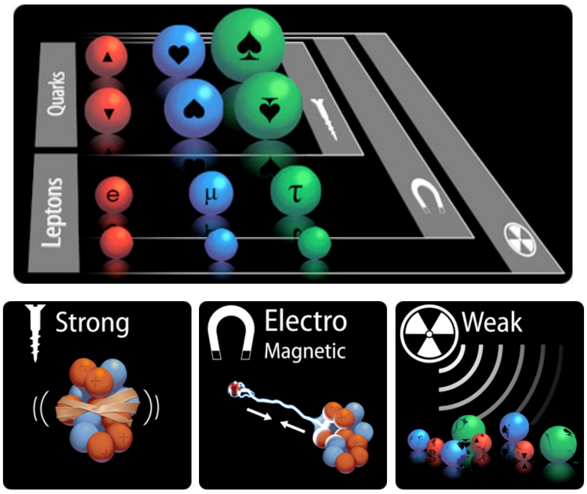
Physics 3:

Particle Physics

Review

March 13th 2008



- Natural Units
- Relativistic Dynamics
- Anti-matter
- Quarks, Leptons & Hadrons
- Feynman Diagrams and Feynman Rules
- Decays
- QED, QCD, Weak
- What you don't need to know

The Standard Model

The Standard Model describes more-or-less everything we currently know about particle physics: the matter **particles** and the three of the four forces which describe their **interactions**.

Matter: aka the fermions

Leptons			Charge, e
ν_e $e-$	ν_μ $\mu-$	ν_τ $\tau-$	0 −1
Quarks			
u d	c s	t b	+2/3 −1/3

Two processes used to study interactions:

- Decay: measure partial decay widths and lifetimes
- Scattering: measure cross sections

Forces

- Interactions are propagated by the exchange of bosons

Interaction	Bosons	Q, e
Strong	gluons, g	0
Electro-magnetic	photon, γ	0
Weak	$W^{\pm}$, Z^0	0, ±1
Gravity	?	?

Relativistic Dynamics

Relativistic Dynamics is used to describe kinematics in **decays** and **scattering**.

- Four momentum:

$$\underline{\underline{p}} = (E/c, p_x, p_y, p_z) = (E/c, \vec{p})$$

- If we square four-momentum:

$$\underline{\underline{p}}^2 = \frac{E^2}{c^2} - \vec{p}\cdot\vec{p} = m^2c^2$$ we get the mass squared!

- In **decay** the four-momentum is conserved $\sum \underline{\underline{p}}_{\text{initial}} = \sum \underline{\underline{p}}_{\text{final}}$
  e.g. in $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$

$$\underline{\underline{p}}_{\mu} = \underline{\underline{p}}_{e} + \underline{\underline{p}}_{\overline{\nu}_e} + \underline{\underline{p}}_{\nu_\mu}$$ Square both sides: $$m_\mu^2 c^2 = (\underline{\underline{p}}_{e} + \underline{\underline{p}}_{\overline{\nu}_e} + \underline{\underline{p}}_{\nu_\mu})^2$$

- In a **scattering** the four-momentum is conserved $\sum \underline{\underline{p}}_{\text{initial}} = \sum \underline{\underline{p}}_{\text{final}}$
  e.g. $e^+e^- \rightarrow \mu^+\mu^-$ $$\underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-} = \underline{\underline{p}}_{\mu^+} + \underline{\underline{p}}_{\mu^-}$$

- In a scattering, the square of the initial four momentum is s. Energy in the Centre of Mass frame is $\sqrt{s}$, *e.g.* $s = (\underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-})^2$

In both decay and scattering: boson transfers momentum from initial to final state!

