Physics 3:

# Particle Physics

## Lecture 8: The Weak Force Continued

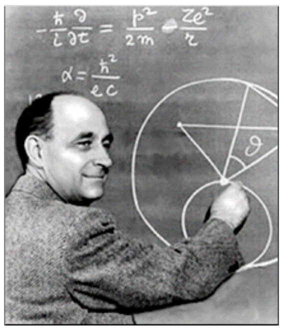
March 6th 2008


- Fermi Theory
- Beta Decay
- Muon and tau decay
- Lepton Universality
- Weak interactions of Quarks
- Weak Decays of Hadrons



# Fermi Theory

### Weak Interactions at Low Momentum Transfer



For muon decay, and many other weak processes:

$$\mathcal{M} \propto \frac{g_w^2}{(\underline{\underline{q}}^2 - m_W^2)}$$

At low momentum transfer $\underline{\underline{q}}^2 \ll m_W^2$

$$\mathcal{M} \rightarrow \propto \frac{g_w^2}{m_W^2}$$

Introduce **Fermi coupling constant:**

$$G_F \propto \frac{g_w^2}{m_W^2} \qquad G_F = \frac{\sqrt{2}\, g_w^2}{8\, m_W^2}$$

- Dimension $[E]^{-2}$
- From experimental measurements: $G_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}$

Measurements of $G_F \,\&\, M_W \Rightarrow g_w = 0.66 \Rightarrow \alpha_w = \frac{g_w^2}{4\pi} = \frac{1}{29} > \alpha_{\mathrm{EM}} = \frac{1}{137}$

- Recall, from problem sheet 2, Q4, range of $W$ boson: $\Delta x \approx \frac{\hbar}{\Delta p} = \frac{\hbar}{m_W c} = 0.002\ \mathrm{fm}$

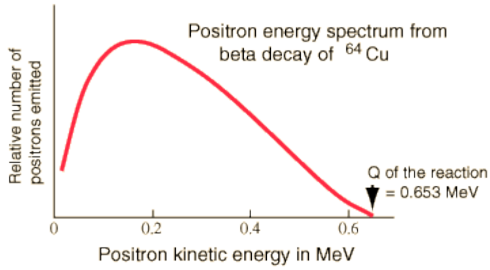
Weak interaction not intrinsically weak - appears weak due to large boson masses.

# Beta Decay

**Weak Nuclear Decay**

$\beta^+$ decay $(A,Z) \rightarrow (A,Z-1) + e^+ + \nu_e$

$\beta^-$ decay $(A,Z) \rightarrow (A,Z+1) + e^- + \overline{\nu}_e$




**Nuclear Interpretation**

$n \rightarrow p\, e^- \overline{\nu}_e$

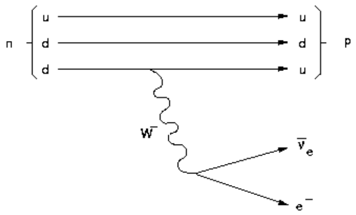
Continuous energy spectrum of $e^{\pm} \Rightarrow$ at least two decay products. This led Pauli to postulate the existence of the neutrino.

**Modern quark level picture**

Decay mediated by exchange of virtual $W^{\pm}$ boson

$d \rightarrow u\, W$

↳ $e^- \overline{\nu}_e$






# Fermi's Golden Rule

Review from Lecture 1, Slide 13

The rate at which a decay or a scattering proceeds is given by **Fermi's Golden Rule:**

$$T_{i \rightarrow f} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \rho$$

Where:

- $\mathcal{M}$ is the matrix element - we will see how these are calculated for different processes in future lectures.
- $\rho$ is the density of states - we will consider this only for some key processes.

- $T$ is related to the cross section of scattering, $\sigma$.
  e.g. $\sigma(e^+e^- \rightarrow \mu^+\mu^-) \propto |\mathcal{M}(e^+e^- \rightarrow \mu^+\mu^-)|^2$.

- $T$ is related to the inverse lifetime of a decay, $\tau$.
  e.g. $\tau(\mu^- \rightarrow e^- \overline{\nu}_e \nu_\mu) \propto 1/|\mathcal{M}(\mu^- \rightarrow e^- \overline{\nu}_e \nu_\mu)|^2$.

