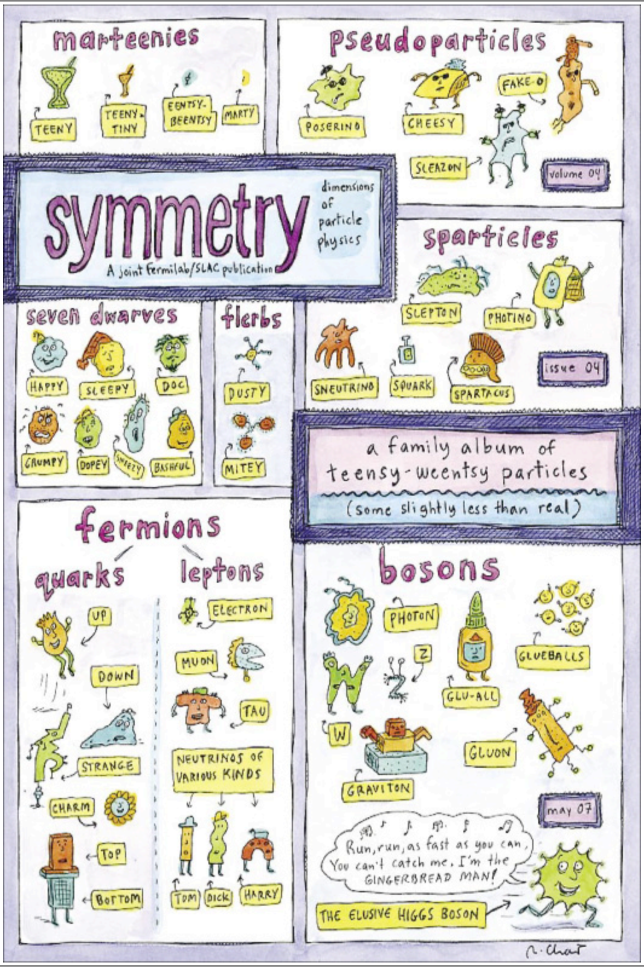


Subatomic Physics:

Particle Physics

Lecture 9: Weak Decays and the Electroweak Model

27th November 2009

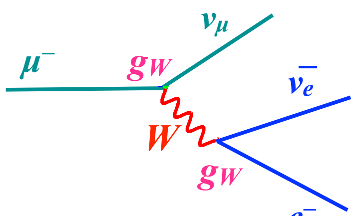
- Interactions of W and Z bosons at low energy
- Electroweak Theory
- Interactions of W and Z bosons at high energy

W and Z boson at low energies

At low energies, $\underline{\underline{q}} \ll m_Z, m_W$:

- The W and Z bosons are virtual:

$$\begin{aligned} \underline{\underline{q}}_Z^2 &= E_Z^2 - \vec{p}_Z \cdot \vec{p}_Z \neq m_Z^2 \\ \underline{\underline{q}}_W^2 &= E_W^2 - \vec{p}_W \cdot \vec{p}_W \neq m_W^2 \end{aligned}$$

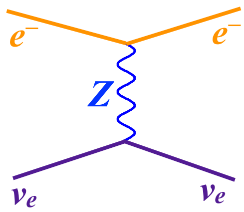


$\mu^- \rightarrow e^- \, \bar{\nu}_e \, \nu_\mu$

$\underline{\underline{q}}_W^2 \neq m_W^2$

- virtual W-bosons are responsible for the decays of the leptons, and the lightest hadrons *e.g.* $\mu^- \rightarrow e^- \, \bar{\nu}_e \, \nu_\mu$

- interactions of virtual Z-boson are not clearly evident, as they look similar to γ interactions (γ interactions are much stronger than Z-boson interactions)



$\nu_e \, e^- \rightarrow \nu_e \, e^-$

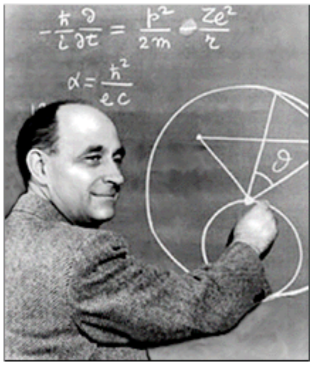
$\underline{\underline{q}}_Z^2 \neq m_Z^2$

