1.2 In a synchrotron accelerator, a charged particle has an energy loss per cycle of
\[ \Delta E \propto \gamma^4 \]
where:
\[ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \]

What is the origin of this energy loss? Calculate the ratio of the energy losses for an electron and proton of a given energy, \( E \). Comment on the significance of your answer. [5]

1.3 Describe what is meant by colour confinement. Briefly discuss the properties of gluons which explain confinement. [5]

3. What interactions are responsible for the following processes? Justify very briefly your answers.
\[
\begin{align*}
\pi^0 &\rightarrow \gamma\gamma \\
\pi^+ &\rightarrow \mu^+\nu_\mu \\
\pi^-p &\rightarrow \Delta(1232) \rightarrow \pi^0n
\end{align*}
\]

4. What are the quark contents of the charmed \( D^0 \) and \( D^{++} \) mesons?
A \( D^{++} \) meson has a mass of 2010 MeV/c\(^2\) and it decays into a \( D^0 \) and a \( \pi^+ \) meson with masses of \( m_{D^0} = 1864.5 \) MeV/c\(^2\) and \( m_\pi = 139.6 \) MeV/c\(^2\), respectively. Calculate the energy of the \( \pi^+ \) in the rest frame of the \( D^{++} \). [5]

3. Explain briefly what neutrino oscillations are, and give an example of experimental evidence for their existence. What does the observation of the oscillations imply for the masses of the neutrinos? [5]

4. The \( K^0 \) meson has a mass of 497.6 MeV/c\(^2\) and it decays into two charged pions of mass 139.6 MeV/c\(^2\). What is the energy of a pion as observed in the rest frame of the \( K^0 \)?
The \( K^0 \) lifetime is \( 0.89 \times 10^{-10} \) s. State what interaction is responsible for the decay, and justify briefly your answer. [5]
6. An experiment at an electron-positron collider is operating at a centre-of-mass energy $E_{\text{CM}} = 30 \text{ GeV}$ and it measures the ratio of cross sections:

$$R = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)} \approx 3.85.$$  

Draw the lowest order Feynman diagrams for these processes.  

Explain briefly why hadrons are observed rather than quarks.  

Discuss and calculate how the experimental result $R \approx 3.85$ can be accounted for in terms of the different types of quarks and the existence of colour.  

Explain why the process $e^+ e^- \rightarrow q\bar{q}g$ is an experimental signature for the existence of gluons. Why does this process only have a small effect on the value of $R$?  

7. Draw the lowest order Feynman diagram for the decay of a muon $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$. 

Describe the meaning of the symbols and their significance in the following equation:

$$\frac{G_F}{\sqrt{2}} = \frac{g_V}{8M_W^2}.$$  

Discuss the $W$ boson propagator, and why it produces a muon decay rate $\Gamma_\mu = \Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu)$ which is proportional to $G_F^2$.  

Explain why the decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ is allowed and why $\mu^+ \rightarrow e^+ \gamma$ and $\mu^+ \rightarrow e^+ e^- e^+$ are forbidden.  

Cosmic ray muons are produced high in the atmosphere, say at 10 km, and have an energy of about 2 GeV. What is the speed $\beta = v/c$ of such a muon? How far will the muon travel on average before it decays?  

(The muon mass is $m_\mu = 105.7 \text{ MeV/c}^2$, and the muon lifetime is $\tau_\mu = 2.197 \mu s$, respectively.)  

Describe lepton universality in weak decays and apply it to find a relation between the decay rates of the tau decays $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$ and $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$.  

II.3 The lifetime of the muon ($\mu^-$) is $2.20 \times 10^{-6}$ s. The lifetime of the tau-lepton ($\tau^-$) is $2.91 \times 10^{-13}$ s.

(a) What force is responsible for the decays of these leptons? [2]

(b) Write down the main decay mode of $\mu^-$. Why don’t muons decay into hadrons? [3]

(c) What are the allowed decays of $\tau^-$ into quarks and leptons? Draw two Feynman diagrams, one representing the decay of a $\mu^-$ and one representing the decay of a $\tau^-$. [5]

(d) The Fermi coupling constant, $G_F$, can be written as:

$$G_F = \frac{\sqrt{2} g_w^2}{8 m_W^2}$$

where $m_W$ is the mass of the $W$-boson and $g_w$ is the weak coupling constant. Find a relationship between the width of the muon ($\Gamma_\mu$), $G_F$ and the mass of the muon ($m_\mu$) in the form: $\Gamma_\mu = K G_F^a m_\mu^b$, where $K$ is a dimensionless constant. Hence explain the relationship between the muon and tau-lepton lifetimes. [10]

7. What are antiparticles? Describe the interpretation of these given by Feynman. Considering the case of muons and antimuons, comment on the relations between the charge and the lepton family number, $L_\mu$, of particles and antiparticles. [6]

Draw the lowest order Feynman diagram for the electromagnetic process $e^+ e^- \rightarrow \mu^+ \mu^-$. Neglecting the spins of the initial and final state particles, state the main features of the matrix element, $\mathcal{M}$, for this process. Show that the cross section, $\sigma$, for this process must have the form $\sigma \propto \alpha^2$ where $\alpha = \frac{e^2}{4\pi\hbar c}$ is the fine structure constant. [6]

At the PETRA accelerator, electrons and positrons were colliding head-on. Using four-momentum conservation, show that the centre-of-mass energy, $\sqrt{s}$, for total energies of $E = 17.5$ GeV for both electron and positron beams is equal to $\sqrt{s} = 2E = 35$ GeV. [4]

The exact cross section for $e^+ e^- \rightarrow \mu^+ \mu^-$ is $\sigma = \frac{87 \text{nb}}{s^{[\text{GeV}^2]}}$. Using the above relation, and the collider luminosity $\mathcal{L} = 1 \times 10^{31} \text{ cm}^{-2}\text{s}^{-1}$, calculate the number of $e^+ e^- \rightarrow \mu^+ \mu^-$ events observed in 30 days assuming that the accelerator was working during half of this time. Note that 1 barn (b) = $10^{-24} \text{ cm}^2$. [4]