# Decay Modes

Review from Lecture 2, Slide 9

Particles can have more than one **decay mode**. e.g. The $K_S$ meson decays 99.9% of the time in one of two ways:

$$K_S \to \pi^+\pi^-,\, K_S \to \pi^0\pi^0$$

- Each decay mode has its own matrix element, $\mathcal{M}$. Fermi's Golden Rule gives us the **partial decay width** for each decay mode:

$$\Gamma(K_S \to \pi^+\pi^-) \propto |\mathcal{M}(K_S \to \pi^+\pi^-)|^2 \qquad \Gamma(K_S \to \pi^0\pi^0) \propto |\mathcal{M}(K_S \to \pi^0\pi^0)|^2$$

- The **total decay width** is equal to the sum of the decay widths for all the allowed decays.

$$\Gamma(K_S) = \Gamma(K_S \to \pi^0\pi^0) + \Gamma(K_S \to \pi^+\pi^-)$$

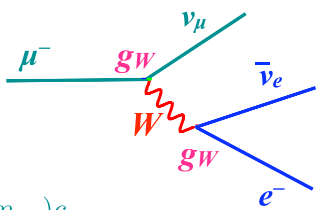
- The **branching ratio**, **BR**, is the fraction of time a particle decays to a particular final state:

$$BR(K_S \to \pi^+\pi^-) = \frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S)} \qquad BR(K_S \to \pi^0\pi^0) = \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S)}$$



# Muon Decay

**How does a muon $\mu^-$ decay?**




- Must decay into lighter particles: $e^-$, $\gamma$, $\nu$.
  In particular, all hadrons are heavier than $m_\mu$.

$L_e$, $L_\mu$, $L_\tau$ conservation ⇒ only lowest order decay is $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

Maximum four-momentum transferred by $W$ boson is $\underline{\underline{q}} = (m_\mu - m_{\nu_\mu})c$

$$\mathcal{M} \propto \frac{g_W^2}{\underline{\underline{q}}^2 - m_W^2}$$

$$\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu) \propto |\mathcal{M}|^2 \propto \frac{g_W^4}{(\underline{\underline{q}}^2 - m_W^2)^2} \to \frac{g_W^4}{m_W^4} \propto G_F^2$$

Only 1 decay mode:

- Partial decay width, $\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$ = total decay width, $\Gamma_\mu = \hbar/\tau_\mu \propto G_F^2$

To calculate a value for $\Gamma_\mu$, need to know the density of states, $\rho$.
Use dimensional analysis:

➡ $\Gamma$ has dimensions of energy, $[E]$;

➡ $G_F^2$ has dimensions $[E]^{-4}$

To balance dimensions, use $m_\mu$ (only other energy/mass in problem): $\Gamma_\mu = K\, G_F^2\, m_\mu^5$

where $K$ is a dimensionless constant

(full calculation gives $\Gamma_\mu = G_F^2\, m_\mu^5/(192\,\pi^3)$ )

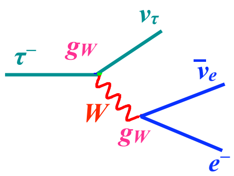
# Tau Decay

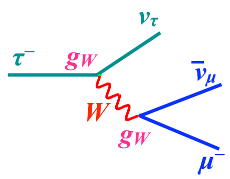
| Decay Mode | BR |
|---|---|
| $\tau^- \to e^- \bar{\nu}_e \nu_\tau$ | 17.8% |
| $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ | 17.4% |
| $\tau^- \to$ hadrons $+ \nu_\tau$ | 64.7% |

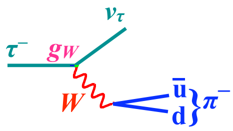
$m_\tau = 1.777$ GeV/c$^2$ $> m_\mu, m_\pi, m_\rho, \ldots$

More than one final state possible.

- e.g. $\tau^- \to e^- \bar{\nu}_e \nu_\tau$, $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$, $\tau^- \to \pi^- \bar{\nu}_\mu \nu_\tau$




$$\mathcal{M}(\tau \to e\nu_e\nu_\tau) \propto \frac{g_W^2}{q^2 - m_W^2} \qquad \mathcal{M}(\tau \to \mu\nu_\mu\nu_\tau) \propto \frac{g_W^2}{q^2 - m_W^2}$$

$\tau^- \to e^- \bar{\nu}_e \nu_\tau$ and $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ have same matrix element as $\mu^- \to e^- \bar{\nu}_e \nu_\tau$:

$$\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu) = K\, G_F^2\, m_\mu^5$$
$$\Gamma(\tau^- \to e^-\bar{\nu}_e\nu_\tau) = K\, G_F^2\, m_\tau^5$$

