.

- Two important quantities for Lorentz transformations:

$$\beta = v/c \qquad \gamma(v) = 1/\sqrt{1-\beta^2}$$

- Four-momentum of a particle: $\underline{\underline{p}} = (E/c,\, p_x,\, p_y,\, p_z) = (E/c, \vec{p})$
- Energy of a particle $\quad E^2 = \vec{p}^{\,2}c^2 + m^2c^4 \qquad E = \gamma m c^2$
- Scalar product of 4-momentum: $(\underline{\underline{p}})^2 = (E/c)^2 - \vec{p}^{\,2} = m^2c^2$
- Particles with m=**0** travel at the speed of light

Natural Units

Lorentz boosts: $\gamma = E/m \qquad \gamma\beta = |\vec{p}|/m \qquad \beta = |\vec{p}|/E$

Four momentum: $\underline{\underline{p}} = (E,\, p_x,\, p_y,\, p_z) = (E, \vec{p})$

Invariant mass $\quad (\underline{\underline{p}})^2 = E^2 - \vec{p}^{\,2} = m^2$

Particle Decay

- Almost all particles decay. *e.g.* K_S meson can decay as:

$$K_S \to \pi^+\pi^-$$

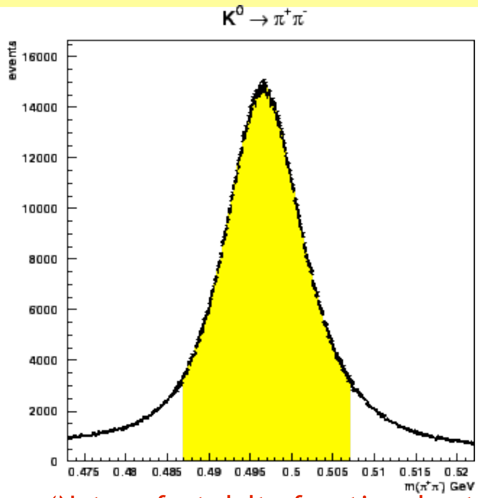
- We can reconstruct the mass of a particle using the kinematics of the decay products.

$$\sum \underline{\underline{p}}_{\text{initial}} = \sum \underline{\underline{p}}_{\text{final}}$$

$$\underline{\underline{p}}(K_S) = \underline{\underline{p}}(\pi^+) + \underline{\underline{p}}(\pi^-)$$

  - squaring each side ...

$$\begin{aligned}(M(K_S))^2 &= \left(\underline{\underline{p}}(\pi^+) + \underline{\underline{p}}(\pi^-)\right)^2 \\ &= \left(\underline{\underline{p}}(\pi^+)\right)^2 + \left(\underline{\underline{p}}(\pi^-)\right)^2 + 2\underline{\underline{p}}(\pi^+)\cdot\underline{\underline{p}}(\pi^-) \\ &= \left(m(\pi^+)\right)^2 + \left(m(\pi^-)\right)^2 + 2\,E(\pi^+)E(\pi^-) - 2\,\vec{p}(\pi^+)\cdot\vec{p}(\pi^-)\end{aligned}$$



(Not perfect delta function due to experimental uncertainties)

- $M(K_S)$ is reconstructed **invariant mass** of K_S

Particle Lifetime

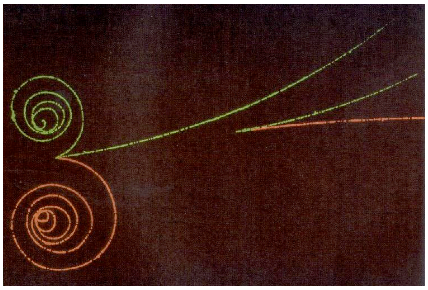
- **Particle lifetime**, τ, the time taken for the sample to reduce to $1/e$ of original sample.
- Different forces have different typical lifetimes.
- Also define **total decay width**, $\Gamma \equiv \hbar/\tau$.

$$\frac{dN}{dt} = -\frac{t}{\tau} = -\frac{\Gamma}{\hbar}t$$

$$\Rightarrow \quad N(t) = N_0 \exp(-t/\tau) = N_0 \exp(-\Gamma t/\hbar)$$

Force	Typical Lifetime
Strong	10^{-20} - 10^{-23} s
Electromag	10^{-20} - 10^{-16} s
Weak	10^{-13} - 10^{3} s

- In its own rest frame particle travels $v\tau = \beta c\tau$ before decaying.
- In the lab, time is dilated by γ.
- If τ is large enough, energetic particles travel a measurable distance $L = \gamma\beta c\tau$ in lab.

