Subatomic Physics:

Particle Physics Handout 8

The Weak Force

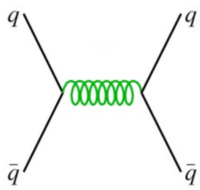
- Weak interactions
- W and Z interactions at low energy
- Fermi theory
- Electroweak theory
- W and Z bosons at high energy

QCD Summary

QCD: Quantum Chromodynamics is the quantum description of the strong force.

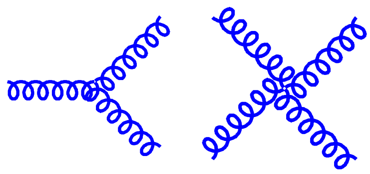
Only quarks feel the strong force.

Gluons are the propagator of the strong force



Quarks and gluons carry **colour charge.**

Gluons self-interact:



- Electromagnetic coupling constant α **decreases** as a charged particles get further apart.
- Strong coupling constant α_S **increases** as further apart quarks become.

Hadrons can be described as consisting of **partons**: quarks and gluons, which interact independently

Colour Confinement
energy required to separate quarks $\rightarrow \infty$
quarks are confined to hadrons

Quarks and gluons produced in collisions hadronise: hadrons are produced.
The decay products of the hadrons appear in the detector as **jets.**

Introduction to the Weak Force

The weak force is responsible for some of the most important phenomena:

- Decays of the muon and tau leptons
- Neutrino interactions
- Decays of the lightest mesons and baryons
- Radioactivity, nuclear fission and fusion

Characteristics of Weak Processes:

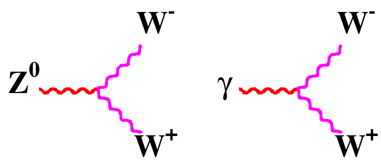
- Long lifetimes 10^{-13} - 10^{3} s
- Small cross sections 10^{-13} mb

Boson	$W^{\pm}$	Z^0
Mass GeV/c^2	80.4	91.2
charge, e	±1	0
spin	$1\hbar$	$1\hbar$

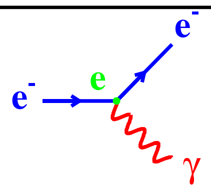
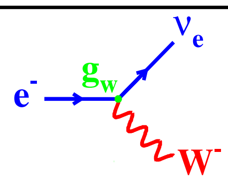
Weak Force is propagated by massive W^+, W^- and Z^0 bosons

- The interactions of $W^{\pm}$ and Z^0 are different (related by symmetry of the weak interaction)

- $W^{\pm}$ and Z^0 can interact with each other
- $W^{\pm}$ and γ can interact (as $W^{\pm}$ bosons are charged)



Weak Vertices

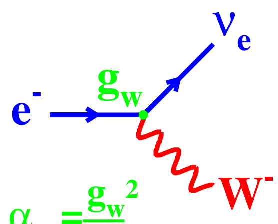
QED	W-boson
mediated by the exchange of virtual photons	mediated by the exchange of W boson
acts on all charged particles	acts on all quark and leptons
coupling strength $\propto e \propto \sqrt{\alpha}$	coupling strength $\propto g_W \propto \sqrt{\alpha_W}$
propagator term: $1/(q^2-m_\gamma^2) = 1/q^2$	propagator term: $1/(q^2-m_W^2)$
For many processes: $\mathcal{M} \propto e^2/q^2$	For many processes: $\mathcal{M} \propto g_W^2/(q^2-m_W^2)$
	

Recall: matrix element, $\mathcal{M}$, is the amplitude of a process.
Scattering cross section, $\sigma \propto \mathcal{M}^2$. Decay width, $\Gamma \propto \mathcal{M}^2$

