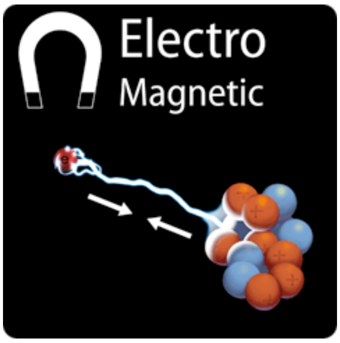
Subatomic Physics:

Particle Physics

Lecture 3: Quantum Electro-Dynamics & Feynman Diagrams

6th November 2009



- Antimatter
- Feynman Diagram and Feynman Rules
- Quantum description of electromagnetism
- Virtual Particles
- Yukawa Potential for QED

Schrödinger and Klein Gordon

- Quantum mechanics describes momentum and energy in terms of operators:

$$\hat{E} = i\hbar\frac{\partial}{\partial t} \qquad \hat{\vec{p}} = -i\hbar\vec{\nabla}$$

- $E=p^2/2m$ gives time-dependent ***Schödinger***: $-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)$
- The solution with a definite energy, E: $\Psi_E(\vec{r},t) = \psi_E(\vec{r})\exp\{-iEt/\hbar\}$
- However for particles near the speed of light $E^2=p^2c^2+m^2c^4$ ⇒

$$-\hbar^2\frac{\partial^2}{\partial t^2}\Psi(\vec{r},t) = -\hbar^2c^2\nabla^2\Psi(\vec{r},t) + m^2c^4\Psi(\vec{r},t)$$

- Solutions with a fixed energy, $E_p=+(p^2c^2+m^2c^4)^{½}$, and three-momentum, p:

$$\Psi(\vec{r},t) = N\exp\{i(\vec{p}\cdot\vec{r} - E_pt)/\hbar\}$$

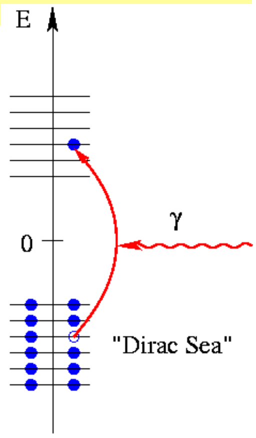
- Also solutions with a negative energy, $E_n=-E_p = -(p^2c^2+m^2c^4)^{½}$, and momentum, $-p$:

$$\Psi^*(\vec{r},t) = N^*\exp\{i(-\vec{p}\cdot\vec{r} + E_pt)/\hbar\}$$

- Negative energy solutions are a direct result of $E^2=p^2c^2+m^2c^4$.
- We interpret these as **anti-particles**

Klein-Gordon equation is non-examinable

Antimatter

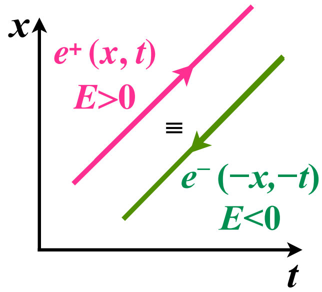


Klein-Gordon equation predicts negative energy solutions.

Dirac Interpretation:

- The vacuum is composed of negative energy levels with $E<0$. Each level is filled with two electrons of opposite spin: the “Dirac Sea”.
- A “hole” in the sea with charge $-e$ and $E<0$, appears as a state with charge $+e$ and $E>0$.
- This idea lead Dirac to predict the **positron**, discovered in 1931.

Feynman-Stueckelberg Interpretation:



- negative energy particles moving backwards in space and time correspond to...
- positive energy antiparticles moving forward in space and time

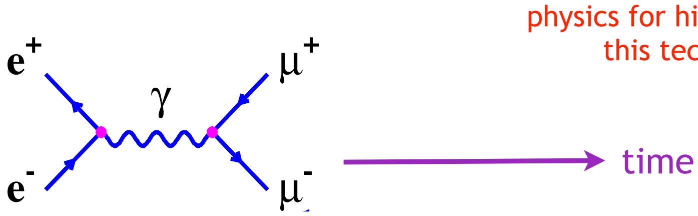
$$\Psi_{e^-}(-\underline{r},-t) \quad \propto \quad \exp -i/\hbar\left\{(-E)(-t)-(-\underline{p})\cdot(-\underline{r})\right\}$$

$$\Psi_{e^+}(+\underline{r},+t) \quad \propto \quad \exp -i/\hbar\left\{(+E)(+t)-(+\underline{p})\cdot(+\underline{r})\right\}$$

Feynman Diagrams

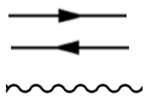
- A Feynman diagram is a pictorial representation of a particular process (decay or scattering) at a **particular order in perturbation theory**.
- Feynman diagrams can be used to **represent** and **calculate** the **matrix elements**, $\mathcal{M}$, for scattering and decays.
- Feynman diagrams are very useful and **powerful** tools. We will use them a lot in this course. We use them a lot in our research!