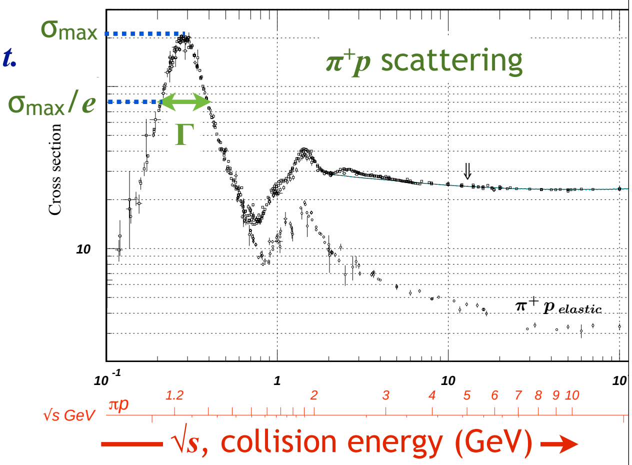


Particle Width

- Lifetimes can be very short, *e.g.* lifetime of Δ^{++} baryon (**uuu**) is 5×10^{-24} s.
- Heisenburg Uncertainty Principle:

$$\Delta E \Delta t \approx \hbar$$

- Very short lifetime gives small Δt.
- $\Delta E \approx \hbar/\Delta t$ is significant $\Rightarrow$ measurable width
- Mass of short lived particles (*e.g.* Δ^{++}) is **not fixed**.
- Mass has most-likely value, but can take on a range of values.



Decay Modes

Particles can have more than one **decay mode**. e.g. The K_S meson decays 99.9% of the time in one of two ways:

$$K_S \to \pi^+\pi^-, K_S \to \pi^0\pi^0$$

- Each decay mode has its own matrix element, $\mathcal{M}$. Fermi's Golden Rule gives us the **partial decay width** for each decay mode:

$$\Gamma(K_S \to \pi^+\pi^-) \propto |\mathcal{M}(K_S \to \pi^+\pi^-)|^2 \qquad \Gamma(K_S \to \pi^0\pi^0) \propto |\mathcal{M}(K_S \to \pi^0\pi^0)|^2$$

- The **total decay width** is equal to the sum of the decay widths for all the allowed decays.

$$\Gamma(K_S) = \Gamma(K_S \to \pi^0\pi^0) + \Gamma(K_S \to \pi^+\pi^-)$$

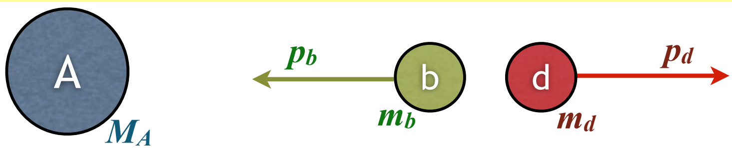
- The **branching ratio**, **BR**, is the fraction of time a particle decays to a particular final state:

$$BR(K_S \to \pi^+\pi^-) = \frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S)} \qquad BR(K_S \to \pi^0\pi^0) = \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S)}$$

Review: Decay Kinematics

Decay of an unstable particle at rest:

A → *b d*



$$\underline{\underline{p}}_A = (M_A, 0) \qquad \underline{\underline{p}}_b = (E_b, \vec{p}_b) \quad \underline{\underline{p}}_d = (E_d, \vec{p}_d)$$

Before After

- Four-momentum conservation:

$$\underline{\underline{p}}_A = \underline{\underline{p}}_b + \underline{\underline{p}}_d \quad \Rightarrow \quad \underline{\underline{p}}_b = \underline{\underline{p}}_A - \underline{\underline{p}}_d \quad \Rightarrow \vec{p_b} = -\vec{p_d}$$

$$(\underline{\underline{p}}_b)^2 = (\underline{\underline{p}}_A)^2 + (\underline{\underline{p}}_d)^2 - 2\underline{\underline{p}}_A \cdot \underline{\underline{p}}_d = M_A^2 + m_d^2 - 2M_A E_d = m_b^2$$

$$\Rightarrow E_d = \frac{M_A^2 + m_d^2 - m_b^2}{2M_A}$$

For moving particles, apply appropriate Lorentz boost.

