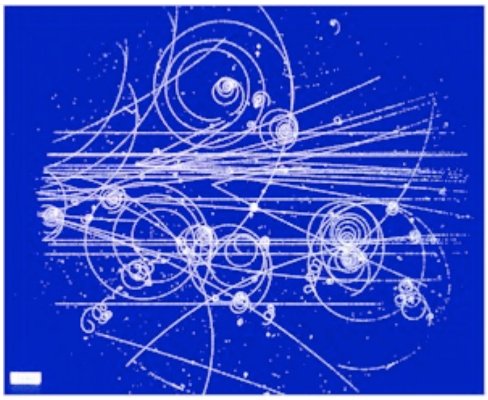
Subatomic Physics:

Particle Physics Lecture 2

Introduction to Measurements in Particle Physics



- Measuring properties of particles and interactions
- Particle quantum numbers and conservation laws
- Review of relativistic dynamics
- Natural Units
- Fermi's Golden Rule
- Measurements in scattering and cross sections

Introduction: Measurements in Particle Physics

Properties of the particles
- ➡ Static properties
- ➡ Dynamic properties

Properties of interactions
- ➡ Decay properties
- ➡ Scattering properties

Static Particle Properties
- Mass, m, Charge, Q
- Magnetic moment
- Spin and Parity, J^{π}
- Colour charge, weak hypercharge
- More quantum numbers...

Dynamic Particle Properties
- Energy, E
- Momentum, $\underline{p}$

Particle Decays
- Particle lifetime, τ, and decay width, Γ
- Allowed and forbidden decays → conservation laws

Particle Scattering
- Total cross section, σ.
- Differential cross section, $d\sigma/d\Omega$
- Collision Luminosity, $\mathcal{L}$
- Event Rate, N

Quark and Lepton Flavour Quantum Numbers

- **Lepton Number**, L: Total number of leptons – total number of anti-leptons
  - ➡ Electron number, L_e
  - ➡ Muon number, L_μ
  - ➡ Tau number, L_τ
  - $L = L_e + L_\mu + L_\tau$

$$L_e = N(e^-) - N(e^+) + N(\nu_e) - N(\bar{\nu}_e)$$
$$L_\mu = N(\mu^-) - N(\mu^+) + N(\nu_\mu) - N(\bar{\nu}_\mu)$$
$$L_\tau = N(\tau^-) - N(\tau^+) + N(\nu_\tau) - N(\bar{\nu}_\tau)$$

- **Quark Number**, N_q: Total number of quarks – total number of anti-quarks
  - ➡ Up quark number, N_u: *e.g.* $N_u = N(u) - N(\bar{u})$
  - ➡ Down quark number, N_d
  - ➡ Strange quark number, N_s
  - ➡ Charm quark number, N_c
  - ➡ Bottom quark number, N_b
  - ➡ Top quark number, N_t
  - $N_q = N_u + N_d + N_s + N_c + N_b + N_t$

- The lepton flavour quantum numbers (L, L_e, L_μ, L_τ) are conserved in **all** Standard Model interactions: strong, electromagnetic and weak.
- Quark number (N_q) is also conserved in all interactions.
- [Individual quark flavours ($N_u, N_d, N_s, N_c, N_b, N_t$) are conserved in strong and electromagnetic interactions. They are not (necessarily) conserved in weak interactions.]

Conservation Laws

Noether's Theorem: Every symmetry of nature has a conservation law associated with it, and vice-versa.

Emmy Noether
1882-1935

- **Energy, Momentum and Angular Momentum**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: translations in time and space; rotations in space
- **Charge conservation e.g. electric charge Q, colour charge**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: gauge transformation - underlying symmetry in QM description of electromagnetism / strong force
- **Lepton Flavour L_e, L_μ, L_τ and total quark number N_q**
  - ➡ Conserved in all interactions
  - ➡ Symmetry: mystery!
- **Quark Flavour $N_u, N_d, N_s, N_c, N_b, N_t$, Parity, π**
  - ➡ Conserved in strong and electromagnetic interactions
  - ➡ Violated in weak interactions
  - ➡ Symmetry: unknown!

Review: Relativistic Dynamics

Please review JH D&R §14

- Two important quantities for Lorentz transformations:

$$\beta = v/c \qquad \gamma(v) = 1/\sqrt{1-\beta^2}$$

- Four-momentum of a particle: $\underline{\underline{p}} = (E/c,\, p_x,\, p_y,\, p_z) = (E/c, \vec{p})$
- Energy of a particle $E^2 = \vec{p}^{\,2}c^2 + m^2c^4 \qquad E = \gamma m c^2$
- Scalar product of 4-momentum: $(\underline{\underline{p}})^2 = (E/c)^2 - \vec{p}^{\,2} = m^2c^2$
- Particles with m=0 travel at the speed of light

Natural Units

Lorentz boosts: $\gamma = E/m \quad \gamma\beta = |\vec{p}|/m \quad \beta = |\vec{p}|/E$

