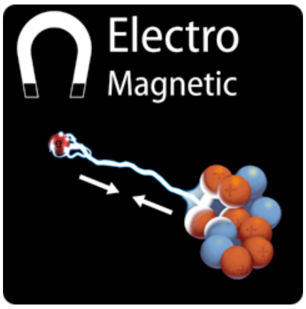
Physics 3:

Particle Physics

Lecture 3: Quantum Electro-Dynamics & Feynman Diagrams

February 18th 2008



* Antimatter
* Feynman Diagram and Feynman Rules
* Quantum description of electromagnetism
* Virtual Particles
* Yukawa Potential for QED

Summary

Natural Units: set $\hbar = c = 1$
- Measure energies in GeV
- Every quantity is measured as a power of energy

Particle Decay
- **Lifetime**, τ, time taken for sample to decrease by $1/e$.
- **Partial width** of decay mode, $\Gamma(A \to x) \propto |\mathcal{M}(A \to x)|^2$
- **Total width** is sum of all possible decay widths, $\Gamma = \hbar/\tau$
- **Branching ratio**, proportion decays to given final state, $BR\,(A \to x) = \Gamma(A \to x)/\Gamma$

Particle Scattering
- **Cross section**, σ, probability for decay to happen. Measured in $\mathrm{b} = 10^{-28}\ \mathrm{m}^2$.
- **Luminosity**, $\mathcal{L}$ is a property of beam.
- **Integrated luminosity**, $\int \mathcal{L} dt$.
- Number of events: $N = \sigma \int \mathcal{L} dt$
- Two types of scattering experiment: collider and fixed target.

Relativistic Kinematics

$$\underline{\underline{p}} = (E, p_x, p_y, p_z) = (E, \vec{p})$$
$$\left(\underline{\underline{p}}\right)^2 = E^2 - \vec{p}^{\,2} = m^2$$

Centre of Mass energy

$$s = (\underline{\underline{p}}_a + \underline{\underline{p}}_b)^2 \qquad E_{\mathrm{CM}} = \sqrt{s}$$

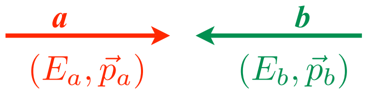
Scattering

Consider a collision between two particles: ***a*** and ***b***.

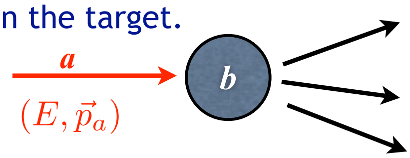
- Elastic collision: ***a*** and ***b*** scatter off each other ***a b*** → ***a b***. *e.g.* $e^+e^-\rightarrow e^+e^-$
- Inelastic collision: new particles are created ***a b*** → ***c d …*** *e.g.* $e^+e^-\rightarrow \mu^+\mu^-$

Two main types of particle physics experiment:

- **Collider experiments** beams of ***a*** and ***b*** are brought into collision. Often $\vec{p}_a = -\vec{p}_b$



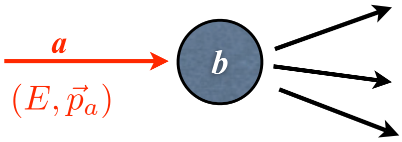
- **Fixed Target Experiments**: A beam of ***a*** are accelerated into a target at rest. ***a*** scatters off ***b*** in the target.

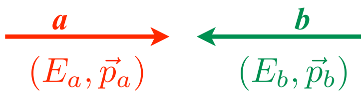


Cross Section

D&R lecture 2

- We have a beam of particles incident on a target (or another beam).





- Flux of incident beam, ***f***: number of particles per unit area per unit time.
- Beam illuminates ***N*** particles in target.
- We measure the **scattering rate**, ***dw/dΩ***, number of particles scattered in given direction, per unit time per unit solid angle, ***dΩ***.

$$\frac{dw}{d\Omega} = fN\frac{d\sigma}{d\Omega}$$

dσ/dΩ is differential cross section

- Integrate over the solid angle, rate of scattering: $w = fN\sigma$
- Define **luminosity**, $\mathcal{L} = fN$
- **Scattering rate** $w = \mathcal{L}\sigma$

Cross Section and Luminosity

- The cross section, σ, measures the how often a scattering process occurs.
- σ is characteristic of a given process, from Fermi's Golden Rule $\sigma \propto |\mathcal{M}|^2$.
- Also depends on the energy of the colliding particles.
- σ measured in units of area. Normally use barn, $1 \text{ b} = 10^{-28}\text{m}^2$.
- **Luminosity**, $\mathcal{L}$, is characteristic of the beam. Measured in units of inverse area per unit time.
- **Integrated luminosity,** $\int \mathcal{L}dt$ is luminosity delivered over a given period. Measured in units of inverse area, usually b^{-1}.