- Example: $\pi^+ \to \mu^+\nu_\mu$ work in rest frame of pion. $m_\nu \approx 0$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = 109.8 \text{ MeV} \quad |\vec{p}_\nu| = |\vec{p}_\mu| = \sqrt{E_\mu^2 - m_\mu^2} = 29.8 \text{ MeV}/c$$

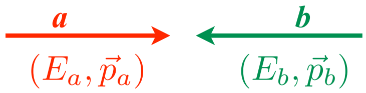
Scattering

Consider a collision between two particles: ***a*** and ***b***.

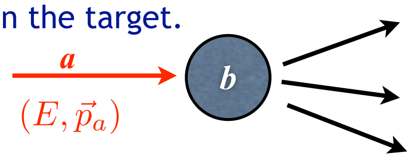
- Elastic collision: ***a*** and ***b*** scatter off each other ***a b*** → ***a b***. *e.g.* $e^+e^- \to e^+e^-$
- Inelastic collision: new particles are created ***a b*** → ***c d*** ... *e.g.* $e^+e^- \to \mu^+\mu^-$

Two main types of particle physics experiment:

- **Collider experiments** beams of ***a*** and ***b*** are brought into collision. Often $\vec{p}_a = -\vec{p}_b$



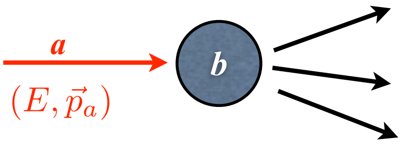
- **Fixed Target Experiments**: A beam of ***a*** are accelerated into a target at rest. ***a*** scatters off ***b*** in the target.

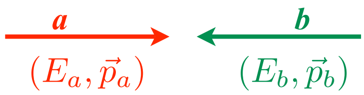


Cross Section

D&R lecture 2

- We have a beam of particles incident on a target (or another beam).





- Flux of incident beam, ***f*** : number of particles per unit area per unit time.
- Beam illuminates ***N*** particles in target.
- We measure the **scattering rate**, ***dw/dΩ***, number of particles scattered in given direction, per unit time per unit solid angle, ***dΩ***.

$$\frac{dw}{d\Omega} = fN\frac{d\sigma}{d\Omega}$$

dσ/dΩ is differential cross section

- Integrate over the solid angle, rate of scattering: $w = fN\sigma$
- Define **luminosity**, $\mathcal{L} = fN$
- **Scattering rate** $w = \mathcal{L}\sigma$

Cross Section and Luminosity

- The cross section, σ, measures the how often a scattering process occurs.
- σ is characteristic of a given process, from Fermi's Golden Rule $\sigma \propto |\mathcal{M}|^2$.
- Also depends on the energy of the colliding particles.
- σ measured in units of area. Normally use barn, $1\ \mathbf{b} = 10^{-28}\mathbf{m}^2$.
- **Luminosity**, $\mathcal{L}$, is characteristic of the beam. Measured in units of inverse area per unit time.
- **Integrated luminosity,** $\int \mathcal{L}dt$ is luminosity delivered over a given period. Measured in units of inverse area, usually $\mathbf{b}^{-1}$.

Force	Typical Cross Sections
Strong	10 mb
Electromag	10^{-2} mb
Weak	10^{-13} mb

- Event rate:

$$w = \mathcal{L}\sigma$$

- Total number of events:

$$N = \sigma \int \mathcal{L}dt$$

Centre of Mass Energy, $\sqrt{s}$

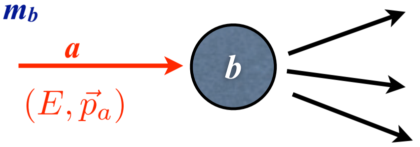
- Define Lorentz-invariant quantity, s: square of sum of four-momentum of incident particles:

$$\begin{aligned} s &= (\underline{p}_a + \underline{p}_b)\cdot(\underline{p}_a + \underline{p}_b) \\ &= (\underline{p}_a)^2 + (\underline{p}_b)^2 + 2\,\underline{p}_a\cdot\underline{p}_b \\ &= m_a^2 + m_b^2 + 2(E_aE_b - |\vec{p}_a||\vec{p}_b|\cos\theta) \end{aligned}$$

- $\sqrt{s} = E_{CM}$ is the energy in centre of momentum frame, energy available to crate new particles!