Richard Feynman receiving the 1967 Noble prize in physics for his invention of this technique.



Conventions

- Time flows from left to right (occasionally upwards)



- Fermions are solid lines with arrows
- Anti-fermion are solid lines with backward pointing arrows.
- Bosons are wavy (or dashed) lines

Use **Feynman Rules** to calculate $\mathcal{M}$ at different orders in perturbation theory.

Quantum Electrodynamics (QED)

QED is the quantum theory of electromagnetic interactions.

Classical electromagnetism:

- Force between charged particle arise from the electric field

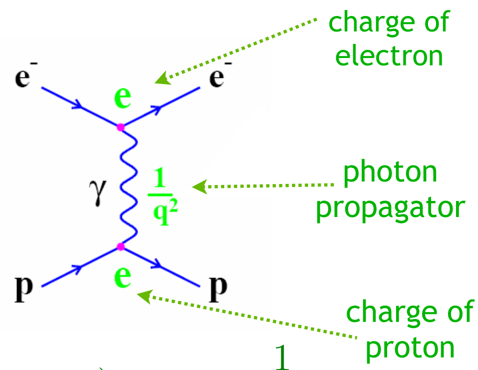
$$\vec{E} = \frac{Q\,\hat{r}}{4\pi\epsilon_0 r^2}$$

- act instantaneously at a distance

Quantum Picture:

- **Force between charged particle described by exchange of photons.**
- Strength of interaction is related to charge of particles interacting.

e.g. electon-proton scattering ***ep→ep*** propagated by the exchange of photons



$$\mathcal{M}(ep \to ep) \propto e \cdot \frac{1}{\underline{\underline{q}}^2 - m_\gamma^2} \cdot e$$

Feynman rules:

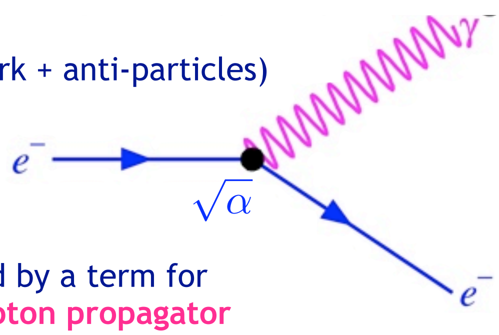
- **Vertex term**: each photon–charged particle interaction gives a factor of fermion charge, ***Q***.
- **Propagator term**: each photon gives a factor of $1/(\underline{\underline{q}}^2 - m_\gamma^2) = 1/\underline{\underline{q}}^2$ where $\underline{\underline{q}}$ is the photon four-momentum.
- Matrix element is proportional to product of **vertex** and **propagator** terms.

Electromagnetic Vertex

Basic electromagnetic process:

- Initial state charged fermion (***e, μ, τ*** or quark + anti-particles)
- Absorption or emission of a photon
- Final state charged fermion

Examples: $e^- \to e^-\gamma$; $e^-\gamma \to e^-$



Mathematically, EM interactions are described by a term for the interaction **vertex** and a term for the **photon propagator**

QED Conservation Laws

- Momentum, energy and charge is conserved at each vertex
- Fermion flavour (***e, μ, τ, u, d ...***) is conserved: *e.g.* $u \to u\gamma$ allowed, $c \to u\gamma$ forbidden

Coupling strength

- Matrix element is proportional to the fermion charge: $\mathcal{M} \propto e$
- Alternatively use the fine structure constant, ***α***

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = \frac{e^2}{4\pi} \approx \frac{1}{137}$$

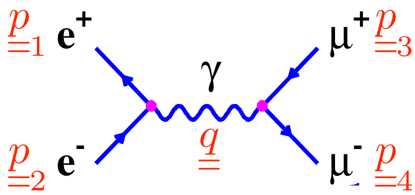
⇒ strength of the coupling at the vertex is $\propto \sqrt{\alpha}$

Virtual Particles

The force between two charged particles is propagated by **virtual photons**.

