Particle Physics: Problem Sheet 5

Weak, electroweak and LHC Physics

1. Draw a quark level Feynman diagram for the decay $K^+ \to \pi^+\pi^0$.

2. Write down all possible decay modes of the W^- boson into quarks and leptons. What is the strength of each of these vertices?

3. By drawing the lowest order Feynman diagrams, show that both charged ($W^\pm$ exchange) and neutral current (Z^0 exchange) contribute to neutrino electron scattering: $\nu_e + e^- \to \nu_e + e^-$.

4. Muons decay with a lifetime of 2.2 μs.

   (a) What boson is responsible for the decay of the muon?

   (b) What are the final decay products?

   (c) Draw a Feynman diagram illustrating this decay.

   (d) What is the matrix element for this decay? How does this relate to the Fermi constant:
   $$G_F = \frac{\sqrt{2}\, g_w^2}{8\, m_W^2} \ ?$$

5. Draw the lowest order Feynman diagram for the decay of a tau into an electron and into a muon: $\tau^- \to e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$, respectively.

   Calculate the matrix element for each of these decays. Compare this to the matrix element for muon decay, in Q4 above.

6. Recall that the rate of a process, such as the decay width, Γ, for a given process is given by Fermi's Golden rule:
   $$\Gamma = \frac{2\pi}{\hbar}|\mathcal{M}|^2\rho$$
   where ρ is the phase space factor. For this question we will use dimensional analysis to understand ρ.

   The decay width Γ is measured in units of energy. Considering muon and tau decays, what is the (energy) dimensions of the Fermi constant, G_F? Therefore, show that the dimension of ρ must be $[E]^5$.

   The only energy scale for muon or tau decay is the mass of the decaying lepton (m_μ in Q4; m_τ in Q5). Use this to calculate the ratio of following decays:
   $$\frac{\Gamma(\mu^- \to e^-\bar{\nu}_e\nu_\mu)}{\Gamma(\tau^- \to e^-\bar{\nu}_\tau\nu_\mu)}$$
   How does this relate to the lifetime of the tau lepton? (The lifetime of the muon is given in Q4, and the masses of the tau and muon are on the particle handout.)

7. Estimate the total decay width, Γ_Z, and the lifetime of the Z^0 boson using:

   - The measurement of the total hadronic width:
     $\Gamma(Z \to \text{hadrons}) = 1744.4$ MeV
   - The measurement of the width into charged leptons:
     $\Gamma(Z \to \ell^+\ell^-) = 84.0$ MeV. ($\ell \equiv \{e, \mu, \tau\}$).

- The prediction for the width into one flavour of neutrino: $\Gamma(Z \to \nu\bar{\nu}) = 167.0$ MeV.

Compare with the observed resonance width, $\Gamma_Z = 2495.2 \pm 2.3$ MeV.

What are the branching ratios of each of the decay modes?

8. The key predications of electroweak theory is that the couplings and the masses of the bosons are related as:

$$e = g_W \sin\theta_W \qquad m_Z^2 = m_W^2/\cos^2\theta_W$$

At high energies, the measured values of the electromagnetic coupling constant, the "weak mixing angle" and the Z-boson mass are $\alpha_{\rm EM}(\underline{\underline{q}}^2 = m_Z^2) = 1/128$, $\sin^2\theta_W(\underline{\underline{q}}^2 = m_Z^2) = 0.23120$ and $m_Z = 91.188$ GeV/c^2.

Calculate the weak coupling constant, α_W, and predicted mass of the W-boson in the electroweak model, and compare this to the measured mass of $m_W = 80.413$ GeV/c^2.

9. **Higgs at the LHC** *We didn't cover this in the lectures, due to lack of time. However, using the handout where nesecarry, you should still be able to do this question.*

At LHC the Higgs boson, H can be produced in association with a W or Z boson, e.g. $pp \to WH, pp \to ZH$.

(a) Draw the Feynman diagram for this process. (Select the correct one from page 10 of lecture handout 9.) What partons are involved in the hard scatter?

(b) What is the minimum effective collision energy, $\sqrt{\hat{s}}$ needed to create a real Higgs boson of mass, $m_H = 120$ GeV/c^2 in association with a Z boson. What are typical values of Feynman-x for this value of $\sqrt{\hat{s}}$?

(c) Looking at the Higgs branching ratio plot on Lecture 10, page 7, what is the most likely decays of a Higgs boson with $m_H = 120$ GeV/c^2?

(d) What are the possible final states for the process $pp \to ZH$ where $m_H = 120$ GeV/c^2? What would this look like in the ATLAS detector?

10. **Feynman diagrams practise**
In each of the following decays try to draw a quark level Feynman diagram and determine which interaction is responsible. The quark content of $D^{\star+}$ is $c\bar{d}$; the quark content of K^{*0} is $\bar{s}d$. The others are given on the particle properties sheet.

$$\begin{aligned} D^{\star+} &\to D^0\pi^+ & \qquad \Sigma^0 &\to \Lambda\gamma \\ D^+ &\to \overline{K}^0\pi^+ & \qquad \tau^- &\to \rho^-\nu_\tau \\ K^{*0} &\to K^+\pi^- & \qquad \pi^0 &\to \gamma\gamma \end{aligned}$$

Hints: The photon only interacts electromagnetically and the neutrino only weak. First establish the relevant quark and lepton flavour quantum numbers of the initial and final states. Quark and lepton flavours are conserved at each vertex for the strong and electromagnetic processes, the weak charged current is flavour changing. Always draw the simplest possible diagrams.

Do your answers agree with the observed lifetimes?