At low energies we see the effect of Z boson mainly in scattering involving neutrinos *e.g.* $\nu_e \, e^- \rightarrow \nu_e \, e^-$

- as γ cannot couple to the neutral neutrinos

Fermi Theory

Weak Interactions at Low Momentum Transfer



For muon decay, and many other weak processes:

$$\mathcal{M} \propto \frac{g_w^2}{(\underline{q}^2 - m_W^2)}$$

At low momentum transfer $\underline{q}^2 \ll m_W^2$

$$\mathcal{M} \to \propto \frac{g_w^2}{m_W^2}$$

Introduce **Fermi coupling constant**:

$$G_F \propto \frac{g_w^2}{m_W^2} \qquad G_F = \frac{\sqrt{2}\, g_w^2}{8\, m_W^2}$$

- Dimension $[E]^{-2}$
- From experimental measurements: $G_F = 1.16637 \times 10^{-5}\ \mathrm{GeV}^{-2}$

Measurements of $G_F \,\&\, M_W \;\Rightarrow\; g_w = 0.66 \;\Rightarrow\; \alpha_w = \frac{g_w^2}{4\pi} = \frac{1}{29} > \alpha_{\mathrm{EM}} = \frac{1}{137}$

- Recall, from problem sheet 2, Q8, range of W boson: $\Delta x \approx \frac{\hbar}{\Delta p} = \frac{\hbar}{m_W c} = 0.002\ \mathrm{fm}$

Weak interaction not intrinsically weak - appears weak due to large boson masses.

Tools of Quantum Mechanics

Review from lecture 2

- Each particle can be described as a **quantum state**, $|\phi\rangle$
- The electromagnetic, weak and strong forces acting on these states can be represented by (three different) **quantum operators**, $\hat{O}$
- Rates of interactions such as **particle lifetimes** and **scattering cross sections** are given by **Fermi's Golden rule:**
- Transition between an initial state $|\phi_i\rangle$ and a final state $|\phi_f\rangle$ are related to the matrix element $\mathcal{M} = V_{fi} = \langle \phi_f \,|\, \hat{O} \,|\, \phi_i \rangle$:

$$\text{Transition probability}, T = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \rho$$

ρ is the denisty of final states: the more final states are allowed the faster the process will happen

- T is related to the cross section of scattering, σ. e.g. $\sigma(e^+e^- \to \mu^+\mu^-) \propto |\mathcal{M}\,(e^+e^- \to \mu^+\mu^-)|^2$.
- T is related to the inverse lifetime of a decay, τ. e.g. $\tau(\mu^- \to e^-\, \bar{\nu}_e\, \nu_\mu) \propto 1/|\mathcal{M}\,(\mu^- \to e^-\, \bar{\nu}_e\, \nu_\mu)|^2$.

Decay Modes

Review from lecture 2

Particles can have more than one **decay mode**. e.g. The K_S meson decays 99.9% of the time in one of two ways:

$$K_S \to \pi^+\pi^-, K_S \to \pi^0\pi^0$$

- Each decay mode has its own matrix element, $\mathcal{M}$. Fermi's Golden Rule gives us the **partial decay width** for each decay mode:

$$\Gamma(K_S \to \pi^+\pi^-) \propto |\mathcal{M}(K_S \to \pi^+\pi^-)|^2 \qquad \Gamma(K_S \to \pi^0\pi^0) \propto |\mathcal{M}(K_S \to \pi^0\pi^0)|^2$$

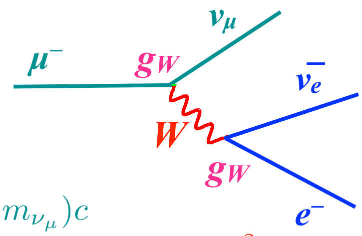
- The **total decay width** is equal to the sum of the decay widths for all the allowed decays.

$$\Gamma(K_S) = \Gamma(K_S \to \pi^0\pi^0) + \Gamma(K_S \to \pi^+\pi^-)$$

- The **branching ratio**, **BR**, is the fraction of time a particle decays to a particular final state:

$$BR(K_S \to \pi^+\pi^-) = \frac{\Gamma(K_S \to \pi^+\pi^-)}{\Gamma(K_S)} \qquad BR(K_S \to \pi^0\pi^0) = \frac{\Gamma(K_S \to \pi^0\pi^0)}{\Gamma(K_S)}$$

Muon Decay



A muon can only decay into lighter particles: e^-, γ, ν.