Force	Typical Cross Sections
Strong	10 mb
Electromag	10^{-2} mb
Weak	10^{-13} mb

- Event rate:

$$w = \mathcal{L}\sigma$$

- Total number of events:

$$N = \sigma \int \mathcal{L}dt$$

Centre of Mass Energy, $\sqrt{s}$

- Define Lorentz-invariant quantity, s: square of sum of four-momentum of incident particles:

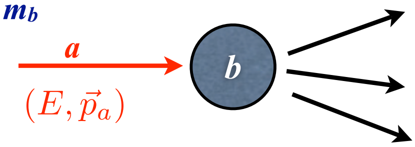
$$\begin{aligned} s &= (\underline{p}_a + \underline{p}_b) \cdot (\underline{p}_a + \underline{p}_b) \\ &= (\underline{p}_a)^2 + (\underline{p}_b)^2 + 2\,\underline{p}_a \cdot \underline{p}_b \\ &= m_a^2 + m_b^2 + 2(E_a E_b - |\vec{p}_a||\vec{p}_b|\cos\theta) \end{aligned}$$

- $\sqrt{s} = E_{\text{CM}}$ is the energy in centre of momentum frame, energy available to crate new particles!

- **Fixed Target Collision,** b is at rest. $E_a \gg m_a, m_b$

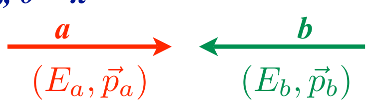
$$s = m_a^2 + m_b^2 + 2E_a m_b \approx 2E_a m_b$$

$$E_{\text{CM}} = \sqrt{2E_a m_b}$$



- **Collider Experiment,** with $E = E_a = E_b \gg m_a, m_b, \theta = \pi$

$$s = 4E^2 \qquad E_{\text{CM}} = 2E$$



Examples

- From 1989 to 1995 the LEP collider at CERN collided electrons and positrons head-on with $E(e^-) = E(e^+) = 45.1$ **GeV.**

$$\begin{aligned} s &= \left(\underline{p}(e^+) + \underline{p}(e^-)\right)^2 \\ &= 2m_e^2 + 2(E^2 - |\vec{p}_{e+}||\vec{p}_{e-}|\cos\theta) \\ &\approx 2(E^2 + |\vec{p}_{e+}||\vec{p}_{e-}|) \\ &\approx 4E^2 \\ E_{\mathrm{CM}} &= 2E = 91.2\ \mathrm{GeV} \end{aligned}$$

- $\sigma(e^+e^- \to \mu^+\mu^-) = 1.9$ **nb** at $E_{CM} = 91.2$ **GeV**
- Total integrated luminosity $\int \mathcal{L}\, dt = 400\ \mathrm{pb}^{-1}$
- $N_{evts}(e^+e^- \to \mu^+\mu^-) = 400{,}000 \times 1.9 = 380{,}000$

- To make hadrons, the LEP electron beam was fired into a Beryllium target.
- Electrons collide with protons and neutrons in Beryllium.

$$\begin{aligned} s &= \left(\underline{p}(e^-) + \underline{p}(p)\right)^2 \\ &= m_e^2 + m_p^2 \\ &\quad +2(E_e E_p - |\vec{p}_e||\vec{p}_p|\cos\theta) \\ &\approx 2(E_e m_p) \end{aligned}$$

$$\begin{aligned} E_{\mathrm{CM}} &= \sqrt{2E_e m_p} \\ &= \sqrt{2 \times 45.1 \times 1} \\ &= 9.5\ \mathrm{GeV} \end{aligned}$$

- In fixed target electron energy is wasted providing momentum to the CM system rather than to make new particles.