Natural Units

- Natural units: set $c = \hbar = 1$
  - All quantities can be expressed as a power of energy.
- Mass, momentum and energy measured in the same units: **MeV** or **GeV**

- Two important quantities for Lorentz transformations:

$$\beta = v/c \qquad \gamma(v) = 1/\sqrt{1-\beta^2}$$

Natural Units

Lorentz boosts: $\gamma = E/m \quad \gamma\beta = |\vec{p}|/m \quad \beta = |\vec{p}|/E$

Four momentum: $\underline{\underline{p}} = (E,\, p_x,\, p_y,\, p_z)$

Invariant mass $\underline{\underline{p}}^2 = E^2 - \vec{p}^2 = m^2$

Matter

Six quarks and six leptons

Matter is grouped into three **generations**.
Each generation consists of:

- 1 lepton with $Q = -1e$
- 1 neutral lepton $Q = 0$ (ν)
- 1 quark with $Q = +2/3e$
- 1 quark with $Q = -1/3e$

Each generation is successively heavier.

1st	2nd	3rd	Q
ν_e	ν_μ	ν_τ	0
$e-$	$\mu-$	$\tau-$	$-1e$
u	c	t	$+2/3e$
d	s	b	$-1/3e$

Quantum Numbers

- **Leptons** L_e, L_μ, L_τ

	L_e	L_μ	L_τ
$\nu_e, e-$	+1	0	0
$\nu_\mu, \mu-$	0	+1	0
$\nu_\tau, \tau-$	0	0	+1

- **Quarks:**
  - Isospin, $I_Z = ½[\mathrm{N(u)} - \mathrm{N(d)} + \mathrm{N}(\bar{\mathrm{d}}) - \mathrm{N}(\bar{\mathrm{u}})]$
  - Baryon number, $\mathcal{B} = 1/3$ for quarks, $\mathcal{B} = -1/3$ for anti-quarks
  - Strangness: S, Charm: C, Bottomness: B, Topness: T - net number of s, c, b, t

    $S = \mathrm{N}(\bar{\mathrm{s}}) - \mathrm{N(s)}$ $C = \mathrm{N(c)} - \mathrm{N}(\bar{\mathrm{c}})$ $B = \mathrm{N}(\bar{\mathrm{b}}) - \mathrm{N(b)}$ $T = \mathrm{N(t)} - \mathrm{N}(\bar{\mathrm{t}})$
  - Every quark carries a colour charge: red, blue or green

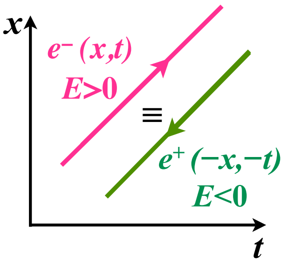
Anti-matter

Every matter particle has an anti-matter partner.

$$E^2 = \vec{p}^{\,2}c^2 + m^2c^4 \Rightarrow E = \pm\sqrt{\vec{p}^{\,2}c^2 + m^2c^4}$$

- Particle is the positive energy solution
- Anti-particle is negative energy solution

Leptons			Charge
ν_e	ν_μ	ν_τ	0
$e-$	$\mu-$	$\tau-$	-1
Quarks			
u	c	t	$+2/3$
d	s	b	$-1/3$



Feynman's interpretation: negative energy particle with charge Q moving backward in space & time appears as positive energy particle with charge $-Q$ moving forward in space & time.

Anti-leptons			Charge
$\bar{\nu}_e$	$\bar{\nu}_\mu$	$\bar{\nu}_\tau$	0
$e+$	$\mu+$	$\tau+$	+1
Anti-quarks			
$\bar{\mathrm{u}}$	$\bar{\mathrm{c}}$	$\bar{\mathrm{t}}$	$-2/3$
$\bar{\mathrm{d}}$	$\bar{\mathrm{s}}$	$\bar{\mathrm{b}}$	$+1/3$

Anti-matter particle has:

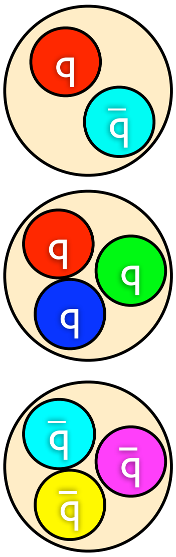
- Opposite electric charge, opposite colour charge
- Same mass & lifetime
- Opposite $\mathcal{B}, S, C, B, T, I_Z, L_e, L_\mu$ & L_τ

Hadrons

Free quarks are never observed.