- In particular, all hadrons are heavier than m_μ.

L_e, L_μ, L_τ conservation ⇒ only lowest order decay is $\mu^- \to e^- \bar{\nu}_e \nu_\mu$

Maximum four-momentum transferred by W boson is $\underline{\underline{q}} = (m_\mu - m_{\nu_\mu})c$

$$\mathcal{M} \propto \frac{g_W^2}{\underline{\underline{q}}^2 - m_W^2}$$

$$\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu) \propto |\mathcal{M}|^2 \propto \frac{g_W^4}{(\underline{\underline{q}}^2 - m_W^2)^2} \to \frac{g_W^4}{m_W^4} \propto G_F^2$$

As there's only 1 decay mode:

- the partial width, $\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu)$ = total decay width, $\Gamma_\mu = \hbar/\tau_\mu \propto G_F^2$

To calculate a value for Γ_μ, need to know the density of states, ρ.
Use dimensional analysis:

➡ Γ has dimensions of energy, $[E]$;

➡ G_F^2 has dimensions $[E]^{-4}$

To balance dimensions, use m_μ (only other energy/mass in problem): $\Gamma_\mu = K\, G_F^2\, m_\mu^5$

where K is a dimensionless constant

(full calculation gives $\Gamma_\mu = G_F^2\, m_\mu^5/(192\,\pi^3)$)

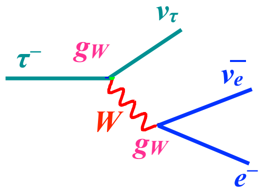
Tau Lepton Decay

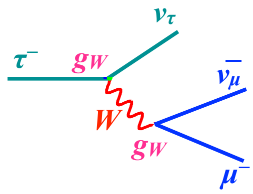
Decay Mode	BR
$\tau^- \to e^- \bar{\nu}_e \nu_\tau$	17.8%
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$	17.4%
$\tau^- \to \text{hadrons} + \nu_\tau$	64.7%

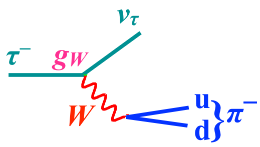
$m_\tau = 1.777\ \text{GeV}/c^2 > m_\mu, m_\pi, m_\rho, \ldots$

More than one final state possible.

- e.g. $\tau^- \to e^- \bar{\nu}_e \nu_\tau$, $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$, $\tau^- \to \pi^- \bar{\nu}_\mu \nu_\tau$







$$\mathcal{M}(\tau \to e\nu_e\nu_\tau) \propto \frac{g_W^2}{q^2 - m_W^2} \qquad \mathcal{M}(\tau \to \mu\nu_\mu\nu_\tau) \propto \frac{g_W^2}{q^2 - m_W^2}$$

$\tau^- \to e^- \bar{\nu}_e \nu_\tau$ and $\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$ have same matrix element as $\mu^- \to e^- \bar{\nu}_e \nu_\tau$:

$$\begin{aligned} \Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) &= K\, G_F^2\, m_\mu^5 \\ \Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau) &= K\, G_F^2\, m_\tau^5 \end{aligned}$$

- From measured branching ratios:

$$\begin{aligned} \Gamma_\tau &= \Gamma(\tau^- \to e^-\, \bar{\nu}_e\, \nu_\tau)/0.178 \\ \Gamma_\mu &= \Gamma(\mu^- \to e^-\, \bar{\nu}_e\, \nu_\mu) \end{aligned}$$

Rearranging: $$\frac{\Gamma(\tau^- \to e^- \bar{\nu}_e\, \nu_\tau)}{\Gamma(\mu^- \to e^- \bar{\nu}_e\, \nu_\mu)} = \frac{0.178\,\Gamma_\tau}{\Gamma_\mu} = \frac{0.178\,\tau_\mu}{\tau_\tau} = \frac{m_\tau^5}{m_\mu^5}$$