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Nuclear and Particle Physics Junior Honours:

Particle Physics

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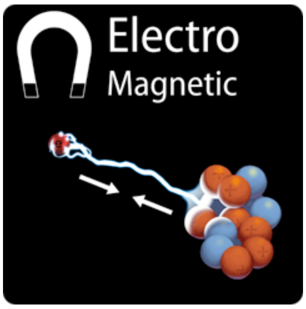

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Schrödinger and Klein Gordon

- Quantum mechanics describes momentum and energy in terms of operators:

$$\hat{E} = i\hbar\frac{\partial}{\partial t} \qquad \hat{\vec{p}} = -i\hbar\vec{\nabla}$$

- $E=p^2/2m$ gives time-dependent ***Schödinger***: $-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)$
- The solution with a definite energy, E: $\Psi_E(\vec{r},t) = \psi_E(\vec{r})\exp\{-iEt/\hbar\}$
- However for particles near the speed of light $E^2=p^2c^2+m^2c^4$ ⇒

$$-\hbar^2\frac{\partial^2}{\partial t^2}\Psi(\vec{r},t) = -\hbar^2c^2\nabla^2\Psi(\vec{r},t) + m^2c^4\Psi(\vec{r},t)$$

Klein Gordon Equation

- Solutions with a definite energy, $E_p=+(p^2c^2+m^2c^4)^{1/2}$, and three-momentum, p:

$$\Psi(\vec{r},t) = N\exp\{i(\vec{p}\cdot\vec{r} - E_pt)/\hbar\}$$

- Also solutions with a negative energy, $E_n=-E_p = (p^2c^2+m^2c^4)^{1/2}$, and momentum, $-p$:

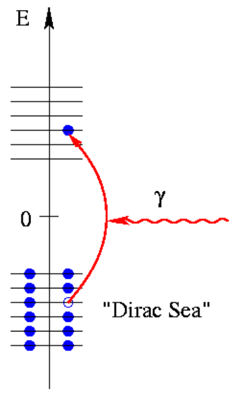
$$\Psi^*(\vec{r},t) = N^*\exp\{i(-\vec{p}\cdot\vec{r} + E_pt)/\hbar\}$$

- Negative energy solutions are a direct result of $E^2=p^2c^2+m^2c^4$.
- We interpret these as **anti-particles**

Klein Gordon equation is non-examinable

Antimatter

Klein-Gordon equation predicts negative energy solutions.

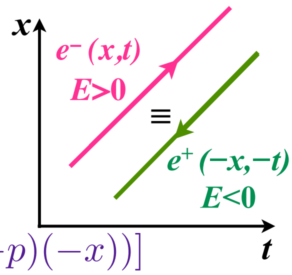


Dirac Interpretation:

- The vacuum is composed of negative energy levels with $E<0$. Each level is filled with two electrons of opposite spin: the "Dirac Sea".
- A "hole" in the sea with charge $-e$ and $E<0$, appears as a state with charge $+e$ and $E>0$.
- This idea lead Dirac to predict the **positron**, discovered in 1931.

Feynman-Stueckelberg Interpretation:

- negative energy particles moving backwards in space and time correspond to...
- positive energy antiparticles moving forward in space and time

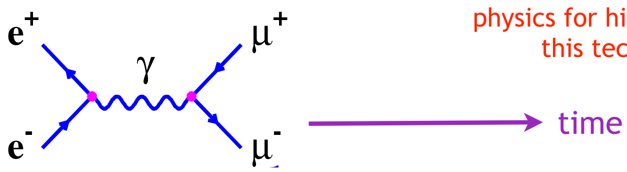


$$\Psi_{e^-}(-x,-t) \propto \exp\left[-i/\hbar\left((-E)(-t)-(-p)(-x)\right)\right]$$
$$\Psi_{e^+}(x,t) \propto \exp\left[-i/\hbar\left(Et-px\right)\right]$$

Feynman Diagrams

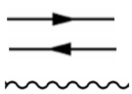
- A Feynman diagram is a pictorial representation of a particular process (decay or scattering) at a particular order in perturbation theory.
- Feynman diagrams can be used to **represent** and **calculate** the **probability amplitudes,** $\mathcal{M}$, for scattering and decays.
- Feynman diagrams are very useful and **powerful** tools. We will use them a lot in this course. We use them a lot in our research!

Richard Feynman receiving the 1967 Noble prize in physics for his invention of this technique.