Quarks are always found in bound colour-neutral states:

- Mesons: a quark and an anti-quark
- Baryons: three quarks
- Anti-baryons: three anti-quarks



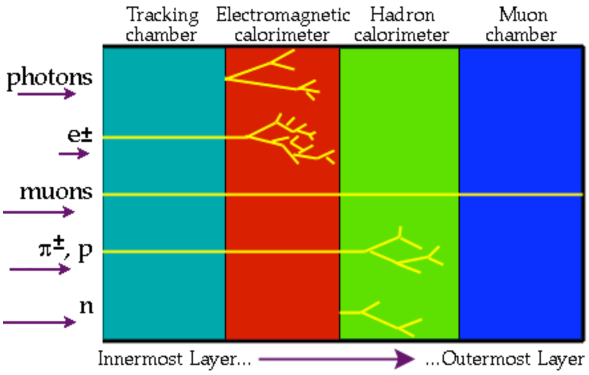
Colour confinement

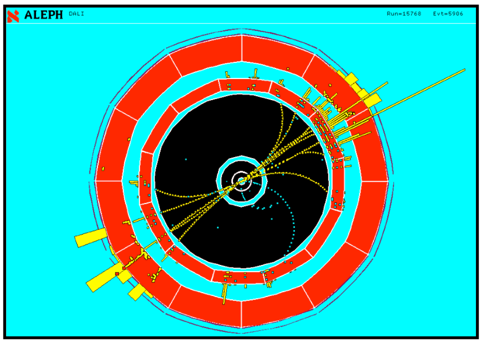
- The quarks are confined to hadrons due to strong force
- gluon self-interactions
- coupling constant α_S increases as quarks become further apart

Interactions

- Consider the interactions of the individual quarks

Detector Signals





- Charged particles leave several position measurements in the tracking detector. Positions are joined up to trace out a ‘track’, used to reconstruct the momentum.
- Energies of electrons, photons and hadrons are absorbed in calorimeter, allowing energy to be measured.
- Neutrinos do not interact at all in detector. Observed imbalance in momentum.
- Quarks “hadronise”, producing series of hadrons. Appear in detector as narrow “jet” of particles.

Forces & Interactions

Three forces to consider: strong (QCD), electromagnetic (QED) & weak.

- Weak force has two bosons: W and Z

Forces are propagated by the exchange of bosons.

- Bosons exchange four momentum, $\underline{\underline{q}}$, between the initial and final state

Strength of interaction is acts on some properties of the particle, *e.g.* electromagnetic force is couples to electric charges of interacting particles

Strong	exchange of gluons	couples to colour charge
Electromagnetic	exchange of photons	couples to electric charge
Weak Neutral Current	exchange of Z^0 boson	couples to **all fermions**
Weak Charged Current	exchange of $W^\pm$ boson	couples to **all fermions**

The exchanged bosons are often **virtual** (as opposed to real).

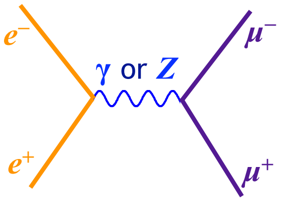
Virtual: square of four momentum is not mass squared: $\underline{\underline{q}}^2 = E^2 - \vec{p}\cdot\vec{p} \neq m^2_{\text{boson}}$

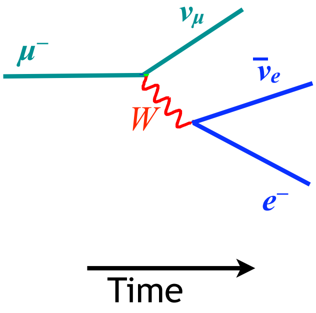
Allowed by HUP; we can never directly detect virtual bosons: only their effects.

Feynman Diagrams

Feynman diagrams are used to **illustrate** and **calculate** rates of decays and scattering.