Lepton Universality

- We've shown $\Gamma_\mu = K\, G_F^2\, m_\mu^5$ $\left(\Gamma_\mu = G_F^2\, m_\mu^5/(192\,\pi^3)\right)$ and $\dfrac{m_\tau^5}{m_\mu^5} = \dfrac{0.178\,\tau_\mu}{\tau_\tau}$

- Experimental measurements:
  - $\tau_\mu = 2.19703 \times 10^{-6}$ s $m_\mu = 105.65837\ \text{MeV}/c^2$ $m_\tau = 1777.0\ \text{MeV}/c^2$
  - used to extract G_F (and g_W) $\Rightarrow$ $G_F = 1.16637(1) \times 10^{-5}\ \text{GeV}^{-2}$

- Predict the lifetime of the tau-lepton:

$$\tau_\tau = \text{BR}(\tau \to e^-\nu_e\nu_\tau)\, \tau_\mu \frac{m_\mu^5}{m_\tau^5} = 2.91 \times 10^{-13}\ \text{s}$$

- Compare to measured $\tau_\tau = (2.906 \pm 0.011) \times 10^{-13}$ s

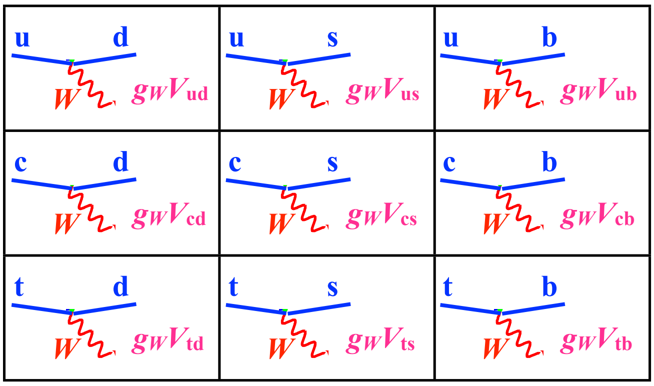
The relationship between the tau and muon lifetimes illustrates **lepton universality**.

- Coupling of to W-boson to all leptons is equal = g_W
- electrons, muons and taus all interact identically
- interact with the **same bosons** with **same coupling strength**

Weak Interactions of Quarks

In general, any vertex W-(Q=+2/3 e quark)-(Q=−1/3 e quark) is valid.

- W-boson coupling to quarks suppressed by a flavour-dependent factor V or V_{CKM}



(Known as the "CKM matrix" - values from experimental measurements)

V_{ud}=0.974	V_{us}=0.227	V_{ub}=0.004
V_{cd}=0.230	V_{cs}=0.972	V_{cb}=0.042
V_{td}=0.008	V_{ts}=0.041	V_{tb}=0.999

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Largest couplings are within a generation:

d ↔ u s ↔ c b ↔ t

Electroweak Theory

We've seen already that wherever a γ boson can be exchanged a Z can also be exchanged:

- The weak and electromagnetic force are linked.
- At short distances (or high energies) the strength of the electromagnetic force and the weak force are comparable. Can be related by a parameter, **sin θ**$_W$

$$e = g_W \sin\theta_W$$

The weak and electromagnetic interactions are manifestations of a underlying force: **the electroweak force.**

- Couplings of the γ, W (and Z) bosons are related: $e = g_W \sin\theta_W$
- Mass of the W and Z bosons are related: $m_Z^2 = m_W^2/\cos^2\theta_W$

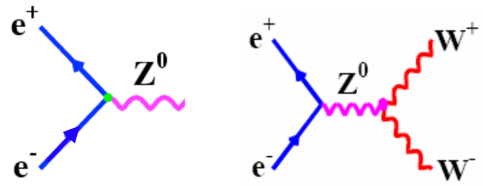
Just three fundamental parameters required to describe:

- couplings of W, Z and γ to quarks and leptons
- masses of the W, Z, γ bosons
- interactions of the W, Z, γ bosons with each other