- From measured branching ratios:

$$\Gamma_\tau = 0.178\,\Gamma(\tau^- \to e^-\,\bar{\nu}_e\,\nu_\tau)$$
$$\Gamma_\mu = \Gamma(\mu^- \to e^-\,\bar{\nu}_e\,\nu_\mu)$$

Rearranging:
$$\frac{\Gamma(\tau^- \to e^-\bar{\nu}_e\,\nu_\tau)}{\Gamma(\mu^- \to e^-\bar{\nu}_e\,\nu_\tau)} = \frac{0.178\,\Gamma_\tau}{\Gamma_\mu} = \frac{0.178\,\tau_\mu}{\tau_\tau} = \frac{m_\tau^5}{m_\mu^5}$$



# Lepton Universality

- We've shown $\Gamma_\mu = K\,G_F^2\,m_\mu^5$ $\left(\Gamma_\mu = G_F^2\,m_\mu^5/(192\,\pi^3)\right)$ and $\frac{m_\tau^5}{m_\mu^5} = \frac{0.178\,\tau_\mu}{\tau_\tau}$
- Experimental measurements:
  - $\tau_\mu = 2.19703 \times 10^{-6}$ s $\quad m_\mu = 105.65837$ MeV/c$^2$ $\quad m_\tau = 1777.0$ MeV/c$^2$
  - used to extract $G_F$ (and $g_W$) $\Rightarrow$ $G_F = 1.16637(1) \times 10^{-5}$ GeV$^{-2}$
- Predict the lifetime of the tau-lepton:

$$\tau_\tau = \mathrm{BR}(\tau \to e^-\nu_e\nu_\tau)\,\tau_\mu\,\frac{m_\mu^5}{m_\tau^5} = 2.91 \times 10^{-13}\ \mathrm{s}$$

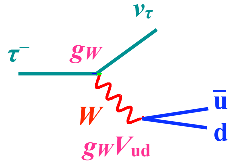
- Compare to measured $\tau_\tau = (2.906 \pm 0.011) \times 10^{-13}$ s

The relationship between the tau and muon lifetimes illustrates **lepton universality**.

- Coupling of to $W$-boson to all leptons is equal = $g_W$
- electrons, muons and taus all interact identically
- interact with the **same bosons** with **same coupling strength**

# Hadronic Decays of Tau

In general, any vertex $W$-($Q$=+2/3 $e$ quark)-($Q$=−1/3 $e$ quark) is valid.

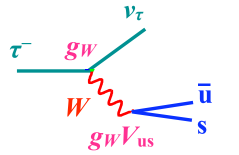



**In $\tau$ decay: the virtual $W^-$ boson can decay into $\bar{u}$+d or $\bar{u}$+s**

- Other pairs of quarks form hadrons which are too heavy

$$\mathcal{M}(\tau \to \bar{u}d\nu_\tau) \propto \frac{g_W^2 V_{ud}}{q^2 - m_W^2}$$

Coupling at $W$-u-d ($g_W V_{ud}$), $W$-e-$\nu$ ($g_W$), $W$-u-s ($g_W V_{us}$) vertices not equal




- $g_W V$ is coupling to **one colour** of quark
- $\Gamma$ is enhanced number of quark colours, $N_c$
- $q^2 \ll m_W^2$

$$\mathcal{M}(\tau \to \bar{u}s\nu_\tau) \propto \frac{g_W^2 V_{us}}{q^2 - m_W^2}$$

*e.g.* $$\Gamma(\tau^- \to K^-\nu_\tau) \approx \Gamma(\tau \to \bar{u}s\nu_\tau) \propto N_c \frac{g_W^4 V_{us}^2}{m_W^4}$$

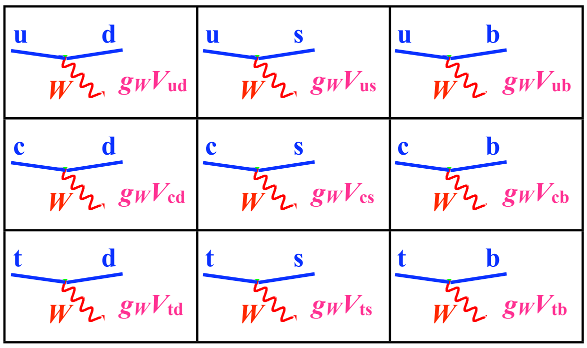
(would need to know density of states to fully calculate $\Gamma$, depends on both $m_\tau$ and $m_K$)



# Weak Interactions of Quarks

In general, any vertex $W$-($Q$=+2/3 $e$ quark)-($Q$=−1/3 $e$ quark) is valid.