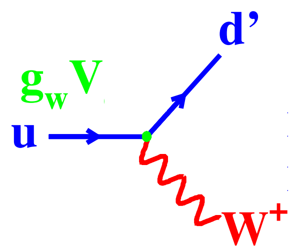
Interactions of the $W^{\pm}$ boson

- Known as "charged current interactions"
- $W^{\pm}$ boson interacts with **all** fermions (all quarks and leptons)
- Charged current **changes the flavour of the fermion**:
  - *e.g.* electron emitting an W-boson can't remain an electron - violates conservation of charge!
    - an electron turns into a electron neutrino
    - an up quark turns into a down quark

      and vice versa!
- Coupling strength at every vertex $\propto g_W$ (more about quarks later)
- Propagator term describing the W-boson $\propto \dfrac{1}{(\underline{\underline{q}}^2 - m_W^2)}$
  - $\underline{\underline{q}}$ is the four-momentum transferred by the W-boson



$$\alpha_w = \frac{g_w^{\ 2}}{4\pi}$$



Allowed Flavour Changes

At a W-boson vertex:

- Lepton numbers: L_e, L_μ and L_τ, is conserved:

  Allowed lepton flavour changes: $e^- \leftrightarrow \nu_e \quad \mu^- \leftrightarrow \nu_\mu \quad \tau^- \leftrightarrow \nu_\tau$

- Total Quark Number, N_q, is conserved
- Individual quark flavour numbers: N_u, N_d, N_s, N_c, N_b, N_t are **not** conserved

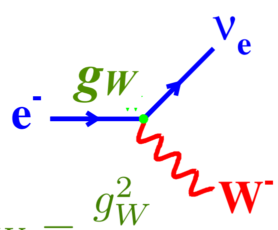
  Allowed quark flavour changes:

  (Q=+2/3 e quark) $\leftrightarrow$ (Q=−1/3 e quark)

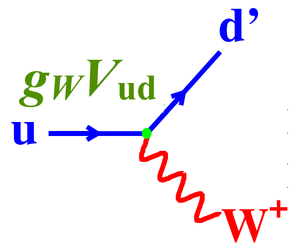
  (d s b) $\leftrightarrow$ (u c t)

- Each of the nine possible quark flavour changes has a different coupling strength: e.g. $g_W V_{ud}$ for **u** to **d** quarks (Vs are terms in **CKM** matrix - more later)
- Main quark flavour changes are within a generation:

  d $\leftrightarrow$ **u**   **s** $\leftrightarrow$ **c**   **b** $\leftrightarrow$ **t**

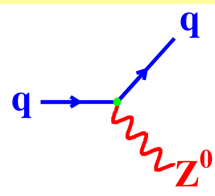


$$\alpha_W = \frac{g_W^2}{4\pi}$$

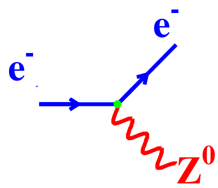


Interactions of the Z^0 boson

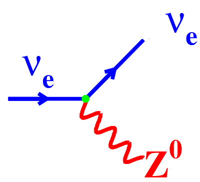
- Known as "neutral current interactions"
- Acts on **all** fermions - (all quarks and leptons)
- Neutral current **conserves** flavour of the fermion
- No allowed fermion flavour changes



- Propagator term $\propto \dfrac{1}{(\underline{q}^2 - m_Z^2)}$



- Coupling strength depends on fermion flavour - we won't consider this in this course



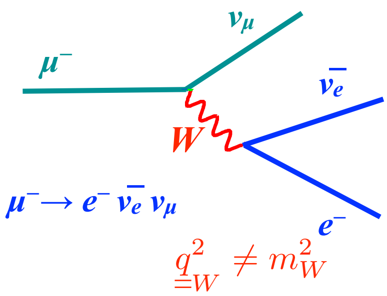
Anywhere a photon could be exchanged a Z^0 boson can be exchanged.
(Almost vice-versa, except Z^0 boson also has neutrino interactions too!)

Electromagnetic and weak neutral current interactions are linked!

