The Standard Model

The Standard Model describes more-or-less everything we currently know about particle physics: the matter particles and the three of the four forces which describe their interactions.

Matter: aka the fermions

<table>
<thead>
<tr>
<th>Leptons</th>
<th>Charge, $e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_e$</td>
<td>$e^-$</td>
</tr>
<tr>
<td>$v_\mu$</td>
<td>$\mu^-$</td>
</tr>
<tr>
<td>$v_\tau$</td>
<td>$\tau^-$</td>
</tr>
<tr>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Quarks</th>
<th>Charge, $e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>c</td>
</tr>
<tr>
<td>d</td>
<td>s</td>
</tr>
<tr>
<td>t</td>
<td>b</td>
</tr>
<tr>
<td>+2/3</td>
<td>-1/3</td>
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</tbody>
</table>

Two processes used to study interactions:
- Decay: measure partial decay widths and lifetimes
- Scattering: measure cross sections

Forces
- Interactions are propagated by the exchange of bosons

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Bosons</th>
<th>$Q, e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>gluons, $g$</td>
<td>0</td>
</tr>
</tbody>
</table>
| Electro-
  magnetic | photon, $\gamma$ | 0      |
| Weak         | $W^\pm$, $Z^0$ | 0, $\pm 1$ |
| Gravity      | ?            | ?      |
Relativistic Dynamics

Relativistic Dynamics is used to describe kinematics in **decays** and **scattering**.

- **Four momentum:**
  \[ p = (E/c, p_x, p_y, p_z) = (E/c, \vec{p}) \]

- If we square four-momentum:
  \[ p^2 = \frac{E^2}{c^2} - \vec{p} \cdot \vec{p} = m^2 c^2 \] we get the mass squared!

- In **decay** the four-momentum is conserved e.g. in \( \mu \rightarrow e^- \bar{\nu}_e \nu_\mu \)
  \[ p_{\mu} = p_{e^-} + p_{\bar{\nu}_e} + p_{\nu_\mu} \]
  Square both sides: \( m_{\mu}^2 c^2 = (p_{e^-} + p_{\bar{\nu}_e} + p_{\nu_\mu})^2 \)

- In **a scattering** the four-momentum is conserved e.g. \( e^+ e^- \rightarrow \mu^+ \mu^- \)
  \[ p_{e^+} + p_{e^-} = p_{\mu^+} + p_{\mu^-} \]

- In a scattering, the square of the initial four momentum is \( s \). Energy in the Centre of Mass frame is \( \sqrt{s} \), e.g. \( s = (p_{e^+} + p_{e^-})^2 \)

In both decay and scattering: boson transfers momentum from initial to final state!

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Natural Units

- **Natural units**: set \( c = \hbar = 1 \)
  - All quantities can be expressed as a power of energy.
  - Mass, momentum and energy measured in the same units: **MeV** or **GeV**

- **Two important quantities for Lorentz transformations**:
  \[ \beta = v/c \quad \gamma(v) = 1/\sqrt{1 - \beta^2} \]

**Natural Units**

- **Lorentz boosts**:
  \[ \gamma = E/m \quad \gamma \beta = |\vec{p}|/m \quad \beta = |\vec{p}|/E \]

- **Four momentum**:
  \[ p = (E, p_x, p_y, p_z) \]

- **Invariant mass**:
  \[ p^2 = E^2 - \vec{p}^2 = m^2 \]
**Quantum Numbers**

- **Leptons**: $e^-, \mu^-, \tau^-$
- **Quarks:**
  - Isospin, $I_Z = \# [N(u) - N(d) + N(d^\prime) - N(u^\prime)]$
  - Baryon number, $B = 1/3$ for quarks, $B = -1/3$ for anti-quarks
  - Strangeness: $S$, Charm: $C$, Bottomness: $B$, Topness: $T$ - net number of $s, c, b, t$
    
    $S = N(\bar{s}) - N(s) \quad C = N(c) - N(\bar{c}) \quad B = N(\bar{b}) - N(b) \quad T = N(t) - N(\bar{t})$
    
    - Every quark carries a colour charge: red, blue or green

**Matter**

**Six quarks and six leptons**

Matter is grouped into three generations.

Each generation consists of:

- 1 lepton with $Q = -1e$
- 1 neutral lepton $Q = 0$ ($\nu$)
- 1 quark with $Q = +2/3e$
- 1 quark with $Q = -1/3e$

Each generation is successively heavier.

<table>
<thead>
<tr>
<th>Quantum Numbers</th>
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</thead>
<tbody>
<tr>
<td><strong>Leptons</strong> $L_e, L_\mu, L_\tau$</td>
</tr>
<tr>
<td><strong>Quarks:</strong></td>
</tr>
<tr>
<td>- Isospin, $I_Z = # [N(u) - N(d) + N(d^\prime) - N(u^\prime)]$</td>
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<table>
<thead>
<tr>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_e$</td>
<td>$v_\mu$</td>
<td>$v_\tau$</td>
<td>0</td>
</tr>
<tr>
<td>$e^-$</td>
<td>$\mu^-$</td>
<td>$\tau^-$</td>
<td>$-1e$</td>
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</tbody>
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<th>$c$</th>
<th>$t$</th>
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<td>$d$</td>
<td>$s$</td>
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**Anti-matter**

Every matter particle has an anti-matter partner.

