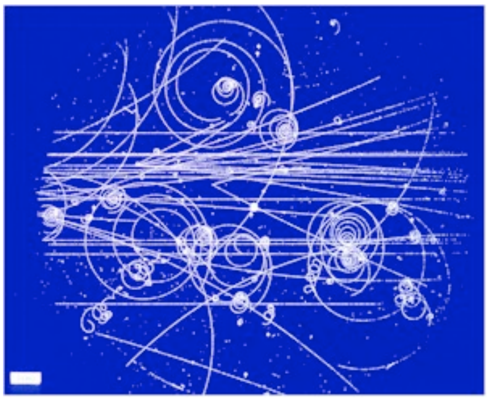
Subatomic Physics:

Particle Physics

Lecture 2: Practical Particle Physics

3rd November 2009

What can we measure at the LHC, and how do we interpret that in term of fundamental particles and interactions?



* Particle Properties and Quantum Numbers
* Natural Units
* Relativistic kinematics
* Decay Properties
* Scattering Properties

Introduction: Measurements in Particle Physics

We saw last time that particles generally do two things:

- **Scatter** *e.g.* collisions of protons on protons at LHC
  collision of cosmic rays in atmosphere
- **Decay** *e.g.* decay of particles produced at LHC
  decay of cosmic ray muons

➡ Measurements of scatterings and decays are used to infer properties of the particles and interactions.

➡ Can also measure directly some properties of the longer-lived particles.

Static Particle Properties

- Mass, m, Charge, Q
- Magnetic moment
- Spin and Parity, J^{π}

Particle Scattering

- Total cross section, σ.
- Differential cross section, $d\sigma/d\Omega$
- Collision Luminosity, $\mathcal{L}$
- Event Rate, N

Particle Decays

- Particle lifetime, τ, and decay width, Γ
- Allowed and forbidden decays → conservation laws

Tools of Quantum Mechanics

from introductory Subatomic slides

- Each particle can be described as a **quantum state**, $|\phi\rangle$
- The electromagnetic, weak and strong forces acting on these states can be represented by (three different) **quantum operators**, $\hat{O}$
- Rates of interactions such as **particle lifetimes** and **scattering cross sections** are given by **Fermi's Golden rule:**
- Transition between an initial state $|\phi_i\rangle$ and a final state $|\phi_f\rangle$ are related to the matrix element $\mathcal{M} = V_{fi} = \langle \phi_f \,|\, \hat{O} \,|\, \phi_i \rangle$:

$$\text{Transition probability}, T = \frac{2\pi}{\hbar}|\mathcal{M}|^2 \rho$$

- T is related to the cross section of scattering, σ. e.g. $\sigma(e^+e^- \to \mu^+\mu^-) \propto |\mathcal{M}\,(e^+e^- \to \mu^+\mu^-)|^2$.
- T is related to the inverse lifetime of a decay, τ. e.g. $\tau(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \propto 1/|\mathcal{M}\,(\mu^- \to e^- \bar{\nu}_e \nu_\mu)|^2$.

We will see how to calculate $\mathcal{M}$ in future lectures

Quark and Lepton Flavour Quantum Numbers

- **Lepton number**, L: Total number of leptons – total number of anti-leptons
  - ➡ Electron number, L_e
  - ➡ Muon number, L_μ
  - ➡ Tau number, L_τ
  - $L = L_e + L_\mu + L_\tau$

$$L_e = N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e)$$
$$L_\mu = N(\mu^-) - N(\mu^+) + N(\nu_\mu) - N(\bar{\nu}_\mu)$$
$$L_\tau = N(\tau^-) - N(\tau^+) + N(\nu_\tau) - N(\bar{\nu}_\tau)$$

- **Quark Number**, N_q: Total number of quarks – total number of anti-quarks
  - ➡ Up quark number, N_u: *e.g.* $N_u = N(u) - N(\bar{u})$
  - ➡ Down quark number, N_d
  - ➡ Strange quark number, N_s
  - ➡ Charm quark number, N_c
  - ➡ Bottom quark number, N_b
  - ➡ Top quark number, N_t
  - $N_q = N_u + N_d + N_s + N_c + N_b + N_t$