W and Z bosons at low energy

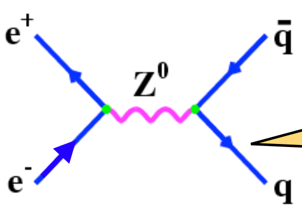
- If the momentum of the W or Z boson, $\underline{q} \ll m_Z, m_W$ the bosons are virtual:

$$\begin{aligned} \underline{q}_Z^2 &= E_Z^2 - \vec{p}_Z \cdot \vec{p}_Z \neq m_Z^2 \\ \underline{q}_W^2 &= E_W^2 - \vec{p}_W \cdot \vec{p}_W \neq m_W^2 \end{aligned}$$



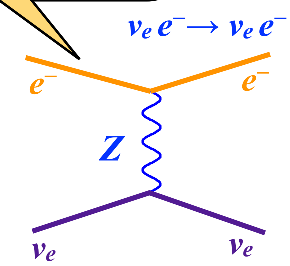
- Virtual W-bosons are responsible for the decays of the leptons, and the lightest hadrons *e.g.* $\mu^- \to e^- \bar{\nu}_e \nu_\mu$
- virtual W-boson interactions can also be described by **Fermi Theory**

- Many interactions of virtual Z-bosons are not clearly evident, as the same interaction can take place due to γ exchange.



$e^+e^- \to q\bar{q}$ scattering can be due to Z or γ

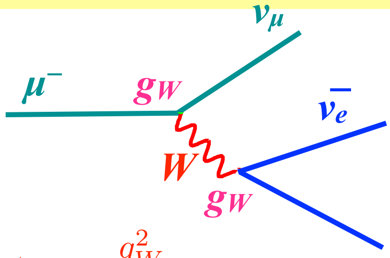
Interactions involving ν must be due to Z (or W)



- At low energies we see the effect of Z boson mainly in scattering involving neutrinos *e.g.* $\nu_e e^- \to \nu_e e^-$
  - as γ cannot couple to the neutral neutrinos

Fermi Theory

W-boson interactions at Low Momentum Transfer



$$\mathcal{M} \propto \frac{g_W^2}{q^2 - m_W^2}$$

For muon decay, and many other weak processes:

$$\mathcal{M} \propto \frac{g_w^2}{(q^2 - m_W^2)}$$

At low momentum transfer $q^2 \ll m_W^2$

$$\mathcal{M} \rightarrow \propto \frac{g_w^2}{m_W^2}$$

Introduce **Fermi coupling constant:**

$$G_F \propto \frac{g_w^2}{m_W^2} \qquad G_F = \frac{\sqrt{2}\, g_w^2}{8\, m_W^2}$$

- Dimension $[E]^{-2}$
- From experimental measurements: $G_F = 1.16637 \times 10^{-5}$ GeV^{-2}

Measurements of $G_F \,\&\, M_W \;\Rightarrow\; g_w = 0.66 \;\Rightarrow\; \alpha_w = \frac{g_w^2}{4\pi} = \frac{1}{29} > \alpha_{\rm EM} = \frac{1}{137}$

- Recall, from problem sheet 2, Q7, range of W boson: $\Delta x \approx \frac{\hbar}{\Delta p} = \frac{\hbar}{m_W c} = 0.002$ fm

Weak interaction not intrinsically weak - appears weak due to large boson masses.

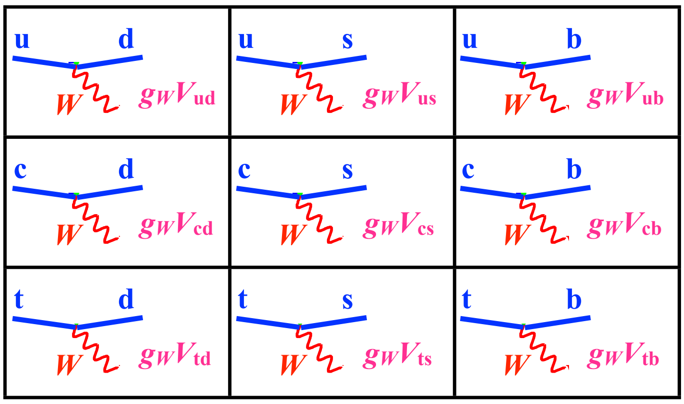
W interactions with quarks and leptons

Lepton universality:

- Coupling of to W-boson to all leptons is equal = g_W
- electrons, muons and taus all interact identically
- interact with the **same bosons** with **same coupling strength**

Quark interactions:

- In general, any vertex W-(Q=+2/3 e quark)-(Q=−1/3 e quark) is valid.
- W-boson coupling to quarks suppressed by a flavour-dependent factor V



Known as the **CKM matrix.** (values from experimental measurements)

V_{ud}=0.974	V_{us}=0.227	V_{ub}=0.004
V_{cd}=0.230	V_{cs}=0.972	V_{cb}=0.042
V_{td}=0.008	V_{ts}=0.041	V_{tb}=0.999

Largest couplings are within a generation:

d ↔ u s ↔ c b ↔ t

Electroweak Theory

We've seen already that wherever a γ boson can be exchanged a Z can also be exchanged:

- The weak and electromagnetic force are linked.
- At short distances (or high energies) the strength of the electromagnetic force and the weak force are comparable. Can be related by a parameter, $\sin\theta_W$

$$e = g_W \sin\theta_W$$

The weak and electromagnetic interactions are manifestations of a underlying force: **the electroweak force.**

- Couplings of the γ, W (and Z) bosons are related: $e = g_W \sin\theta_W$
- Mass of the W and Z bosons are related: $m_Z^2 = m_W^2/\cos^2\theta_W$

Just three fundamental parameters are required to describe:

- couplings of W, Z and γ to quarks and leptons
- masses of the W, Z, γ bosons
- interactions of the W, Z, γ bosons with each other

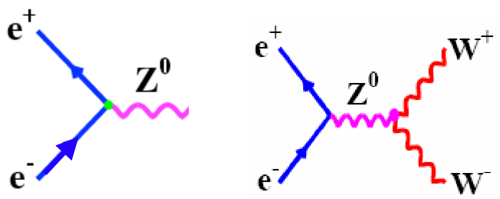
Normally formulated in terms of most accurately measured parameters: e, G_F, m_Z

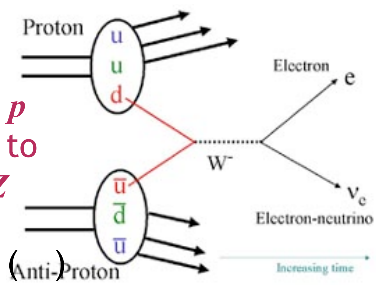
W and Z boson at high energies

Using a collider, we can create high enough energies to make **real** W and Z bosons:

$$\underline{\underline{q}} \sim m_Z, m_W$$

- e.g. LEP collider $e^+e^-\to Z$, $e^+e^-\to W^+W^-$
- e.g. at the LHC: $pp\to Z+X$, $pp\to W+X$





quarks within p (or $\bar{p}$) interact to make W or Z

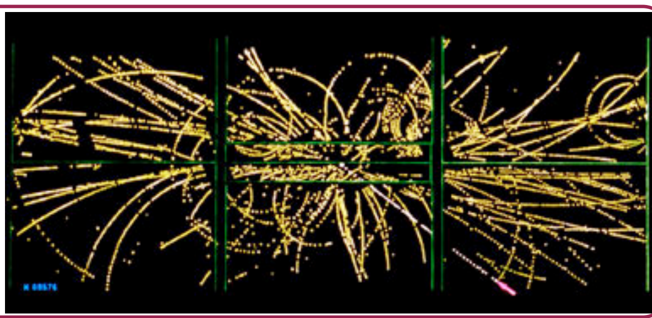
Can study the properties of the W and Z bosons in detail.

- masses, lifetime/width, coupling strengths, decay modes, spin...

