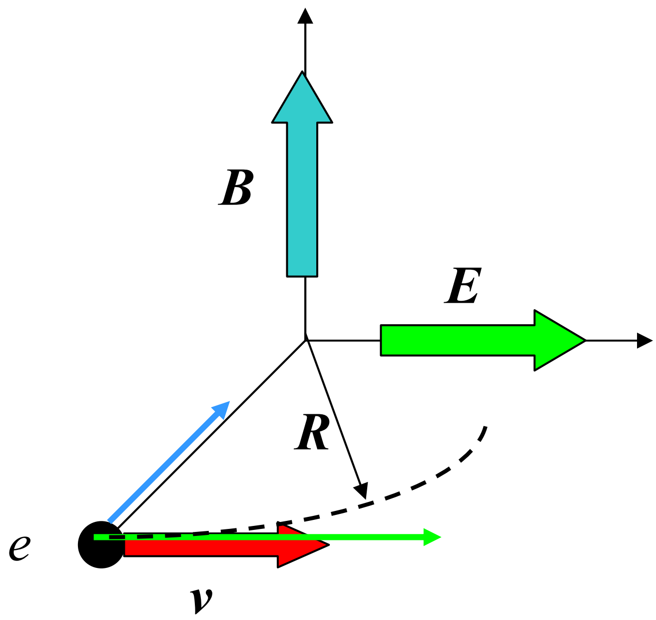
Particle Physics: Problem Sheet 3
Accelerators & Detectors

1. In a synchrotron accelerator a particle of charge q moves in a circular orbit of radius R under the influence of a magnetic field, $\vec{B}$, perpendicular to the direction of the particle, as shown in the diagram. Show that the magnitude of the momentum of the charged particle is given by: $|\vec{p}| = q|\vec{B}|R$.



   Show that if the charge of the particle is e, R is measured in meters, B is measured in Tesla and p is measured in GeV/c, this can be written as $p_T \approx 0.3BR$, where p_T represents the momentum transverse to the magnetic field.

2. The luminosity of a proton-proton collider such as the LHC can be estimated as:

$$\mathcal{L} = \frac{N_p^2\, n_\mathrm{b}\, f_\mathrm{rev}}{A}$$

   where: N_p is the number of protons per bunch, n_b is the number of bunches per beam, f_rev is the revolution frequency and A is the effective area of the collision.

   Calculate f_rev for the LHC.

   At design specifications, the LHC will collect an integrated luminosity of $\int \mathcal{L}dt = 100\ \mathrm{fb}^{-1}$. Assuming that the LHC runs for 10^7 seconds per year, calculate the instantaneous LHC luminosity.

   Use this to estimate the effective area, and hence radius, of the proton beam.

3. The Tevatron collides protons and anti-protons with $E_p = E_{\bar{p}} = 1.96$ TeV. Both the proton and anti-proton beams have 36 bunches per beam. There are $N_p = 2.5 \times 10^{11}$ and $N_{\bar{p}} = 1 \times 10^{11}$ particles per bunch. What is the energy stored in the Tevatron beams?

4. In a synchrotron accelerator, why do charged particles loose energy? The energy loss per turn is:

$$\Delta E = \frac{q^2 \beta^3 \gamma^4}{3\epsilon_0 \rho}$$

   The LEP and LHC synchrotrons were built in the same tunnel ($\rho_{\mathrm{LEP}} = \rho_{\mathrm{LHC}} = 4300$ m). At LEP the energy of the electrons was $E_e = 45.2$ GeV; at LHC the energy of the protons will be $E_p = 7000$ GeV. What is the ratio of the energy loss at LEP and LHC?

5. Write down the formula for the decay length of relativistic particle.

   The closest part of the ATLAS detector is 5cm from the interaction point. What particles live long enough to be detected in the ATLAS detector? You need only consider leptons and hadrons.

6. Following on from the previous question.... Describe briefly how these particles appear in a typical collider detector, such as the ATLAS detector.

7. Cosmic ray muons are produced at the top of the atmosphere. As they travel through matter, muons loose energy to ionisation. The energy loss for muons can be described by $dE/dx \approx 2.0\ \mathrm{MeVg^{-1}cm^2}$.

   How much energy does a muon with three-momentum, $p_\mu = 5$ GeV/c lose by ionisation before reaching sea level?

   Hint: the total energy loss, $\Delta E = dE/dx \times x$, where x is the thickness of the atmosphere in g/cm^2. The mass thickness of the atmosphere can be inferred from the pressure at sea level, $P = 1$ atm $= 10^5\ \mathrm{kgm^{-1}s^{-2}}$, by assuming the density to be constant.