- Lepton number, electron number, muon number and tau number (L, L_e, L_μ, L_τ) are conserved in **all** interactions: strong, electromagnetic and weak.
- Quark number (N_q) is also conserved in all interactions.
- Individual quark numbers ($N_u, N_d, N_s, N_c, N_b, N_t$) are conserved in strong and electro-magnetic interactions. They are not (necessarily) conserved in weak interactions.

Conservation Laws

Emmy Noether
1882-1935

Noether's Theorem: Every symmetry of nature has a conservation law associated with it, and vice-versa.

- **Energy, Momentum and Angular Momentum**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: translations in time and space; rotations in space
- **Charge conservation Q**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: gauge transformation - underlying symmetry in QM description of electromagnetism
- **Lepton Number L_e, L_μ, L_τ and total quark number N_q**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: mystery!
- **Quark Flavour Number, $N_u, N_d, N_s, N_c, N_b, N_t$, Parity, π**
  - ➡ Conserved in strong and electromagnetic interactions
  - ➡ Violated in weak interactions
  - ➡ Symmetry: unknown!

Natural Units I

kg m s
SI units: [M] [L] [T]

GeV c $\hbar$
Natural units: [Energy] [velocity] [action]

- For everyday physics SI units are a natural choice: $\mathbf{M_{(SH\ student)} \sim 75kg}$.
- Not so good for particle physics: $M_{\mathrm{proton}} \sim 10^{-27}\mathrm{kg}$

- PP chooses a different basis - **Natural Units**, based on:
  - ★ quantum mechanics ($\hbar$);
  - ★ relativity (c);
  - ★ appropriate unit of energy $\mathbf{1\ GeV = 10^9\ eV = 1.60 \times 10^{-10}\ J}$

Energy	GeV	Time	$(\mathrm{GeV}/\hbar)^{-1}$
Momentum	GeV/c	Length	$(\mathrm{GeV}/\hbar c)^{-1}$
Mass	GeV/c^2	Area	$(\mathrm{GeV}/\hbar c)^{-2}$

Natural Units II

Simplify even further by ★ measuring speeds relative to c

★ measuring action/angular momentum/spin relative to $\hbar$

Equivalent to setting $c = \hbar = 1$! All quantities are expressed in powers of GeV

Energy	GeV	Time	GeV^{-1}
Momentum	GeV	Length	GeV^{-1}
Mass	GeV	Area	GeV^{-2}

Convert to SI units by reintroducing missing factors of $\hbar$ and c

- Example: Area = 1 GeV^{-2}

$$[L]^2 = [E]^{-2}[\hbar]^n[c]^m = [E]^{-2}\,[E]^n[T]^n\,[L]^m[T]^{-m} \qquad n = 2, m = 2$$

Area (in SI units) = $1\ \mathrm{GeV}^{-2} \times \hbar^2 c^2 = 3.89 \times 10^{-32}\ \mathrm{m}^2 = 0.389\ \mathrm{mb}$

Other common units:

- Masses and energies measured in MeV
- cross section measured in barn, $\mathbf{b} \equiv 10^{-28}\ \mathrm{m}^2$
- lengths in $\mathbf{fm} = 10^{-15}\ \mathrm{m}$
- electric charge in units of e

Two useful relations: $\hbar c = 197\ \mathrm{MeV\ fm}$ $\quad \hbar = 6.582 \times 10^{-22}\ \mathrm{MeV\ s}$