- e.g. muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$
- e.g. $e^+e^- \to \mu^+\mu^-$ scattering





Use the Feynman Rules to calculate the matrix element, $\mathcal{M}$, from diagram

- For decay the partial width of the decay, Γ, is proportional to $\mathcal{M}^2$
- For scattering the cross section, σ, is proportional to $\mathcal{M}^2$

Use four momentum conservation to calculate boson four momentum, $\underline{\underline{q}}$

- Muon decay

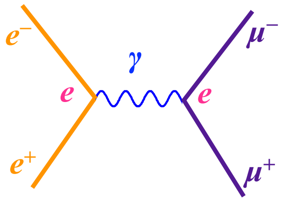
$$\underline{\underline{q}} = \underline{\underline{p}}_\mu - \underline{\underline{p}}_{\nu_\mu} = \underline{\underline{p}}_e + \underline{\underline{p}}_{\nu_e}$$

- $e^+e^- \to \mu^+\mu^-$ scattering

$$\underline{\underline{q}} = \underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-} = \underline{\underline{p}}_{\mu^+} + \underline{\underline{p}}_{\mu^-}$$

Feynman Rules

The matrix element, $\mathcal{M}$, is the amplitude, per unit time, for a given process to happen.



We calculate $\mathcal{M}$ from:
- the vertex couplings at the vertex
- the boson propagator term

The Feynman Rules

- Write down the **coupling at the each vertex:**
  - *charge of the fermion* (for EM), g_S (for QCD)
  - g_W (for ***W-ℓ-ν*** vertex), $g_W V_{qq'}$ (for ***W-q-q'*** vertex),
- Work out the four-momentum transferred by the boson, $\underline{q}$
- Write down the **propagator term** for each boson: $1/(\underline{q}^2 - m^2_{\text{boson}})$

$\mathcal{M}$ is proportional to vertex couplings and propagator terms *e.g.*

$$\mathcal{M}(e^+e^- \to \mu^+\mu^-) = e^2/\underline{q}^2$$

- If process involves hadrons: consider interactions of the constituent quarks

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Decays

We use **decays** and **scattering cross section** to understand interactions.

- A decay can only occur if $m_{\text{initial}} > \sum m_{\text{final}}$
- The stronger the interaction, the quicker the particle will decay.

Measurable quantities:

- **lifetime:** τ Dimensions: time.
- **total width:** $\Gamma = \hbar/\tau$ Dimensions: energy.
- **Partial width of decay mode** *e.g.* $\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)$

$$\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) \propto \left(\mathcal{M}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)\right)^2$$

Force	Typical Lifetimes
Strong	10^{-20} - 10^{-23} s
EM	10^{-20} - 10^{-16} s
Weak	10^{-13} - 10^{3} s

- The **total width** is the sum of **all** the individual decay modes *e.g.*

$$\Gamma_\tau = \Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) + \Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) + \Gamma(\tau^- \to \nu_\tau + \text{hadrons})$$

- The **branching ratio** is the fraction of time a particle decays into a particular final state, *e.g.*

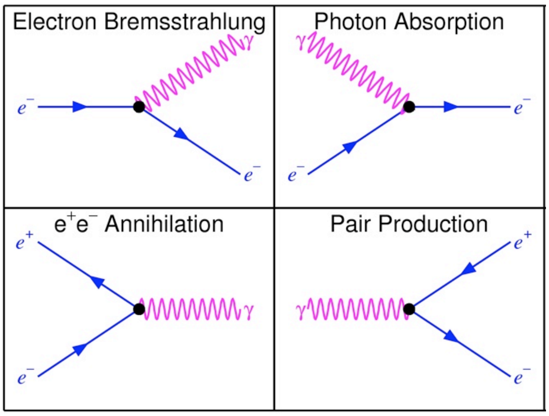
$$\text{BR}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = \frac{\Gamma(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{\Gamma_\tau}$$

- The sum of all possible branching ratios adds to 1.

Quantum Electrodynamics

QED is quantum theory of electromagnetic interactions.