Discovery of W and Z bosons at CERN in 1983 at the Sp$\bar{p}$S collider

$p\bar{p}$ collider with $E_p = E_{\bar{p}} = 270$ GeV

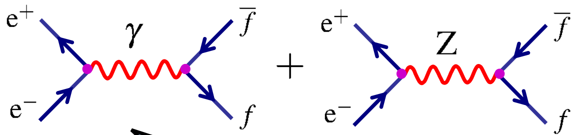
$p\bar{p}\to W^-\to e^-\bar{\nu}_e$ event at UA1 experiment



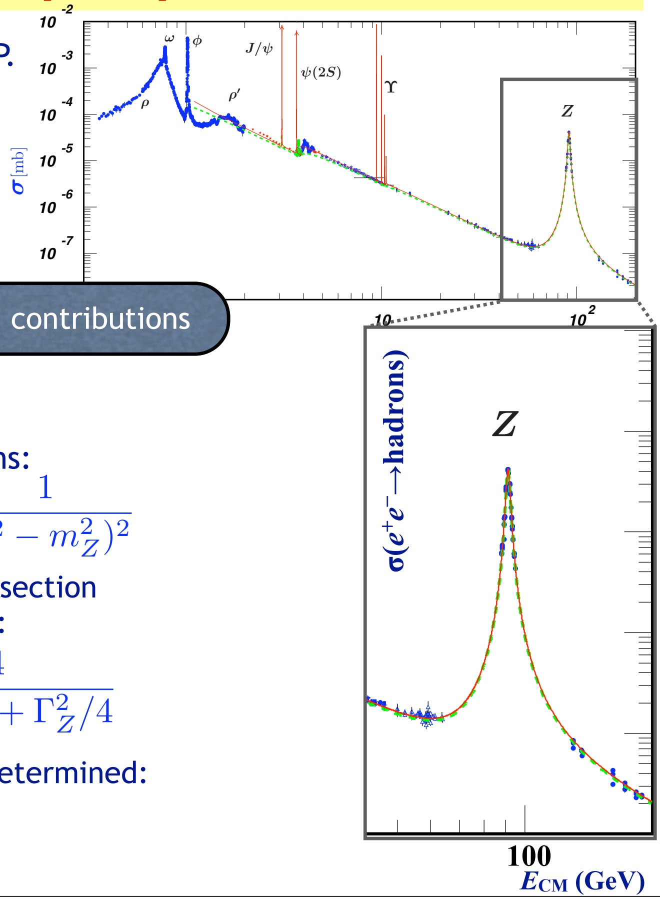
Measuring Z^0 properties

Most studies of the Z boson were made at LEP.

- Main process was $e^+e^- \rightarrow$ **hadrons:**



Mixture of EM (γ) & Weak (Z) contributions



- At low E_{CM} mainly γ interactions.
- At $E_{CM}=\sqrt{s}\sim m_Z$, mainly Z-boson interactions:

$$\sigma(e^+e^- \rightarrow Z \rightarrow f\bar{f}) \propto \frac{1}{(\underline{q}^2 - m_Z^2)^2}$$

- Aside: for an massive boson, shape of cross section also depends on total decay width $\Gamma_Z = \hbar/\tau_Z$:

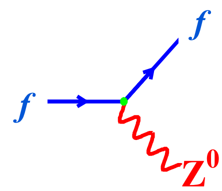
$$\sigma(E=\underline{q}^2) = \sigma_{\max}\frac{\Gamma_Z^2/4}{(\underline{q}^2 - m_Z)^2 + \Gamma_Z^2/4}$$

- Measurements of $\sigma(e^+e^- \rightarrow \text{hadrons})$ at LEP determined:
  - $m_Z = 91.188 \pm 0.002$ GeV/c^2
  - $\Gamma_Z = 2.4953 \pm 0.002$ GeV

Interactions of the Z boson

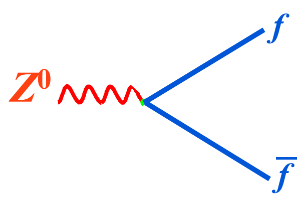
Z boson interacts with **all** quarks and leptons.