Review: Relativistic Dynamics

Please review JH D&R §14

- Two important quantities for Lorentz transformations:

$$\beta = v/c \qquad \gamma(v) = 1/\sqrt{1-\beta^2}$$

- Four-momentum of a particle: $\underline{\underline{p}} = (E/c,\, p_x,\, p_y,\, p_z) = (E/c, \vec{p})$
- Energy of a particle $\quad E^2 = \vec{p}^{\,2}c^2 + m^2c^4 \qquad E = \gamma mc^2$
- Scalar product of 4-momentum: $(\underline{\underline{p}})^2 = (E/c)^2 - \vec{p}^{\,2} = m^2c^2$
- Particles with m=0 travel at the speed of light

Natural Units

Lorentz boosts: $\gamma = E/m \qquad \gamma\beta = |\vec{p}|/m \qquad \beta = |\vec{p}|/E$

Four momentum: $\underline{\underline{p}} = (E,\, p_x,\, p_y,\, p_z) = (E, \vec{p})$

Invariant mass $\quad (\underline{\underline{p}})^2 = E^2 - \vec{p}^{\,2} = m^2$

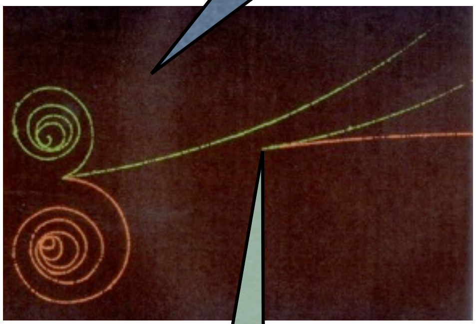
Particle Decay Introduction

Most fundamental particles and hadrons decay.

We can measure:

- **particle lifetime**, τ: average time taken particle to decay
- **decay width,** $\Gamma \equiv \hbar/\tau$, measured in units of energy
- **decay length,** L: average distance travelled before decaying
- **branching ratio, decay modes**: particles in final state, how often a given final state occurs
- **decay kinematics** $\sum \underline{p}_{\text{initial}} = \sum \underline{p}_{\text{final}}$

"Tracks" left by charged particles



Here: one invisible (neutral) particle has decayed into two particles

Look at the following examples:

- decay of π^+ meson into muon: $\pi^+ \to \mu^+ \nu_\mu$
- decay of K_S mesons into two pions: $K_S \to \pi^+\pi^-, K_S \to \pi^0\pi^0$

Particle Lifetime

- **Particle lifetime**, τ, the time taken for the sample to reduce to $1/e$ of original sample.
- Different forces have different typical lifetimes.
- Also define **total decay width**, $\Gamma \equiv \hbar/\tau$.

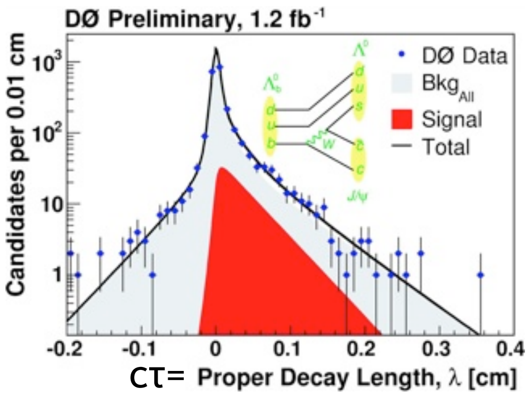
$$\frac{dN}{dt} = -\frac{N}{\tau} = -\frac{\Gamma}{\hbar}N$$

$$\Rightarrow \quad N(t) = N_0 \exp(-t/\tau) = N_0 \exp(-\Gamma t/\hbar)$$

Force	Typical Lifetime
Strong	10^{-20} - 10^{-23} s
Electromag	10^{-20} - 10^{-16} s
Weak	10^{-13} - 10^{3} s

- In its own rest frame particle travels $v\tau = \beta c\tau$ before decaying.
- In the lab, time is dilated by γ.
- If τ is large enough, energetic particles travel a measurable distance $L = \gamma\beta c\tau$ in lab.