Conventions

- Time flows from left to right (occasionally upwards)



- Fermions are solid lines with arrows
- Anti-fermion are solid lines with backward pointing arrows.
- Bosons are wavy (or dashed) lines

We'll apply the **Feynman Rules** to calculate $\mathcal{M}$ at different orders in perturbation theory.

Quantum Electrodynamics (QED)

QED is the quantum theory of electromagnetic interactions.

Classical electromagnetism:

- Force between charged particle arise from the electric field

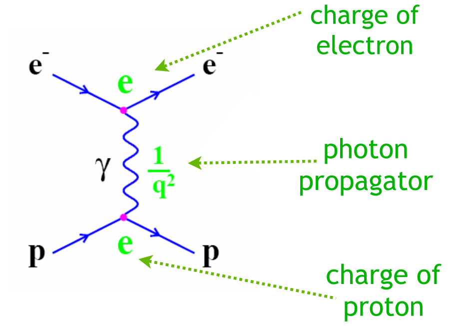
$$\vec{E} = \frac{Q\,\hat{r}}{4\pi\epsilon_0 r^2}$$

- act instantaneously at a distance

Quantum Picture:

- **Force between charged particle described by exchange of photons.**
- Strength of interaction is related to charge of particles interacting.

e.g. electon-proton scattering ***ep→ep*** propagated by the exchange of photons



$$\mathcal{M} \propto e \cdot 1/\underline{q}^2 \cdot e$$

Feynman rules:

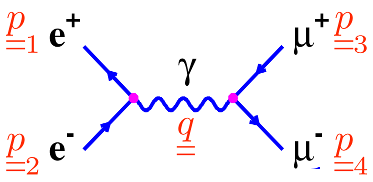
- **Vertex term:** each photon–charged particle interaction gives a factor of fermion charge, ***Q***.
- **Propagator term:** each photon gives a factor of $1/\underline{q}^2$ where $\underline{q}$ is the photon four-momentum.
- Matrix element is proportional to product of **vertex** and **propagator** terms.

Virtual Particles

The force between two charged particles is propagated by **virtual photons**.

- A particle is virtual when its four-momentum squared, does not equal its rest mass: $m_X^2 \neq E_X^2 - \vec{p}_X^{\,2}$
- Allowed due to **Heisenberg Uncertainty Principle**: can borrow energy to create particle if energy ($\Delta E = mc^2$) repaid within time (Δt), where $\Delta E \Delta t \approx \hbar$

Example: electron-positron scattering creating a muon pair: $e^+e^- \to \mu^+\mu^-$.



$$\underline{q}^2 \neq m_\gamma^2$$

- Four momentum conservation:

$$\underline{p}_1 + \underline{p}_2 = \underline{p}_3 + \underline{p}_4$$

- Momentum transferred by the photon is:

$$\underline{q} = (\underline{p}_1 + \underline{p}_2) = (\underline{p}_3 + \underline{p}_4)$$

- Squaring,

$$\begin{aligned} \underline{q}^2 &= (\underline{p}_1)^2 + (\underline{p}_2)^2 + 2\underline{p}_1 \cdot \underline{p}_2 \\ &= 2m_e^2 + 2(E_1E_2 - \vec{p}_1 \cdot \vec{p}_2) \; > \; 0 \end{aligned}$$

- In QED interactions mass of photon propagator is non-zero.
- Only intermediate photons may be virtual. Final state ones must be real!

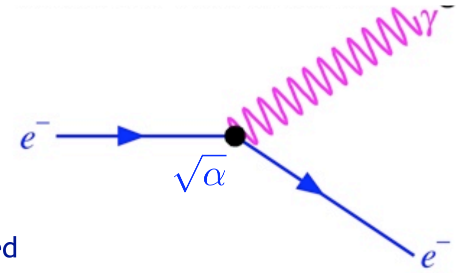
Electromagnetic Vertex

Basic electromagnetic process:

- Initial state fermion
- Absorption or emission of a photon
- Final state fermion

Examples: $e^- \to e^-\gamma$, $e^-\gamma \to e^-$

All electromagnetic interactions are described by the **vertex** and a **photon propagator**



QED Conservation Laws

- Momentum, energy and charge is conserved at all vertices
- Fermion flavour (e, μ, τ, u, d ...) is conserved: e.g. $u \to u\gamma$ allowed, $c \to u\gamma$ forbidden
- Parity, π, is conserved.