- All charged particles interact via QED
- All interactions are described by fermion-fermion-photon (γ) vertex:
  - Fermion emits or absorbs a photon
  - **γ→fermion anti-fermion** or **fermion anti-fermion→γ**
- **Fermion flavour does not change** when it emits or absorbs a photon *e.g.* an e^- remains an e^-, **b**-quark remains a **b**-quark

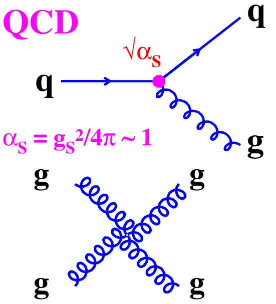


- Strength of vertex is proportional to charge of fermion
- Cross sections, decay width ∝ $\mathcal{M}^2$ write in powers of α

$$\alpha = \frac{e^2}{4\pi\epsilon_0} \approx \frac{1}{137}$$

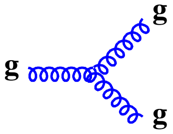
QED conserves:
$Q, I_Z, S, C, B, T, \mathcal{B}, L_e, L_\mu, L_\tau$

Quantum Chromodynamics

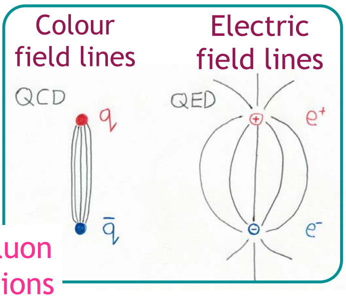


QCD is quantum theory of strong interactions.

- Acts on colour charged *i.e.* only quarks and gluons interact via QCD
- quark-quark-gluon vertex:
  - A quark emits or absorbs a gluon
  - **gluon→quark + anti-quark** or **quark + anti-quark→gluon**
- Quark flavour does not change, but colour charge changes



- As gluons also carry colour charge, the gluons interact with other gluons



- Potential between two quarks is:

$$V_{\rm QCD}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr$$

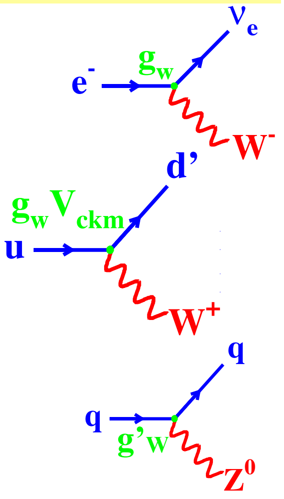
quark-gluon interaction

gluon-gluon interactions

- Almost impossible to pull two quarks apart: **colour confinement**

QCD conserves:
$Q, I_Z, S, C, B, T, \mathcal{B}, L_e, L_\mu, L_\tau$

Weak Interactions



Weak Force is propagated by massive $W^\pm$ and Z^0 bosons

Weak force interacts on all quarks and leptons.

Charged current **changes the flavour of the fermion**:

- Allowed flavour changes: $\mathcal{B}$, L_e, L_μ and L_τ conserved

$$e^-\leftrightarrow\nu_e \quad \mu^-\leftrightarrow\nu_\mu \quad \tau^-\leftrightarrow\nu_\tau \quad e^+\leftrightarrow\bar{\nu}_e \quad \mu^+\leftrightarrow\bar{\nu}_\mu \quad \tau^+\leftrightarrow\bar{\nu}_\tau$$

$$(Q{=}+2/3 \text{ quark}) \leftrightarrow (Q{=}-1/3 \text{ quark})$$

$$(Q{=}-2/3 \text{ anti-quark}) \leftrightarrow (Q{=}+1/3 \text{ anti-quark})$$

Strength of charged current:

- Leptons vertices, universal coupling: g_W
- Quark vertices, depends on quark flavour *e.g.* for **W-u-d:** $g_W V_{ud}$

Neutral current no **fermion flavour change.**

Handy hint: neutrinos are only involved in weak interactions.