- A particle is virtual when its four-momentum squared, does not equal its rest mass: $m_X^2 \neq E_X^2 - \vec{p}_X^2$
- Allowed due to **Heisenberg Uncertainty Principle**: can borrow energy to create particle if energy (**$\Delta E = mc^2$**) repaid within time (**Δt**), where **$\Delta E \Delta t \approx \hbar$**

Example: electron-positron scattering creating a muon pair: $e^+e^- \rightarrow \mu^+\mu^-$.



$$\underline{\underline{q}}^2 \neq m_\gamma^2$$

- Four momentum conservation:
$$\underline{\underline{p}}_1 + \underline{\underline{p}}_2 = \underline{\underline{p}}_3 + \underline{\underline{p}}_4$$
- Momentum transferred by the photon is:
$$\underline{\underline{q}} = (\underline{\underline{p}}_1 + \underline{\underline{p}}_2) = (\underline{\underline{p}}_3 + \underline{\underline{p}}_4)$$
- Squaring,
$$\begin{aligned} \underline{\underline{q}}^2 &= (\underline{\underline{p}}_1)^2 + (\underline{\underline{p}}_2)^2 + 2\underline{\underline{p}}_1 \cdot \underline{\underline{p}}_2 \\ &= 2m_e^2 + 2(E_1E_2 - \vec{p}_1 \cdot \vec{p}_2) > 0 \end{aligned}$$

- In QED interactions mass of photon propagator is non-zero.
- Only intermediate photons may be virtual. Final state ones must be real!

Perturbation Theory

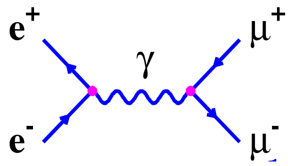
- QED is formulated from time dependent **perturbation theory**.
- Perturbation series: break up the problem into a piece we can solve exactly plus a small correction.

- *e.g.* for $e^+e^- \rightarrow \mu^+\mu^-$ scattering.
  - Many more diagrams have to be considered for a accurate prediction of **σ**($e^+e^- \rightarrow \mu^+\mu^-$).
- As α is small the lowest order in the expansion dominates, and the series quickly converges!

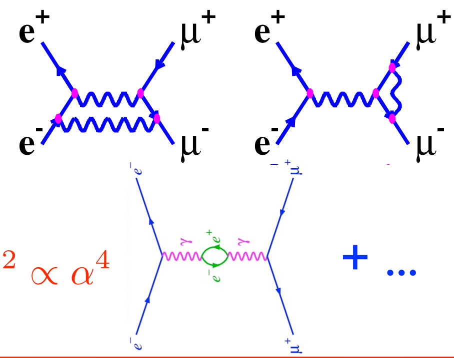
- For most of the course, we will only consider lowest order contributions to processes.

Lowest Order

$|\mathcal{M}|^2 \propto \alpha^2$

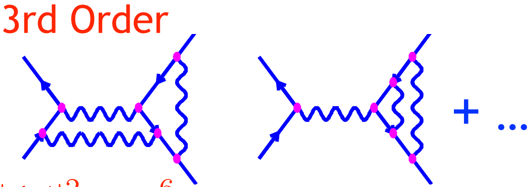


2nd Order



$|\mathcal{M}|^2 \propto \alpha^4$

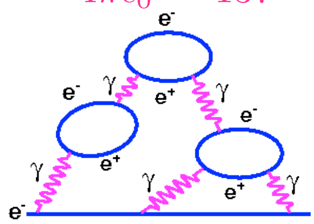
3rd Order



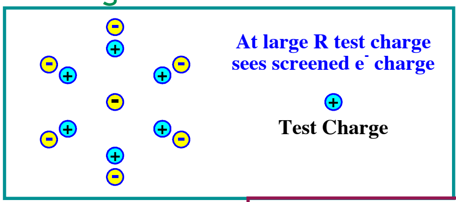
$|\mathcal{M}|^2 \propto \alpha^6$

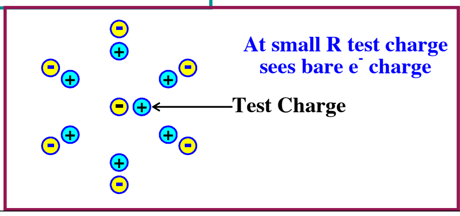
QED Coupling Constant

- Strength of interaction between electron and photon $\propto \alpha = \frac{e^2}{4\pi\epsilon_0} \approx \frac{1}{137}$
- However, α is not really a constant...
- An electron is never alone:
  - it emits virtual photons, these can convert to electron positron pairs...

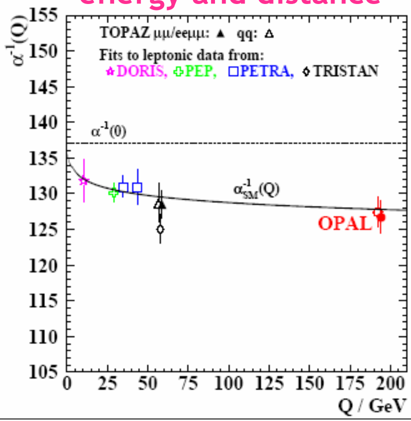


- Any test charge will feel the e^+e^- pairs: true charge of the electron is **screened**.
- At higher energy (shorter distances) the test charge can see the “bare” charge of the electron.