Coupling strength

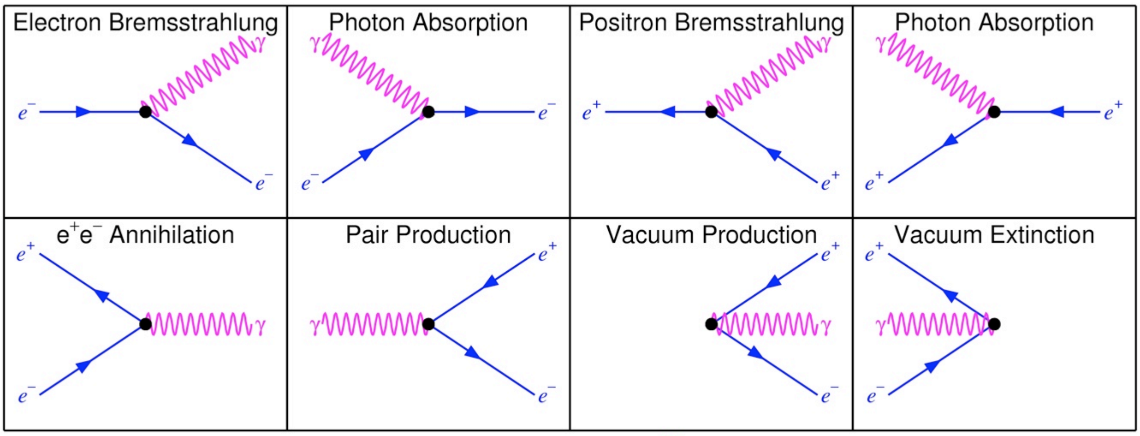
Matrix element is proportional to the fermion charge: $\mathcal{M} \propto e$

Alternatively use the fine structure constant, α

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c} = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

$\Rightarrow$ strength of the coupling at the vertex is $\propto \sqrt{\alpha}$

Basic QED Processes

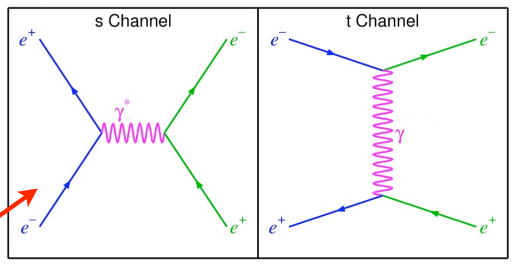


Time

- All of these described by the same basic vertex term, $\propto Q$
- None of above processes is physical as they violate energy-momentum conservation:

$$p_\gamma^2 = (p_{e_1} - p_{e_2})^2 \neq m_\gamma^2$$

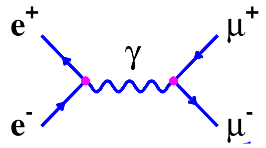
- Join two together to get a real processes



Perturbation Theory

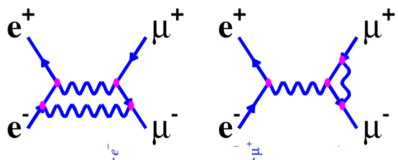
- QED is formulated from time dependent **perturbation theory**.
- Perturbation series: break up the problem into a piece we can solve exactly plus a small correction.
- *e.g.* for $e^+e^-\rightarrow\mu^+\mu^-$ scattering.
  - Many more diagrams have to be considered for a accurate prediction of $\sigma(e^+e^-\rightarrow\mu^+\mu^-)$.
- As α is small the lowest order in the expansion dominates, and the series quickly converges!
- For most of the course, we will only consider lowest order contributions to processes.

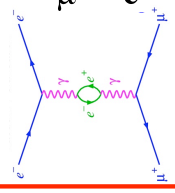
Lowest Order



$|\mathcal{M}|^2 \propto \alpha^2$

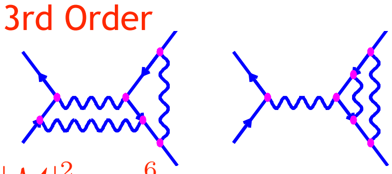
2nd Order





$|\mathcal{M}|^2 \propto \alpha^4$ + ...

3rd Order



+ ...

$|\mathcal{M}|^2 \propto \alpha^6$

Yukawa Potential

The quantum and classical descriptions of electromagnetism should agree.