$E^2 = p^2 c^2 + m^2 c^4 \Rightarrow E = \pm \sqrt{p^2 c^2 + m^2 c^4}$

- Particle is the positive energy solution
- Anti-particle is negative energy solution

Feynman’s interpretation:

negative energy particle with charge $Q$ moving backward in space & time appears as positive energy particle with charge $-Q$ moving forward in space & time.

<table>
<thead>
<tr>
<th>Anti-leptons</th>
<th>Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{v}_e$</td>
<td>0</td>
</tr>
<tr>
<td>$e^+$</td>
<td>+1</td>
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<td>$\bar{u}$</td>
</tr>
<tr>
<td>$\bar{d}$</td>
</tr>
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</table>

Anti-matter particle has:

- Opposite electric charge, opposite colour charge
- Same mass & lifetime
- Opposite $B, S, C, B, T, I_Z, L_e, L_\mu, L_\tau$
**Hadrons**

**Free quarks are never observed.**
Quarks are always found in bound colour-neutral states:
- Mesons: a quark and an anti-quark
- Baryons: three quarks
- Anti-baryons: three anti-quarks

**Colour confinement**
- The quarks are confined to hadrons due to strong force
- Gluon self-interactions
- Coupling constant $\alpha_s$ increases as quarks become further apart

**Interactions**
- Consider the interactions of the individual quarks

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**Detector Signals**

- Charged particles leave several position measurements in the tracking detector. Positions are joined up to trace out a 'track', used to reconstruct the momentum.
- Energies of electrons, photons and hadrons are absorbed in calorimeter, allowing energy to be measured.
- Neutrinos do not interact at all in detector. Observed imbalance in momentum.
- Quarks “hadronise”, producing series of hadrons. Appear in detector as narrow “jet” of particles.
Forces & Interactions

Three forces to consider: strong (QCD), electromagnetic (QED) & weak.

- Weak force has two bosons: $W$ and $Z$

Forces are propagated by the exchange of bosons.

- Bosons exchange four momentum, $q$, between the initial and final state

Strength of interaction is acts on some properties of the particle, e.g. electromagnetic force is couples to electric charges of interacting particles

<table>
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<tr>
<th>Strong</th>
<th>exchange of gluons</th>
<th>couples to colour charge</th>
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<tbody>
<tr>
<td>Electromagnetic</td>
<td>exchange of photons</td>
<td>couples to electric charge</td>
</tr>
<tr>
<td>Weak Neutral Current</td>
<td>exchange of $Z^0$ boson</td>
<td>couples to all fermions</td>
</tr>
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<td>Weak Charged Current</td>
<td>exchange of $W^\pm$ boson</td>
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The exchanged bosons are often virtual (as opposed to real).

Virtual: square of four momentum is not mass squared: $q^2 = E^2 - \vec{p} \cdot \vec{p} \neq m^2_{boson}$

Allowed by HUP; we can never directly detect virtual bosons: only their effects.

Feynman Diagrams

Feynman diagrams are used to illustrate and calculate rates of decays and scattering.

- e.g. muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

- e.g. $e^+e^- \to \mu^+\mu^-$ scattering

Use the Feynman Rules to calculate the matrix element, $\mathcal{M}$, from diagram

- For decay the partial width of the decay, $\Gamma$, is proportional to $\mathcal{M}^2$
- For scattering the cross section, $\sigma$, is proportional to $\mathcal{M}^2$

Use four momentum conservation to calculate boson four momentum, $q$

- Muon decay
  \[ q = p_\mu - p_\nu_\mu = p_e + p_\nu_e \]

- $e^+e^- \to \mu^+\mu^-$ scattering
  \[ q = p_{e^+} + p_{e^-} = p_{\mu^+} + p_{\mu^-} \]
Decays

We use decays and scattering cross section to understand interactions.

• A decay can only occur if $m_{\text{initial}} > \sum m_{\text{final}}$
• The stronger the interaction, the quicker the particle will decay.

Measurable quantities:
• lifetime: $\tau$ Dimensions: time.
• total width: $\Gamma = \hbar/\tau$ Dimensions: energy.
• Partial width of decay mode e.g. $\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$

$$\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) \propto (\mathcal{M}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau))^2$$

• The total width is the sum of all the individual decay modes e.g.

$$\Gamma_\tau = \Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) + \Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) + \Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})$$

• The branching ratio is the fraction of time a particle decays into a particular final state, e.g.

$$BR(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) = \frac{\Gamma(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)}{\Gamma_\tau}$$

• The sum of all possible branching ratios adds to 1.