Four momentum: $\underline{\underline{p}} = (E,\, p_x,\, p_y,\, p_z) = (E, \vec{p})$

Invariant mass $(\underline{\underline{p}})^2 = E^2 - \vec{p}^{\,2} = m^2$

Natural Units I

SI units:	[M]	[L]	[T]	Natural units:	[Energy]	[velocity]	[action]
	kg	m	s		GeV	c	$\hbar$

- For everyday physics SI units are a natural choice: **M**(SH student)~**75kg**.
- Not so good for particle physics: $M_{\text{proton}} \sim 10^{-27}$ kg

- PP chooses a different basis - **Natural Units**, based on:
  - ★ quantum mechanics ($\hbar$);
  - ★ relativity (c);
  - ★ appropriate unit of energy **1 GeV = 10^9 eV = 1.60 × 10^{-10} J**

Energy	GeV	Time	$(\text{GeV}/\hbar)^{-1}$
Momentum	GeV/c	Length	$(\text{GeV}/\hbar c)^{-1}$
Mass	GeV/c^2	Area	$(\text{GeV}/\hbar c)^{-2}$

Natural Units II

Simplify even further by ★ measuring speeds relative to c

★ measuring action/angular momentum/spin relative to $\hbar$

Equivalent to setting $c = \hbar = 1$! All quantities are expressed in powers of GeV

Energy	GeV	Time	GeV^{-1}
Momentum	GeV	Length	GeV^{-1}
Mass	GeV	Area	GeV^{-2}

Convert to SI units by reintroducing missing factors of $\hbar$ and c

- Example: Area = 1 GeV^{-2}

$$[L]^2 = [E]^{-2}[\hbar]^n[c]^m = [E]^{-2}\,[E]^n[T]^n\,[L]^m[T]^{-m} \qquad n = 2, m = 2$$

Area (in SI units) = $1\ \mathrm{GeV}^{-2} \times \hbar^2\, c^2 = 3.89 \times 10^{-32}\ \mathrm{m}^2 = 0.389\ \mathrm{mb}$

Other common units:

- Masses and energies measured in MeV
- cross section measured in barn, $\mathbf{b} \equiv 10^{-28}\ \mathrm{m}^2$
- lengths in $\mathbf{fm} = 10^{-15}\ \mathrm{m}$
- electric charge in units of e

Two useful relations: $\hbar c = 197\ \mathrm{MeV\ fm}$ $\qquad \hbar = 6.582 \times 10^{-22}\ \mathrm{MeV\ s}$

Quantum Mechanical Description of Interactions

- Each particle can be described as a **quantum state**, $|\phi\rangle$
- The electromagnetic, weak and strong forces acting on these states can be represented by (three different) **quantum operators**, $\hat{O}$
- Rates of interactions such as **particle lifetimes** and **scattering cross sections** are given by **Fermi's Golden rule:**
- Transition between an initial state $|\phi_i\rangle$ and a final state $|\phi_f\rangle$ are related to the matrix element $\mathcal{M} = V_{fi} = \langle \phi_f \,|\, \hat{O} \,|\, \phi_i \rangle$:

$$\text{Transition probability}, T = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \rho$$

- T is related to the cross section of scattering, σ. e.g. $\sigma(e^+e^- \to \mu^+\mu^-) \propto |\mathcal{M}\,(e^+e^- \to \mu^+\mu^-)|^2$.
- T is related to the inverse lifetime of a decay, τ. e.g. $\tau(\mu^- \to e^-\,\bar{\nu}_e\,\nu_\mu) \propto 1/|\mathcal{M}\,(\mu^- \to e^-\,\bar{\nu}_e\,\nu_\mu)|^2$.

We will see how to calculate $\mathcal{M}$ in future lectures

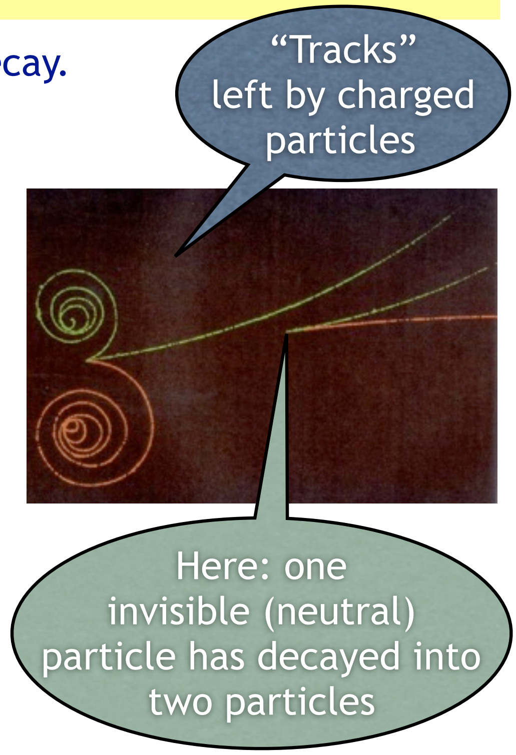
Measuring T (e.g. σ or τ) yields information about the particles ($|\phi\rangle$) and/or the forces ($\hat{O}$)

Particle Decay Introduction

Most fundamental particles and hadrons decay.