Yukawa developed theory whereby exchange of bosons describes force / potential.

- Klein-Gordon equation:

$$-\hbar\frac{\partial^2}{\partial t^2}\Psi(\vec{r},t) = -\hbar^2c^2\nabla^2\Psi(\vec{r},t) + m^2c^4\Psi(\vec{r},t)$$

- Non-time dependent solutions obey:

$$\nabla^2\Psi(\vec{r}) = \frac{m^2c^2}{\hbar^2}\Psi(\vec{r})$$

- Spherically symmetric solutions of this are:

$$\Psi(|\vec{r}|) = -\frac{g^2}{4\pi r}\exp\left(-\frac{mc}{\hbar}|\vec{r}|\right)$$

- Interpret this as a potential, V, caused by a particle of mass, m.

$$V(r) = -\frac{g^2}{4\pi r}\exp\left(-\frac{r}{R}\right) \quad \text{with } R = \frac{\hbar}{mc}$$

- For electromagnetic force, $m=0$, $g=e$.

$$V_{\mathrm{EM}}(r) = -\frac{e^2}{4\pi r}$$

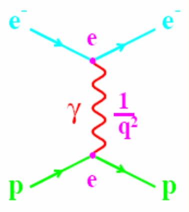
- **Potential felt by a charged particle due to the exchange of a photon.**

QED Scattering Examples

- **Elastic electron-proton scattering:** $e^-p \to e^-p$

$$\mathcal{M} \propto \frac{e^2}{\underline{\underline{q}}^2} = \frac{4\pi\alpha}{\underline{\underline{q}}^2}$$

$$\sigma \propto |\mathcal{M}|^2 \propto \frac{e^4}{\underline{\underline{q}}^4} = \frac{16\pi^2\alpha^2}{\underline{\underline{q}}^4}$$



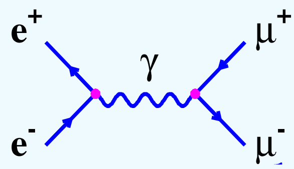
- Momentum transferred to photon from e^-:

$$\begin{aligned}\underline{\underline{q}}^2 &= (\underline{\underline{p}}_f - \underline{\underline{p}}_i)^2 = \underline{\underline{p}}_f^2 + \underline{\underline{p}}_i^2 - 2\underline{\underline{p}}_f \cdot \underline{\underline{p}}_i \\ &= 2m_e^2 - 2(E_f E_i - |\vec{p}_f||\vec{p}_i|\cos\theta) \\ &\approx -4E_f E_i \sin^2(\theta/2)\end{aligned}$$

- **Rutherford scattering:** e^- Au $\to e^-$ Au, can neglect recoil of the gold atoms: $E=E_i=E_f$

$$\sigma \propto \frac{Z^2\pi^2\alpha^2}{E^4\sin^4(\theta/2)}$$

- Inelastic $e^-e^+ \to \mu^+\mu^-$



- Momentum transferred by photon:

$$\underline{\underline{q}}^2 = (\underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-})^2 = s$$

$$\mathcal{M} \propto \frac{e^2}{s^2} = \frac{4\pi\alpha}{s^2}$$

- For this situation need full density of states, ρ, (which we won't do…)

$$\sigma = \frac{16\pi E^2}{3}|\mathcal{M}|^2 = \frac{4\pi\alpha^2}{3s}$$

Summary

Relativistic quantum mechanics predicts negative energy particles: **antiparticles.** Two interpretations:

- a negative energy particle travelling backwards in time.
- a 'hole' in a vacuum filled with negative energy states.

Quantum Electro Dynamics (QED) is the quantum mechanical description of the electromagnetic force.

Electromagnetic force propagated by virtual photons: $\underline{\underline{q}}^2 \neq m_\gamma^2$

Feynman diagrams can be used to **illustrate** QED processes. Use Feynman rules to calculate the matrix element, $\mathcal{M}$.

All QED interactions are described by a **fermion-fermion-photon vertex**:

- Strength of the vertex is the charge of the fermion, Q_f.
- Fermion flavour and energy-momentum are conserved at vertex.

The **photon propagator** ~$1/\underline{\underline{q}}^2$ where $\underline{\underline{q}}$ is the 4-momentum transferred by the photon.

$\mathcal{M}$ is proportional to product of vertex and propagator terms.