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Quantum Electrodynamics

QED is quantum theory of electromagnetic interactions.
- All charged particles interact via QED.
- All interactions are described by fermion-fermion-photon (γ) vertex:
  - Fermion emits or absorbs a photon.
  - γ → fermion anti-fermion or fermion anti-fermion → γ.
- Fermion flavour does not change when it emits or absorbs a photon e.g. an e⁻ remains an e⁻, b-quark remains a b-quark.

\[ \alpha = \frac{e^2}{4\pi\epsilon_0} \approx \frac{1}{137} \]

QED conserves:
- Q, I, S, C, B, T, L_ε, L_μ, L_τ

Quantum Chromodynamics

QCD is quantum theory of strong interactions.
- Acts on colour charged i.e. only quarks and gluons interact via QCD.
- quark-quark-gluon vertex:
  - A quark emits or absorbs a gluon.
  - gluon → quark + anti-quark or quark + anti-quark → gluon.
- Quark flavour does not change, but colour charge changes.
- As gluons also carry colour charge, the gluons interact with other gluons.
- Potential between two quarks is:
  \[ V_{QCD}(r) = -\frac{4\alpha_s}{3} \frac{1}{r} + kr \]
  
QCD conserves:
- Q, I, S, C, B, T, L_ε, L_μ, L_τ

\[ \alpha_s = \frac{g_s^2}{4\pi} \approx 1 \]
Weak Interactions

Weak Force is propagated by massive $W^\pm$ and $Z^0$ bosons.

Weak force interacts on all quarks and leptons.

Charged current changes the flavour of the fermion:
- Allowed flavour changes: $B, L_e, L_\mu$, and $L_\tau$ conserved
  
  $e^- \leftrightarrow \nu_e, \mu^- \leftrightarrow \nu_\mu, \tau^- \leftrightarrow \nu_\tau, e^\pm \leftrightarrow \bar{\nu}_e, \mu^\pm \leftrightarrow \bar{\nu}_\mu, \tau^\pm \leftrightarrow \bar{\nu}_\tau$

  ($Q=+2/3$ quark) $\leftrightarrow$ ($Q=-1/3$ quark)

  ($Q=-2/3$ anti-quark) $\leftrightarrow$ ($Q=+1/3$ anti-quark)

Strength of charged current:
- Leptons vertices, universal coupling: $g_W$
- Quark vertices, depends on quark flavour e.g. for $W$-u-d: $g_W V_{ud}$

Neutral current no fermion flavour change.

Handy hint: neutrinos are only involved in weak interactions.

Weak force conserves:

$q, B, L_e, L_\mu, L_\tau$

Weak Interactions at Low Energy

If four-momentum $q$ transferred by a $W$-boson is small, $q \ll M_W^2$ use Fermi constant, $G_F$, to describe rate of process:

$$G_F = \frac{\sqrt{2} g_W^2}{8 m_W^2}$$

E.g. muon decay has one allowed decay mode $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$

$$\mathcal{M} \propto \frac{g_W^2}{q^2 - m_W^2} \rightarrow \frac{g_W^2}{m_W^2} \propto G_F$$

Decay width: $\Gamma \propto \mathcal{M}^2 \propto G_F^2$

To balance the dimensions use something with dimensions of energy: use $m_\mu$ as lifetime will depend on $m_\mu$.

$$\Gamma_\mu = \Gamma(\mu \rightarrow e^- \bar{\nu}_e \nu_\mu) = K G_F^2 m_\mu^5$$

$K$: dimensionless constant
The Ratio $R$

$$R(E_{CM}) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

At low energies, dominated by $\gamma$ exchange.

As $E_{CM}$ increases can produce more flavours of quark: steps in $R$ observed.
Each flavour of quark can be produced in three colours: red, green, blue.

$R$ is sum over all possible quark states which can be produced: $R = 3 \sum_{q} Q_q^2 \frac{e^4}{s^2}$

no. of colours

What you don’t need to know...

The masses of the particles; they are given on the constant sheet! Except:
- neutrino mass is so small you can always ignore it $m_\nu \approx 0$!
- electron mass so small you can ignore it compared to other masses.
- $W$ and $Z$ bosons are much more massive than all lepton and hadron masses.

The lifetimes of the particles, they will be given if required. But remember typical lifetimes for the different forces.

The quark content of the hadrons. Except, handy to remember:
- proton is $uud$ anti-proton is: $\bar{u}\bar{u}\bar{d}$
- neutron is $udd$ anti-neutron is: $\bar{u}\bar{d}\bar{d}$

You can work out the charge of a particle from its symbol e.g. $Q(\Lambda^{++}) = +2e$
- exceptions:
  - $p$ and $n$ don’t have superscript (but I hope you know the charge of these)
  - quarks have charge $+2/3e$, $-1/3e$