example of a recent measurement of a particle lifetime



$\tau(\Lambda_b) = (1.30 \pm 0.15) \times 10^{-12}$ s

Decay Modes

Particles can have more than one **decay mode**. e.g. The K_S meson decays 99.9% of the time in one of two ways:

$$K_S \rightarrow \pi^+\pi^-, K_S \rightarrow \pi^0\pi^0$$

- Each decay mode has its own matrix element, $\mathcal{M}$. Fermi's Golden Rule gives us the **partial decay width** for each decay mode:

$$\Gamma(K_S \rightarrow \pi^+\pi^-) \propto |\mathcal{M}(K_S \rightarrow \pi^+\pi^-)|^2 \qquad \Gamma(K_S \rightarrow \pi^0\pi^0) \propto |\mathcal{M}(K_S \rightarrow \pi^0\pi^0)|^2$$

- The **total decay width** is equal to the sum of the decay widths for all the allowed decays.

$$\Gamma(K_S) = \Gamma(K_S \rightarrow \pi^0\pi^0) + \Gamma(K_S \rightarrow \pi^+\pi^-)$$

- The **branching ratio**, **BR**, is the fraction of time a particle decays to a particular final state:

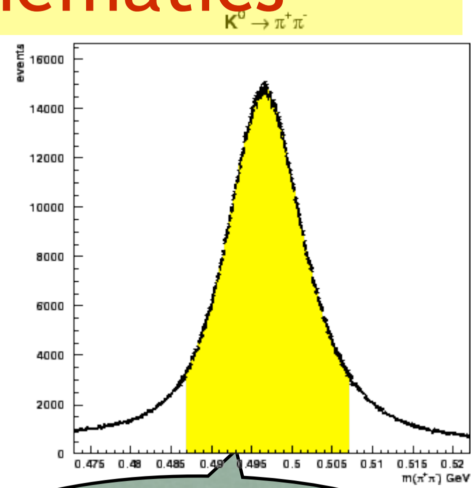
$$BR(K_S \rightarrow \pi^+\pi^-) = \frac{\Gamma(K_S \rightarrow \pi^+\pi^-)}{\Gamma(K_S)} \qquad BR(K_S \rightarrow \pi^0\pi^0) = \frac{\Gamma(K_S \rightarrow \pi^0\pi^0)}{\Gamma(K_S)}$$

Particle Decay Kinematics

- Most particles decay.
  e.g. K_S meson can decay as:
  $$K_S \rightarrow \pi^+\pi^-$$
- Reconstruct the mass of a particle from the momenta of the decay products:

$$\sum \underline{\underline{p}}_{\text{initial}} = \sum \underline{\underline{p}}_{\text{final}}$$

$$\underline{\underline{p}}(K_S) = \underline{\underline{p}}(\pi^+) + \underline{\underline{p}}(\pi^-)$$



Invariant mass from $K_S \rightarrow \pi^+\pi^-$ decay. Not perfect due to limited accuracy of measurements

squaring each side ...

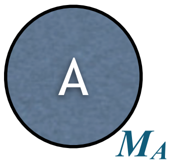
$$\begin{aligned}
(M(K_S))^2 &= \left(\underline{\underline{p}}(\pi^+) + \underline{\underline{p}}(\pi^-)\right)^2 \\
&= \left(\underline{\underline{p}}(\pi^+)\right)^2 + \left(\underline{\underline{p}}(\pi^-)\right)^2 + 2\underline{\underline{p}}(\pi^+) \cdot \underline{\underline{p}}(\pi^-) \\
&= \left(m(\pi^+)\right)^2 + \left(m(\pi^-)\right)^2 + 2\,E(\pi^+)E(\pi^-) - 2\,\vec{p}(\pi^+) \cdot \vec{p}(\pi^-)
\end{aligned}$$

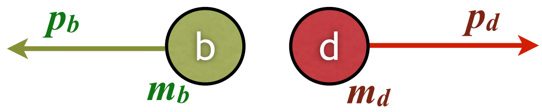
- ***M(K)*** is reconstructed **invariant mass** of K_S

Decay Kinematics II

Decay of an unstable particle at rest:

A → *b d*





$\underline{\underline{p}}_A = (M_A, 0)$

Before

$\underline{\underline{p}}_b = (E_b, \vec{p}_b)$ $\underline{\underline{p}}_d = (E_d, \vec{p}_d)$

After

- Four-momentum conservation:

$$\underline{\underline{p}}_A = \underline{\underline{p}}_b + \underline{\underline{p}}_d \quad \Rightarrow \quad \underline{\underline{p}}_b = \underline{\underline{p}}_A - \underline{\underline{p}}_d \quad \Rightarrow \vec{p}_b = -\vec{p}_d$$

$$(\underline{\underline{p}}_b)^2 = (\underline{\underline{p}}_A)^2 + (\underline{\underline{p}}_d)^2 - 2\underline{\underline{p}}_A \cdot \underline{\underline{p}}_d = M_A^2 + m_d^2 - 2M_A\, E_d = m_b^2$$

$$\Rightarrow E_d = \frac{M_A^2 + m_d^2 - m_b^2}{2M_A}$$

For moving particles, apply appropriate Lorentz boost.

- Example: $\pi^+ \rightarrow \mu^+ \nu_\mu$ work in rest frame of pion. $m_\nu \approx 0$

$$E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi} = 109.8 \text{ MeV} \quad |\vec{p}_\nu| = |\vec{p}_\mu| = \sqrt{E_\mu^2 - m_\mu^2} = 29.8 \text{ MeV}/c$$

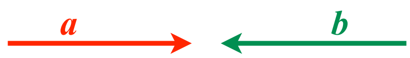
Scattering

Consider a collision between two particles: ***a*** and ***b***.

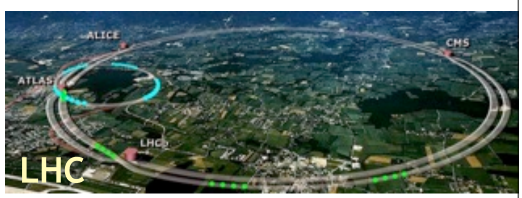
- Elastic collision: ***a*** and ***b*** scatter off each other ***a b*** → ***a b***. *e.g.* $e^+e^- \rightarrow e^+e^-$
- Inelastic collision: new particles are created ***a b*** → ***c d …*** *e.g.* $e^+e^- \rightarrow \mu^+\mu^-$

Two main types of particle physics experiment:

- **Collider experiments** beams of ***a*** and ***b*** are brought into collision. Often $\vec{p}_a = -\vec{p}_b$

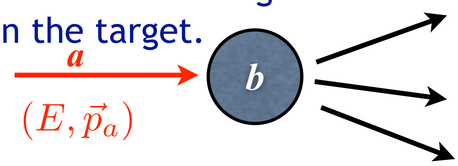


$(E_a, \vec{p}_a)$ $(E_b, \vec{p}_b)$



p-p collider

- **Fixed Target Experiments**: A beam of ***a*** are accelerated into a target at rest. ***a*** scatters off ***b*** in the target.



$(E, \vec{p}_a)$

Measuring Scattering

- **The cross section**, σ, measures the how often a scattering process occurs.
- σ is characteristic of a given process (force) from Fermi's Golden Rule $\sigma \propto |\mathcal{M}|^2$ and energy of the colliding particles.
- σ measured in units of area. Normally use barn, $1\ \mathrm{b} = 10^{-28}\mathrm{m}^2$.
- **Luminosity**, $\mathcal{L}$, is characteristic of the beam. Measured in units of inverse area per unit time.
- **Integrated luminosity**, $\int \mathcal{L}dt$ is luminosity delivered over a given period. Measured in units of inverse area, usually b^{-1}.
- What, and how often, particles are created in the final state.