No change of quark or lepton flavour at Z boson vertex.



Z boson decay

- Z can decay into any fermion–anti-fermion pair, $f\bar{f}$.
- **except** top quark pair, as $m_Z < 2m_t$



$Z\rightarrow e^+e^-$	$Z\rightarrow \nu_e\bar{\nu}_e$	$Z\rightarrow u\bar{u}$	$Z\rightarrow d\bar{d}$
$Z\rightarrow \mu^+\mu^-$	$Z\rightarrow \nu_\mu\bar{\nu}_\mu$	$Z\rightarrow c\bar{c}$	$Z\rightarrow s\bar{s}$
$Z\rightarrow \tau^+\tau^-$	$Z\rightarrow \nu_\tau\bar{\nu}_\tau$		$Z\rightarrow b\bar{b}$

Lepton Universality ⇒

- $\mathcal{M}$ for e, μ, τ decay is same ⇒ $\Gamma(Z\rightarrow e^+e^-) \approx \Gamma(Z\rightarrow \mu^+\mu^-) \approx \Gamma(Z\rightarrow \tau^+\tau^-)$
- $\mathcal{M}$ for ν_e, ν_μ, ν_τ decay is same ⇒ $\Gamma(Z\rightarrow \nu_e\bar{\nu}_e) = \Gamma(Z\rightarrow \nu_\mu\bar{\nu}_\mu) = \Gamma(Z\rightarrow \nu_\tau\bar{\nu}_\tau)$

Number of Neutrinos

Total width of the Z-boson (Γ_Z) is sum of all partial widths:

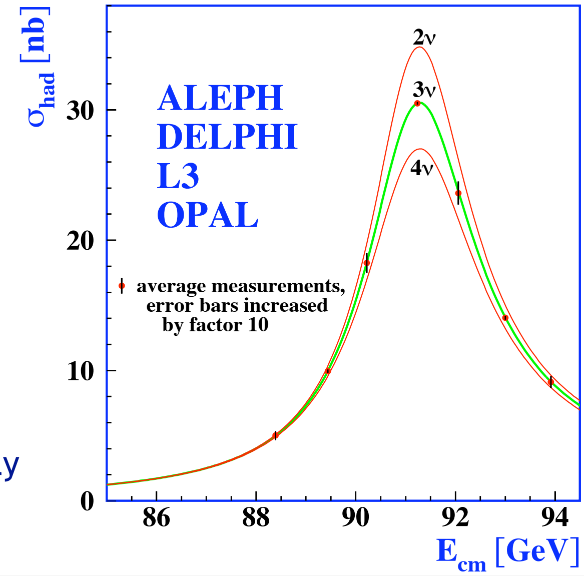
$$\Gamma_Z = \Gamma(Z \to q\bar{q}) + \Gamma(Z \to e^+e^-) + \Gamma(Z \to \mu^+\mu^-) + \Gamma(Z \to \tau^+\tau^-) + N_\nu \Gamma(Z \to \nu\bar{\nu})$$

LEP directly measured:

- partial widths: $\Gamma(Z \to e^+e^-)$, $\Gamma(Z \to \mu^+\mu^-)$, $\Gamma(Z \to \tau^+\tau^-)$, $\Gamma(Z \to \text{hadrons}) = \Gamma(Z \to q\bar{q})$
- total width, Γ_Z

Cannot measure $\Gamma(Z \to \nu\bar{\nu})$ directly as neutrinos leave no signal in the detector.