Normally use three most accurately measured parameters *e.g.* e, G_F, m_Z

W and *Z* boson at high energies

Using a collider, we can create high enough energies to make real *W* and *Z* bosons:



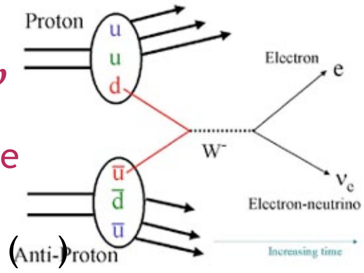
$$\underline{q} \sim m_Z, m_W$$

- e.g. LEP collider $e^+e^-\to Z$, $e^+e^-\to W^+W^-$
- e.g. At Tevatron, Sp$\bar{\text{p}}$S, LHC collider $pp\to Z+X$, $pp\to W+X$

Can study the properties of the *W* and *Z* bosons in detail.

- masses, lifetime/width, coupling strengths, decay modes, spin...

quarks within *p* and *p* (or $\bar{p}$) interact to make *W* or *Z*



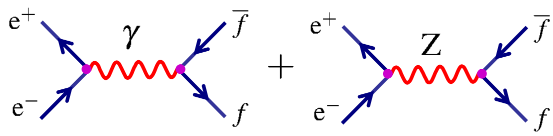
Discovery of *W* and *Z* bosons at CERN in 1983 at the Sp$\bar{\text{p}}$S collider.

$E_p = E_{\bar{p}} = 270$ GeV

$p\bar{p}\to W^-\to e^-\bar{\nu}_e$ event at UA1 experiment

Z^0 resonance

- Both γ and Z contribute to $e^+e^-\to$**hadrons**



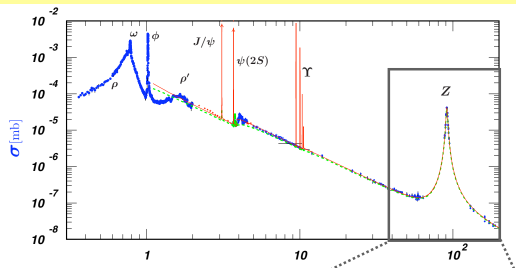
- At low E_{CM} mainly γ interactions
- At $\sqrt{s}\sim m_Z$, mainly Z-boson:

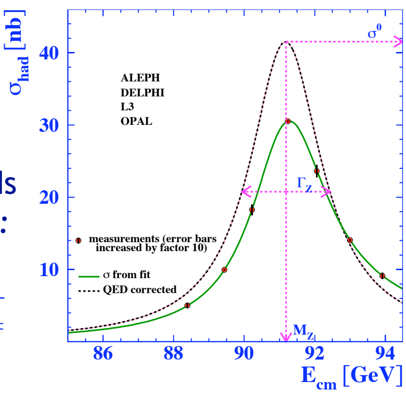
$$\sigma(e^+e^- \to Z \to f\bar{f}) \propto \frac{1}{(\underline{q}^2 - m_Z^2)^2}$$

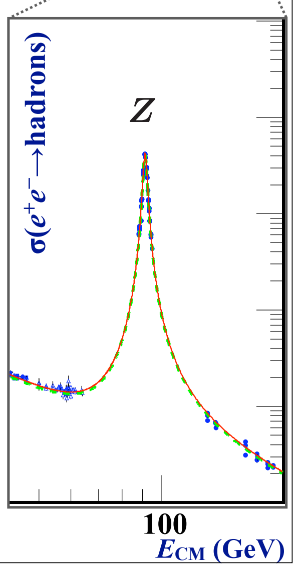
- For a massive boson, $\mathcal{M}$ also depends on total width $\Gamma_Z = \hbar/\tau_Z$. Shape of σ:

$$\sigma(E = \underline{q}^2) = \sigma_{\max} \frac{\Gamma_Z^2/4}{(\underline{q}^2 - m_Z)^2 + \Gamma_Z^2/4}$$

- Measurements at LEP:
  - $m_Z = 91.188 \pm 0.002$ GeV/c^2
  - $\Gamma_Z = 2.4953 \pm 0.002$ GeV

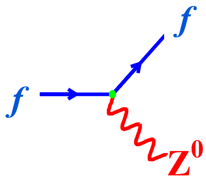




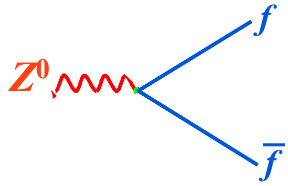


Decays of the Z boson

Z boson interacts with **all** quarks and leptons.
No change of quark or lepton flavour at Z boson vertex.