We can measure:

- **particle lifetime**, τ: average time taken particle to decay
- **decay width,** $\Gamma \equiv \hbar/\tau$, measured in units of energy. Γ (or τ) is characteristic of force causing decay through Fermi's GR: $\Gamma \propto |\mathcal{M}|^2$
- **decay length,** L: average distance travelled before decaying
- **decay modes, branching ratio**: particles in final state, how often a given final state occurs
- **decay kinematics** $\sum \underline{p}_{\text{initial}} = \sum \underline{p}_{\text{final}}$



Use the following examples (next lecture):

- decay of π^+ meson into muon: $\pi^+ \to \mu^+ \nu_\mu$
- decay of K_S mesons into two pions: $K_S \to \pi^+\pi^-, K_S \to \pi^0\pi^0$

Scattering Introduction

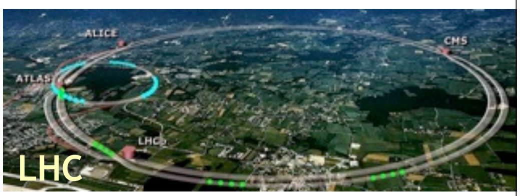
Consider a collision between two particles: a and b.

- Elastic collision: a and b scatter off each other $a\,b \to a\,b$. e.g. $e^+e^- \to e^+e^-$
- Inelastic collision: new particles are created $a\,b \to c\,d$ ... e.g. $e^+e^- \to \mu^+\mu^-$

Two main types of particle physics experiment:

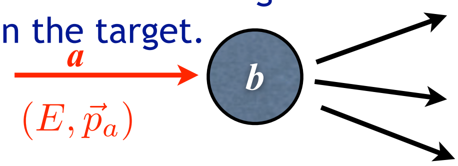
- **Collider experiments** beams of a and b are brought into collision. Often $\vec{p}_a = -\vec{p}_b$

a: $(E_a, \vec{p}_a)$ → ← b: $(E_b, \vec{p}_b)$



p-p collider

- **Fixed Target Experiments**: A beam of a are accelerated into a target at rest. a scatters off b in the target.



Measuring Scattering

- **The cross section**, σ, measures the how often a scattering process occurs.
- σ is characteristic of a given process (force) from Fermi's Golden Rule $\sigma \propto |\mathcal{M}|^2$ and energy of the colliding particles.
- σ measured in units of area. Normally use barn, $\mathbf{1\ b = 10^{-28} m^2}$.
- **Luminosity**, $\mathcal{L}$, is characteristic of the beam. Measured in units of inverse area per unit time.
- **Integrated luminosity**, $\int \mathcal{L} dt$ is luminosity delivered over a given period. Measured in units of inverse area, usually $\mathbf{b^{-1}}$.
- What, and how often, particles are created in the final state.

Force	Typical Cross Sections
Strong	10 mb
Electromag	10^{-2} mb
Weak	10^{-13} mb

- Event rate:

$$w = \mathcal{L}\sigma$$

- Total number of events:

$$N = \sigma \int \mathcal{L} dt$$

Summary of Lecture 2

- **Measurements** in particle physics are used to **understand** and **test** the **underlying theory**.
- Measure properties of particles and their interactions.
- Conserved quantum numbers point to **underlying symmetries**.
- **Rates** of decays and scatterings yield information about the forces and particles involved through the **matrix element.** $\mathcal{M}$.

$$T = \frac{2\pi}{\hbar}|\mathcal{M}|^2\rho$$

Particle Scattering

- Two types of scattering experiment: collider and fixed target.
- **Cross section**, σ, measure of how often a process happens, $\sigma \propto |\mathcal{M}|^2$
- Measured in barn, $\mathbf{b}$: $\mathbf{1\ b = 10^{-28}\ m^2}$.
- Number of events observed is cross section times integrated luminosity ($\int \mathcal{L} dt$) of experiment: $N = \sigma \int \mathcal{L} dt$

Particle Decay

- **Lifetime**, τ, time taken for sample to decrease by $1/e$.
- **Decay width**, $\Gamma = \hbar/\tau \propto |\mathcal{M}|^2$

- Lepton flavour quantum numbers:
  - ➡ L, L_e, L_μ, L_τ
- Quark flavour quantum numbers:
  - ➡ $N_u, N_d, N_s, N_c, N_b, N_t$

Use relativistic kinematics and natural units

$$\underline{p} = (E/c,\, p_x,\, p_y,\, p_z) = (E/c\,, \vec{p}\,)$$

$$\left(\underline{p}\right)^2 = \frac{E^2}{c^2} - \vec{p}^{\,2} = m^2c^2$$