Weak force conserves:
$Q, \mathcal{B}, L_e, L_\mu, L_\tau$

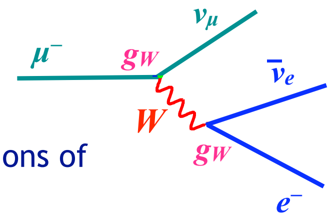
Weak Interactions at Low Energy

If four-momentum $\underline{\underline{q}}$ transferred by a W-boson is small, $\underline{\underline{q}} \ll M_W^2$ use Fermi constant, G_F, to describe rate of process:

$$G_F = \frac{\sqrt{2}\, g_w^2}{8\, m_W^2}$$

e.g. muon decay has one allowed decay mode $\mu^-\to e^-\bar{\nu}_e\,\nu_\mu$

$$\mathcal{M} \propto \frac{g_W^2}{\underline{\underline{q}}^2 - m_W^2} \to \frac{g_W^2}{m_W^2} \propto G_F$$



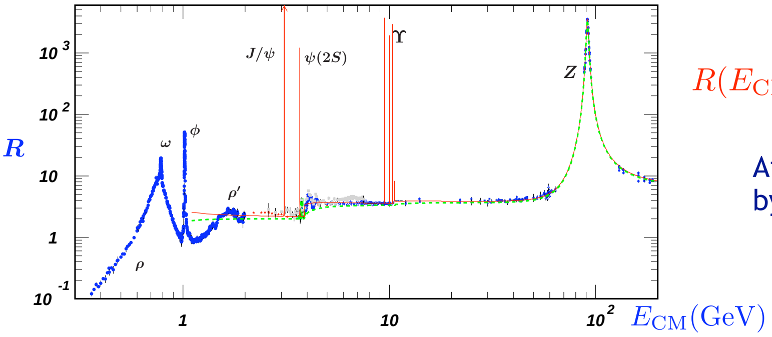
Decay width: $\Gamma \propto \mathcal{M}^2 \propto G_F^2$

To balance the dimensions use something with dimensions of energy: use m_μ as lifetime will depend on m_μ.

$$\Gamma_\mu = \Gamma(\mu \to e^-\bar{\nu}_e\nu_\mu) = KG_F^2\, m_\mu^5$$

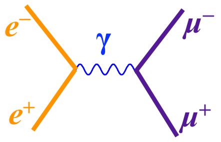
K: dimensionless constant

The Ratio *R*

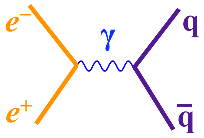


$$R(E_{\mathrm{CM}}) = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

At low energies, dominated by γ exchange



$$\sigma(e^+e^- \to \mu^+\mu^-) \propto e^4/\underline{\underline{q}}^4 = e^4/s^2$$



For each possible quark state (given colour, given flavour)

$$Q_{\mathrm{q}} = \frac{1}{3}, \frac{2}{3}$$

$$\sigma(e^+e^- \to \mathrm{q\bar{q}}) \propto Q_{\mathrm{q}}^2 e^4/\underline{\underline{q}}^4 = Q_{\mathrm{q}}^2 e^4/s^2$$

As E_{CM} increases can produce more flavours of quark: steps in R observed.

Each flavour of quark can be produced in three colours: red, green, blue.

R is sum over all possible quarks states which can be produced: $R = 3\sum_q Q_{\mathrm{q}}^2$

no. of colours

What you don't need to know...

The masses of the particles; they are **given on the constant sheet!** Except:

- neutrino mass is so small you can always ignore it $m_\nu \approx 0$!
- electron mass so small you can ignore it compared to other masses.
- W and Z bosons are much more massive than all lepton and hadron masses.

The lifetimes of the particles, they will be given if required. But remember typical lifetimes for the different forces.

The quark content of the hadrons. Except, handy to remember:

- proton is **uud** anti-proton is: $\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}}$
- neutron is **udd** anti-neutron is: $\bar{\mathrm{u}}\bar{\mathrm{d}}\bar{\mathrm{d}}$

You can work out the charge of a particle from its symbol *e.g.* $Q(\Delta^{++})=+2e$

- exceptions:
  - p and n don't have superscript (but I hope you know the charge of these)
  - quarks have charge $+2/3e$, $-1/3e$