- Z boson can decay into any pair: fermion–anti-fermion, $f\bar{f}$.
- except top quark pair, as $m_Z < 2m_t$



$Z \to e^+e^-$	$Z \to \nu_e\bar{\nu}_e$	$Z \to u\bar{u}$	$Z \to d\bar{d}$
$Z \to \mu^+\mu^-$	$Z \to \nu_\mu\bar{\nu}_\mu$	$Z \to c\bar{c}$	$Z \to s\bar{s}$
$Z \to \tau^+\tau^-$	$Z \to \nu_\tau\bar{\nu}_\tau$		$Z \to b\bar{b}$

Lepton Universality $\Rightarrow$

- $\mathcal{M}$ for e, μ, τ decay is same $\Rightarrow \Gamma(Z\to e^+e^-) \approx \Gamma(Z\to\mu^+\mu^-) \approx \Gamma(Z\to\tau^+\tau^-)$
- $\mathcal{M}$ for ν_e, ν_μ, ν_τ decay is same $\Rightarrow \Gamma(Z\to\nu_e\bar{\nu}_e) = \Gamma(Z\to\nu_\mu\bar{\nu}_\mu) = \Gamma(Z\to\nu_\tau\bar{\nu}_\tau)$

Number of Neutrinos

Total width of the Z-boson (Γ_Z) is sum of all partial widths:

$$\Gamma_Z = \Gamma(Z \to q\bar{q}) + \Gamma(Z \to e^+e^-) + \Gamma(Z \to \mu^+\mu^-) + \Gamma(Z \to \tau^+\tau^-) + N_\nu \Gamma(Z \to \nu\bar{\nu})$$

LEP directly measured:

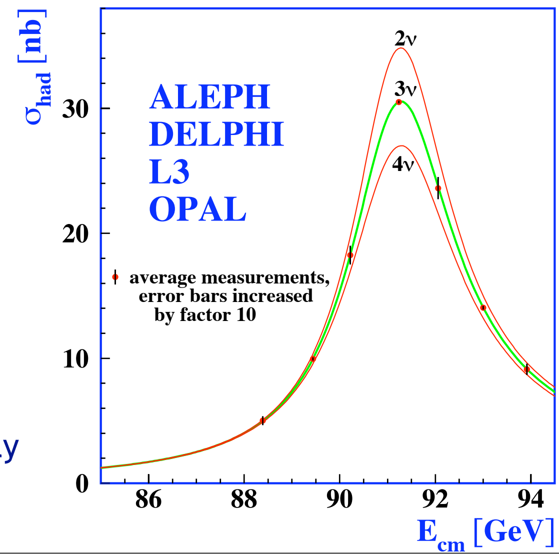
- partial widths: $\Gamma(Z\to e^+e^-)$, $\Gamma(Z\to\mu^+\mu^-)$, $\Gamma(Z\to\tau^+\tau^-)$, $\Gamma(Z\to\text{hadrons}) = \Gamma(Z\to q\bar{q})$
- total width, Γ_Z

Cannot measure $\Gamma(Z \to \nu\bar{\nu})$ directly as neutrinos leave no signal in the detector.

- Can predict $\Gamma(Z \to \nu\bar{\nu})$ using $\mathcal{M}(Z \to \nu\bar{\nu})$
- Use prediction & measurements to find number of neutrinos contributing to Γ_Z
- $N_\nu = 2.999 \pm 0.011$

Consistent with exactly three neutrinos!