α varies as a function of energy and distance



Yukawa Potential

The quantum and classical descriptions of electromagnetism should agree.

Yukawa developed theory whereby exchange of bosons describes force / potential.

- Klein-Gordon equation:

$$-\hbar^2 \frac{\partial^2}{\partial t^2}\Psi(\vec{r},t) = -\hbar^2 c^2 \nabla^2 \Psi(\vec{r},t) + m^2 c^4 \Psi(\vec{r},t)$$

- Non-time dependent solutions obey:

$$\nabla^2 \Psi(\vec{r}) = \frac{m^2c^2}{\hbar^2}\Psi(\vec{r})$$

- Spherically symmetric solutions of this are:

$$\Psi(|\vec{r}|) = -\frac{g^2}{4\pi r}\exp\left(-\frac{mc}{\hbar}|\vec{r}|\right)$$

- Interpret this as a potential, V, caused by a particle of mass, m.

$$V(r) = -\frac{g^2}{4\pi r}\exp\left(-\frac{r}{R}\right) \quad \text{with } R = \frac{\hbar}{mc}$$

- For electromagnetic force, $m=0, g=e$.

$$V_{\mathrm{EM}}(r) = -\frac{e^2}{4\pi r}$$

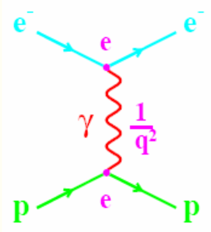
- **Potential felt by a charged particle due to the exchange of a photon.**

QED Scattering Examples

- **Elastic electron-proton scattering: $e^-p \to e^-p$**

$$\mathcal{M} \propto \frac{e^2}{\underline{\underline{q}}^2} = \frac{4\pi\alpha}{\underline{\underline{q}}^2}$$

$$\sigma \propto |\mathcal{M}|^2 \propto \frac{e^4}{\underline{\underline{q}}^4} = \frac{16\pi^2\alpha^2}{\underline{\underline{q}}^4}$$



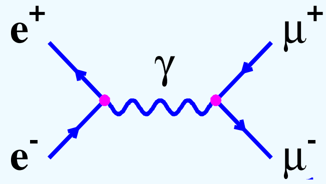
- Momentum transferred to photon from e^-:

$$\begin{aligned} \underline{\underline{q}}^2 &= (\underline{\underline{p}}_f - \underline{\underline{p}}_i)^2 = \underline{\underline{p}}_f^2 + \underline{\underline{p}}_i^2 - 2\underline{\underline{p}}_f \cdot \underline{\underline{p}}_i \\ &= 2m_e^2 - 2(E_f E_i - |\vec{p}_f||\vec{p}_i|\cos\theta) \\ &\approx -4E_f E_i \sin^2(\theta/2) \end{aligned}$$

- **Rutherford scattering: e^- Au $\to e^-$ Au,** can neglect recoil of the gold atoms: $E=E_i=E_f$

$$\sigma \propto \frac{Z^2\pi^2\alpha^2}{E^4 \sin^4(\theta/2)}$$

- **Inelastic $e^-e^+ \to \mu^+\mu^-$**



- Momentum transferred by photon:

$$\underline{\underline{q}}^2 = (\underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-})^2 = s$$

$$\mathcal{M} \propto \frac{e^2}{s^2} = \frac{4\pi\alpha}{s^2}$$

- For this situation need full density of states, ρ, (which we won't do...)

$$\sigma = \frac{16\pi E^2}{3}|\mathcal{M}|^2 = \frac{4\pi\alpha^2}{3s}$$

Summary of Lecture 3

Relativistic quantum mechanics predicts negative energy particles: **antiparticles.** Two interpretations:

- a negative energy particle travelling backwards in time.
- a 'hole' in a vacuum filled with negative energy states.

Quantum Electro Dynamics (QED) is the quantum mechanical description of the electromagnetic force.

Electromagnetic force propagated by virtual photons: $\underline{\underline{q}}^2 \neq m_\gamma^2$

Feynman diagrams can be used to **illustrate** QED processes. Use Feynman rules to calculate the matrix element, $\mathcal{M}$.

All QED interactions are described by a **fermion-fermion-photon vertex:**

- Strength of the vertex is the charge of the fermion, Q_f.
- Fermion flavour and energy-momentum are conserved at vertex.

The **photon propagator** ~$1/\underline{\underline{q}}^2$ where $\underline{\underline{q}}$ is the 4-momentum transferred by the photon.

$\mathcal{M}$ is proportional to product of vertex and propagator terms.