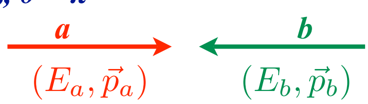
- **Fixed Target Collision,** b is at rest. $E_a >> m_a, m_b$

$$s = m_a^2 + m_b^2 + 2E_am_b \approx 2E_am_b$$
$$E_{\mathrm{CM}} = \sqrt{2E_am_b}$$



- **Collider Experiment,** with $E = E_a = E_b >> m_a, m_b, \theta = \pi$

$$s = 4E^2 \qquad E_{\mathrm{CM}} = 2E$$



Examples

- From 1989 to 1995 the LEP collider at CERN collided electrons and positrons head-on with **$E(e^-) = E(e^+) = 45.1$ GeV.**

$$\begin{aligned} s &= \left(\underline{p}(e^+) + \underline{p}(e^-)\right)^2 \\ &= 2m_e^2 + 2(E^2 - |\vec{p}_{e+}||\vec{p}_{e-}|\cos\theta) \\ &\approx 2(E^2 + |\vec{p}_{e+}||\vec{p}_{e-}|) \\ &\approx 4E^2 \\ E_{\mathrm{CM}} &= 2E = 91.2 \text{ GeV} \end{aligned}$$

- **$\sigma(e^+e^- \to \mu^+\mu^-) = 1.9$ nb** at **$E_{\mathrm{CM}} = 91.2$ GeV**
- Total integrated luminosity **$\int \mathcal{L}\, dt = 400$ pb^{-1}**
- **$N_{\mathrm{evts}}(e^+e^- \to \mu^+\mu^-) = 400{,}000 \times 1.9 = 380{,}000$**

- To make hadrons, the LEP electron beam was fired into a Beryllium target.
- Electrons collide with protons and neutrons in Beryllium.

$$\begin{aligned} s &= \left(\underline{p}(e^-) + \underline{p}(p)\right)^2 \\ &= m_e^2 + m_p^2 \\ &\quad + 2(E_e E_p - |\vec{p}_e||\vec{p}_p|\cos\theta) \\ &\approx 2(E_e m_p) \end{aligned}$$

$$\begin{aligned} E_{\mathrm{CM}} &= \sqrt{2E_e m_p} \\ &= \sqrt{2 \times 45.1 \times 1} \\ &= 9.5 \text{ GeV} \end{aligned}$$

- In fixed target electron energy is wasted providing momentum to the CM system rather than to make new particles.

Summary

Natural Units: set $\hbar = c = 1$

- Measure energies in GeV
- Every quantity is measured as a power of energy

Particle Decay

- **Lifetime**, τ, time taken for sample to decrease by $1/e$.
- **Partial width** of decay mode, $\Gamma(A \to x) \propto |\mathcal{M}(A \to x)|^2$
- **Total width** is sum of all possible decay widths, $\Gamma = \hbar/\tau$
- **Branching ratio**, proportion decays to given final state, $\mathrm{BR}\,(A \to x) = \Gamma(A \to x)/\Gamma$

Particle Scattering

- **Cross section**, σ, probability for decay to happen. Measured in b = 10^{-28} m^2.
- **Luminosity**, $\mathcal{L}$ is a property of beam.
- **Integrated luminosity**, $\int \mathcal{L} dt$.
- Number of events: $N = \sigma \int \mathcal{L} dt$
- Two types of scattering experiment: collider and fixed target.

Relativistic Kinematics

$$\underline{p} = (E, p_x, p_y, p_z) = (E, \vec{p})$$

$$\left(\underline{p}\right)^2 = E^2 - \vec{p}^{\,2} = m^2$$

Centre of Mass energy

$$s = (\underline{p}_a + \underline{p}_b)^2 \qquad E_{\mathrm{CM}} = \sqrt{s}$$