- $W$-boson coupling to quarks suppressed by a flavour-dependent factor $V$ or $V_{CKM}$




(Known as the "CKM matrix" - values from experimental measurements)

| | | |
|---|---|---|
| $V_{ud}$=0.974 | $V_{us}$=0.227 | $V_{ub}$=0.004 |
| $V_{cd}$=0.230 | $V_{cs}$=0.972 | $V_{cb}$=0.042 |
| $V_{td}$=0.008 | $V_{ts}$=0.041 | $V_{tb}$=0.999 |


| | | |
|---|---|---|
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**Largest couplings are within a generation:**

**d ↔ u   s ↔ c   b ↔ t**

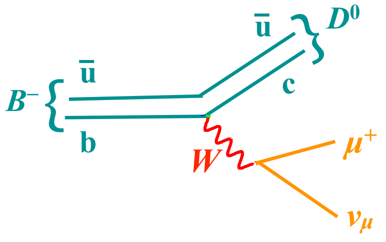
# Weak Hadron Decays

As the strong and electromagnetic forces conserve strangeness, charmness etc. (*S, C, B*) … lightest hadrons with non-zero *S, C, B* quantum numbers **must** decay by weak force!

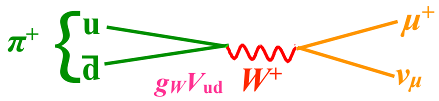
**For interactions of hadrons always consider the interactions of the constituent quarks.**

- *e.g.* $B^- \to D^0 \mu^+ \nu_\mu$

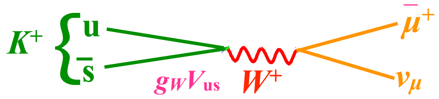
  $\bar{u}b \to \bar{u}c\, \mu^+ \nu_\mu$




- *e.g.* $\pi^+ \to \mu^+ \nu_\mu \quad K^+ \to \mu^+ \nu_\mu$




$$\mathcal{M}(\pi^+ \to \mu^+ \nu_\mu) = \frac{g_W^2\, V_{ud}}{\underline{q}^2 - m_W^2}$$




$$\mathcal{M}(K^+ \to \mu^+ \nu_\mu) = \frac{g_W^2\, V_{us}}{\underline{q}^2 - m_W^2}$$

- predicted ratio of the partial decay widths:

$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = \frac{V_{us}^2}{V_{ud}^2} = 0.055$$

*Confirmed experimentally!*



# Summary

| | |
|---|---|
| The weak force acts on **all** quarks and leptons.<br><br>Two **massive** bosons propagate the weak interaction: $W^\pm$ and $Z^0$. | Weak interactions are characterised by:<br>• Long lifetimes $10^{-13}$ - $10^3$ s<br>• Small cross sections $10^{-13}$ mb |
| **$W^\pm$-boson** interactions **changes** fermion flavour<br><br>$e^- \leftrightarrow \nu_e \quad \mu^- \leftrightarrow \nu_\mu \quad \tau^- \leftrightarrow \nu_\tau$<br>(Q=+2/3 *e* quark) ↔ (Q=−1/3 *e* quark)<br><br>• quark coupling at $W^\pm$ vertex: $g_W V_{CKM}$<br>• lepton coupling at $W^\pm$ vertex: $g_W$<br>• $W^\pm$ propagator term: $1/(\underline{q}^2 - m_W^2)$ | Lepton interactions are universal.<br><br>Quarks interactions not universal.<br>*W*-(Q=+2/3 *e* quark)-(Q=−1/3 *e* quark) coupling is $g_W V_{CKM}$, where $V_{CKM}$ depends on flavour of quark<br><br>Largest couplings within a generation: *Wud*, *Wcs*, *Wtb* |
| **$Z^0$-boson** interactions **conserve** the fermion flavour<br><br>$Z^0$-boson propagator term: $1/(\underline{q}^2 - m_Z^2)$<br><br>$Z^0$-boson interaction is connected to electromagnetic interaction | Fermi theory describes *W*-boson interactions at low momentum transfer $\underline{q}^2 \ll m_W^2$<br>Described by Fermi constant:<br>$G_F \propto g_w^2/m_W^2$ |