- Can predict $\Gamma(Z \to \nu\bar{\nu})$ using $\mathcal{M}(Z \to \nu\bar{\nu})$
- Use prediction & measurements to find number of neutrinos contributing to Γ_Z
- **$N_\nu = 2.999 \pm 0.011$**



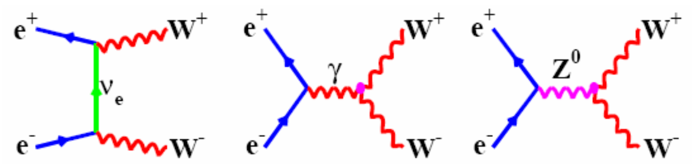
Consistent with exactly three neutrinos!

⇒ Gives us confidence that there are exactly three generations of quarks and leptons

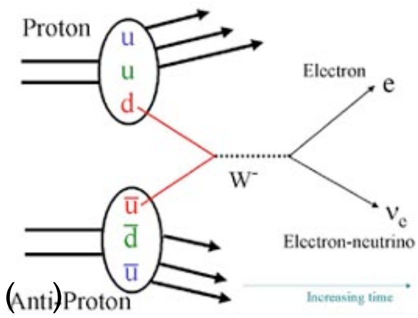
Measuring W-boson properties

At LEP: W-bosons produced in pairs

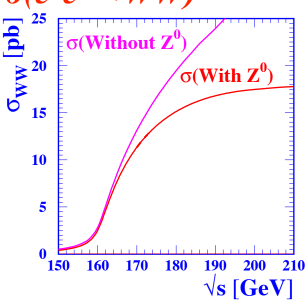
Three possible modes:



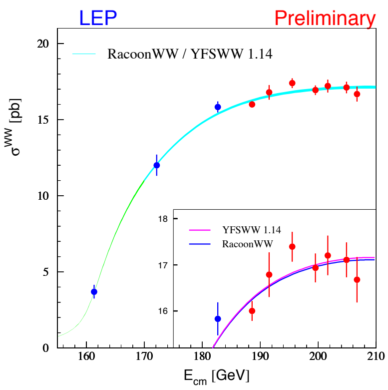
At hadron colliders: W-bosons produced from quarks and anti-quarks



Predicted $\sigma(e^+e^- \to WW)$



$\sigma(e^+e^- \to WW)$ measured at LEP confirms Z^0-W^+-W^- vertex



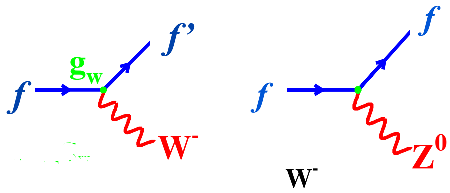
From measurements at LEP & Tevatron:

- $m_W = 80.413 \pm 0.048$ GeV/c^2
- $\Gamma_W = 2.141 \pm 0.041$ GeV

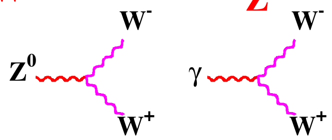
Measurements of W and Z bosons validate the Electroweak Model beautifully

Weak Force Summary

- The weak force acts on **all** quarks and leptons.
- Two **massive** bosons propagate the weak interaction: $W^{\pm}$ and Z^0.
- Interactions characterised by: long lifetimes 10^{-13} - 10^3 s and small cross sections 10^{-13} mb.



Allowed vertices:



At low energies, virtual W and Z bosons responsible for lepton and lightest hadron decays and neutrino scatterings.

Fermi theory describes W-boson interactions for low boson momentum.

Described by Fermi constant:

$$G_F \propto g_w^2/m_W^2$$

- **Lepton interactions are universal:** same coupling to W, Z bosons
- **Quarks interactions not universal:**
  - ➡ W-(Q=+2/3 quark)-(Q=−1/3 quark) coupling is $g_W V$, where V depends on flavours of quark involved.
  - ➡ V is largest within a generation: V_{ud}, V_{cs}, V_{tb}

High energies colliders can produce real W and Z bosons for study. These validate electroweak model *e.g.* Z decays suggest exactly three generations of quarks and leptons.

Electromagnetic & weak are manifestations of a single unified **electroweak interaction**, described by just 3 parameters.