Force	Typical Cross Sections
Strong	10 mb
Electromag	10^{-2} mb
Weak	10^{-13} mb

- Event rate:

$$w = \mathcal{L}\sigma$$

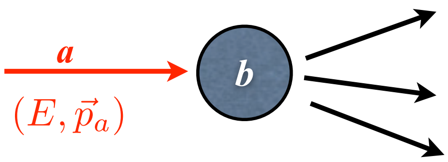
- Total number of events:

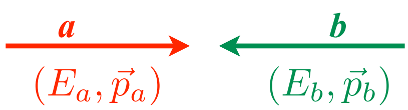
$$N = \sigma \int \mathcal{L}dt$$

Cross Section

see JH D&R §2

- We have a beam of particles incident on a target (or another beam).





- Flux of incident beam, f: number of particles per unit area per unit time.
- Beam illuminates N particles in target.
- We measure the **scattering rate**, $dw/d\Omega$, number of particles scattered in given direction, per unit time per unit solid angle, $d\Omega$.

$$\frac{dw}{d\Omega} = fN\boxed{\frac{d\sigma}{d\Omega}}$$

$d\sigma/d\Omega$ is differential cross section

- Integrate over the solid angle, rate of scattering: $w = fN\sigma$
- Define **luminosity**, $\mathcal{L} = fN$
- **Scattering rate** $w = \mathcal{L}\sigma$

Collision Centre of Mass Energy, $\sqrt{s}$

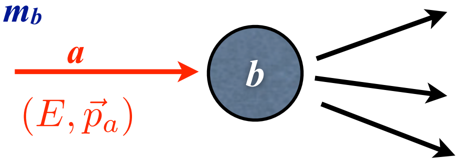
- For a collision define Lorentz-invariant quantity, s: square of sum of four-momentum of incident particles:

$$\begin{aligned} s &= (\underline{\underline{p}}_a + \underline{\underline{p}}_b)\cdot(\underline{\underline{p}}_a + \underline{\underline{p}}_b) \\ &= (\underline{\underline{p}}_a)^2 + (\underline{\underline{p}}_b)^2 + 2\,\underline{\underline{p}}_a\cdot\underline{\underline{p}}_b \\ &= m_a^2 + m_b^2 + 2(E_aE_b - |\vec{p}_a||\vec{p}_b|\cos\theta) \end{aligned}$$

- $\sqrt{s}=E_{CM}$ is the energy in centre of momentum frame, energy available to crate new particles!

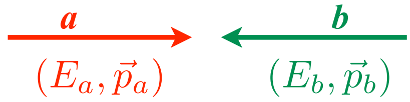
- **Fixed Target Collision,** b is at rest. $E_a \gg m_a, m_b$

$$s = m_a^2 + m_b^2 + 2E_am_b \approx 2E_am_b$$
$$E_{\rm CM} = \sqrt{2E_am_b}$$



- **Collider Experiment,** with $E = E_a = E_b \gg m_a, m_b, \theta = \pi$

$$s = 4E^2 \qquad E_{\rm CM} = 2E$$

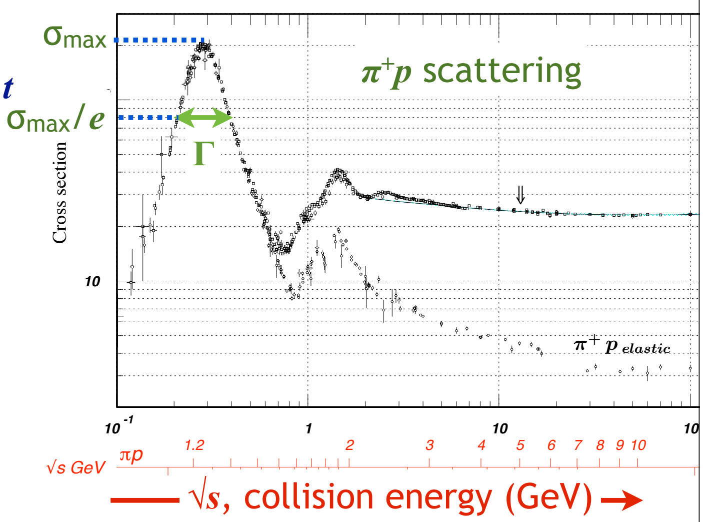


Particle Width

- Lifetimes can be very short, *e.g.* lifetime of Δ^{++} baryon (uuu) is 5×10^{-24} s.
- Heisenburg Uncertainty Principle:

$$\Delta E\Delta t \approx \hbar$$

- Very short lifetime gives small Δt
- $\Delta E \approx \hbar/\Delta t$ is significant $\Rightarrow$ measurable width
- Mass of short lived particles (*e.g.* Δ^{++}) is **not fixed.**
- Mass has most-likely value, but can take on a range of values.



Collision Examples

- The previous collider at CERN collided electrons and positrons head-on with $E(e^-) = E(e^+) = 45.1$ GeV.

$$\begin{aligned} s &= \left(\underline{\underline{p}}(e^+) + \underline{\underline{p}}(e^-)\right)^2 \\ &= 2m_e^2 + 2(E^2 - |\vec{p}_{e+}||\vec{p}_{e-}|\cos\theta) \\ &\approx 2(E^2 + |\vec{p}_{e+}||\vec{p}_{e-}|) \\ &\approx 4E^2 \\ E_{\rm CM} &= 2E \;=\; 91.2\ \text{GeV} \end{aligned}$$

- $\sigma(e^+e^- \to \mu^+\mu^-) = 1.9$ nb at $E_{\rm CM} = 91.2$ GeV
- Total integrated luminosity $\int \mathcal{L}\, dt = 400\ \text{pb}^{-1}$
- $N_{\rm evts}(e^+e^- \to \mu^+\mu^-) = 400{,}000 \times 1.9 = 760{,}000$

- To make hadrons, a **45.1 GeV** electron beam was fired into a Beryllium target.
- Electrons collide with protons and neutrons in Beryllium.

$$\begin{aligned} s &= \left(\underline{\underline{p}}(e^-) + \underline{\underline{p}}(p)\right)^2 \\ &= m_e^2 + m_p^2 \\ &\quad +2(E_e E_p - |\vec{p}_e||\vec{p}_p|\cos\theta) \\ &\approx 2(E_e m_p) \end{aligned}$$

$$\begin{aligned} E_{\rm CM} &= \sqrt{2E_e m_p} \\ &= \sqrt{2 \times 45.1 \times 1} \\ &= 9.5\ \text{GeV} \end{aligned}$$

- In fixed target electron energy is wasted providing momentum to the CM system rather than to make new particles.

Lecture 2 Summary

Natural Units: set $\hbar = c = 1$

- Measure energies in GeV
- Every quantity is measured as a power of energy

Particle Decay

- **Lifetime**, τ, time taken for sample to decrease by $1/e$.
- **Partial width** of decay mode, $\Gamma(A \to x) \propto |\mathcal{M}(A \to x)|^2$
- **Total width** is sum of all possible decay widths, $\Gamma = \hbar/\tau$
- **Branching ratio**, proportion decays to given final state, $BR(A \to x) = \Gamma(A \to x)/\Gamma$

Conserved quantum numbers tell us about the underlying symmetries

Particle Scattering

- **Cross section**, σ, probability for decay to happen. Measured in b = 10^{-28} m^2.
- **Luminosity**, $\mathcal{L}$ is a property of beam (s)
- **Integrated luminosity**, $\int \mathcal{L} dt$.
- Number of events: $N = \sigma \int \mathcal{L} dt$
- Two types of scattering experiment: collider and fixed target.

Relativistic Kinematics

$$\underline{\underline{p}} = (E, p_x, p_y, p_z) = (E, \vec{p})$$

$$\left(\underline{\underline{p}}\right)^2 = E^2 - \vec{p}^{\,2} = m^2$$

Centre of Mass energy

$$s = (\underline{\underline{p}}_a + \underline{\underline{p}}_b)^2 \qquad E_{\rm CM} = \sqrt{s}$$