$\Rightarrow$ Gives us confidence that there are exactly three generations of quarks and leptons

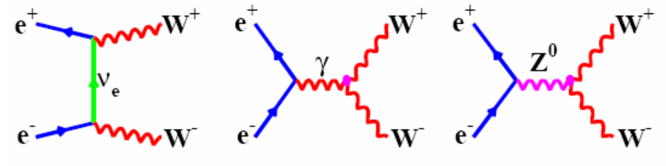


W-boson

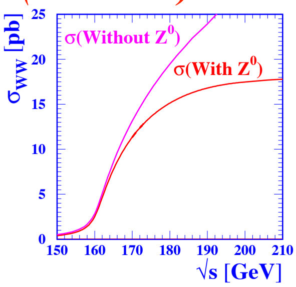
Colliders also produce real W-bosons ⇒ measure properties of the W-boson

At LEP: W-bosons produced in pairs

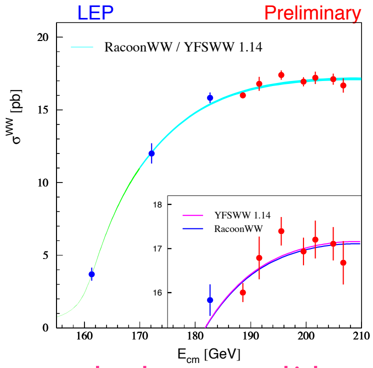
Three possible modes:



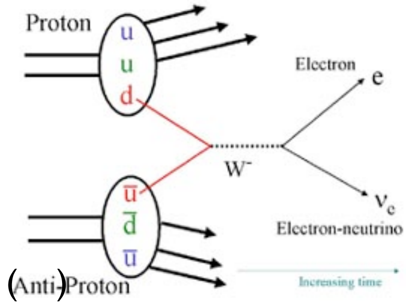
Predicted $\sigma(e^+e^-\to WW)$



$\sigma(e^+e^-\to WW)$ measured at LEP confirms Z^0-W^+-W^- vertex



At Tevatron and LHC: W-bosons produced from quarks and anti-quarks



From measurements at LEP & Tevatron:

- $m_W = 80.413 \pm 0.048$ GeV/c^2
- $\Gamma_W = 2.141 \pm 0.041$ GeV

Measurements of W and Z bosons validate the Electroweak Model beautifully

Summary

The weak force acts on **all** quarks and leptons. Two **massive** bosons propagate the weak interaction: $W^\pm$ and Z^0.	Weak interactions characterised by: • Long lifetimes 10^{-13} - 10^3 s • Small cross sections 10^{-13} mb
At low energies, virtual W and Z bosons responsible for lepton and lightest hadron decays and neutrino scatterings. Fermi theory describes W-boson interactions at low boson momentum transfer $\underline{q}^2 \ll m_W^2$ Described by Fermi constant: $G_F \propto g_w^2/m_W^2$	Lepton interactions are universal. Quarks interactions not universal: W-(Q=+2/3 quark)-(Q=−1/3 quark) coupling is $g_W V_{CKM}$, where V_{CKM} depends on flavour of quark Largest couplings within a generation: Wud, Wcs, Wtb
At high energies colliders can produce real W and Z bosons for study. Studies of Z boson decays suggest only three generations of quarks and leptons.	Electromagnetic & weak are manifestations of a single unified **electroweak interaction.** (just 3 parameters describe interaction!)

Standard Model Interactions

QED	QCD	Weak Neutral Current	Weak Charged Current
quantum theory of EM interactions	quantum theory of strong interactions	quantum theory of weak interactions	
mediated by exchange of virtual photons	mediated by exchange of gluons	mediated by exchange of Z bosons	mediated by exchange of W bosons
acts on all charged particles	acts on quarks only	acts on **all** quarks and leptons	
couples to electric charge	couples to colour charge	does not change quark or lepton flavour	changes quark and leptons flavours
coupling strength $\propto e$ $\propto \sqrt{\alpha}$	coupling strength $\propto g_S$ $\propto \sqrt{\alpha_S}$		coupling strength $\propto g_W$ $\propto \sqrt{\alpha_W}$
propagator: $\frac{1}{q^2}$	propagator: $\frac{1}{q^2}$	propagator: $\frac{1}{(q^2 - m_Z^2)}$	propagator: $\frac{1}{(q^2 - m_W^2)}$