I.2 In a synchrotron accelerator, a charged particle has an energy loss per cycle of:

$$\Delta E \propto \gamma^4$$

where:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \quad .$$

What is the origin of this energy loss? Calculate the ratio of the energy losses for an electron and proton of a given energy, E. Comment on the significance of your answer. **[5]**

I.3 Describe what is meant by *colour confinement.* Briefly discuss the properties of gluons which explain confinement. **[5]**

3. What interactions are responsible for the following processes? Justify very briefly your answers.

$$\pi^0 \to \gamma\gamma$$
$$\pi^+ \to \mu^+\nu_\mu$$
$$\pi^- p \to \Delta(1232) \to \pi^0 n$$

[5]

4. What are the quark contents of the charmed D^0 and D^{*+} mesons?

A D^{*+} meson has a mass of 2010 MeV/c^2 and it decays into a D^0 and a π^+ meson with masses of $m_{D^0} = 1864.5$ MeV/c^2 and $m_\pi = 139.6$ MeV/c^2, respectively. Calculate the energy of the π^+ in the rest frame of the D^{*+}. **[5]**

3. Explain briefly what neutrino oscillations are, and give an example of experimental evidence for their existence. What does the observation of the oscillations imply for the masses of the neutrinos? **[5]**

4. The K^0 meson has a mass of 497.6 MeV/c^2 and it decays into two charged pions of mass 139.6 MeV/c^2. What is the energy of a pion as observed in the rest frame of the K^0?

The K^0 lifetime is 0.89×10^{-10} s. State what interaction is responsible for the decay, and justify briefly your answer. **[5]**

6. An experiment at an electron-positron collider is operating at a centre-of-mass energy $E_{\text{CoM}} = 30$ GeV and it measures the ratio of cross sections:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \approx 3.85\ .$$

Draw the lowest order Feynman diagrams for these processes. **[4]**

Explain briefly why hadrons are observed rather than quarks. **[5]**

Discuss and calculate how the experimental result $R \approx 3.85$ can be accounted for in terms of the different types of quarks and the existence of colour. **[6]**

Explain why the process $e^+e^- \to q\bar{q}g$ is an experimental signature for the existence of gluons. Why does this process only have a small effect on the value of R? **[5]**

7. Draw the lowest order Feynman diagram for the decay of a muon $\mu^- \to e^-\bar{\nu}_e\nu_\mu$. Describe the meaning of the symbols and their significance in the following equation:

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$$

Discuss the W boson propagator, and why it produces a muon decay rate $\Gamma_\mu = \Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu)$ which is proportional to G_F^2. **[7]**

Explain why the decay $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ is allowed and why $\mu^+ \to e^+\gamma$ and $\mu^+ \to e^+e^-e^+$ are forbidden. **[4]**

Cosmic ray muons are produced high in the atmosphere, say at 10 km, and have an energy of about 2 GeV. What is the speed $\beta = v/c$ of such a muon? How far will the muon travel on average before it decays? **[4]**

(The muon mass is $m_\mu = 105.7$ MeV/c^2, and the muon lifetime is $\tau_\mu = 2.197\ \mu$s, respectively.)

Describe lepton universality in weak decays and apply it to find a relation between the decay rates of the tau decays $\tau^- \to e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$. **[5]**

II.3 The lifetime of the muon (μ^-) is 2.20×10^{-6} s. The lifetime of the tau-lepton (τ^-) is 2.91×10^{-13} s.

(a) What force is responsible for the decays of these leptons? **[2]**

(b) Write down the main decay mode of μ^-. Why don't muons decay into hadrons? **[3]**

(c) What are the allowed decays of τ^- into quarks and leptons? Draw two Feynman diagrams, one representing the decay of a μ^- and one representing the decay of a τ^-. **[5]**

(d) The Fermi coupling constant, G_F, can be written as:

$$G_F = \frac{\sqrt{2}\, g_w^2}{8\, m_W^2}$$

where m_W is the mass of the W-boson and g_w is the weak coupling constant.

Find a relationship between the width of the muon (Γ_μ), G_F and the mass of the muon (m_μ) in the form: $\Gamma_\mu = K\, G_F^a\, m_\mu^b$, where K is a dimensionless constant. Hence explain the relationship between the muon and tau-lepton lifetimes. **[10]**

7. What are antiparticles? Describe the interpretation of these given by Feynman. Considering the case of muons and antimuons, comment on the relations between the charge and the lepton family number, L_μ, of particles and antiparticles. **[6]**

Draw the lowest order Feynman diagram for the electromagnetic process $e^+e^- \to \mu^+\mu^-$. Neglecting the spins of the initial and final state particles, state the main features of the matrix element, $\mathcal{M}$, for this process. Show that the cross section, σ, for this process must have the form $\sigma \propto \alpha^2$ where $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$ is the fine structure constant. **[6]**

At the PETRA accelerator, electrons and positrons were colliding head-on. Using four-momentum conservation, show that the centre-of-mass energy, $\sqrt{s}$, for total energies of $E = 17.5$ GeV for both electron and positron beams is equal to $\sqrt{s} = 2E = 35\,\text{GeV}$. **[4]**

The exact cross section for $e^+e^- \to \mu^+\mu^-$ is $\sigma = \frac{87\,\text{nb}}{s\,[\text{GeV}^2]}$. Using the above relation, and the collider luminosity $\mathcal{L} = 1 \times 10^{31}\ \text{cm}^{-2}\text{s}^{-1}$, calculate the number of $e^+e^- \to \mu^+\mu^-$ events observed in 30 days assuming that the accelerator was working during half of this time. Note that $1\,\text{barn (b)} = 10^{-24}\,\text{cm}^2$. **[4]**