Particle Physics: Problem Sheet 2 - Outline Solutions

QED

1. **The simplest vertex in QED is a fermion-fermion-photon vertex. Draw an example Feynman diagram for such a vertex. Write down *all* possible electromagnetic fermion-fermion-photon vertices.**

   All charged leptons and all quarks interact electromagnetically.
   Leptons: $ee\gamma$, $\mu\mu\gamma$,$\tau\tau\gamma$.
   Quarks: uuγ, ddγ, ssγ, ccγ, bbγ, ttγ

   For example, $ee\gamma$ means that an electron, an electron and a gamma meet at the same point in a Feynman diagram. See question 2 to think about all the different processes that this vertex can describe.

   (If you want to think about the colour charges of the quarks in this, then the verticies are: $\mathrm{u^R u^R}\gamma$, $\mathrm{u^B u^B}\gamma$, $\mathrm{u^G u^G}\gamma$ and similarly for the other quarks. So the electromagnetic interaction does not affect the colour charge of the quark.)

2. **Draw the Feynman diagram of $\boldsymbol{\tau^- \rightarrow \tau^-\gamma}$ vertex. What quantities / quantum numbers are conserved at the vertex?**

   Charge, lepton flavour and four momentum are conserved.

   Draw the vertex diagram in all the different orientations that you can. Which processes do each of the diagram correspond to?

   $$\begin{array}{rcl}
   \text{vacuum} & \rightarrow & \tau^+\tau^-\gamma \\
   \tau^- & \rightarrow & \tau^-\gamma \\
   \tau^+ & \rightarrow & \tau^+\gamma \\
   \gamma & \rightarrow & \tau^+\tau^- \\
   \tau^+\tau^- & \rightarrow & \gamma \\
   \tau^+\gamma & \rightarrow & \tau^+ \\
   \tau^-\gamma & \rightarrow & \tau^- \\
   \tau^+\tau^-\gamma & \rightarrow & \text{vacuum}
   \end{array}$$

   What is the relationship between these processes and physical processes?

   All of these processes are not real processes, as the photon is virtual ($(\underline{p}_\gamma)^2 \neq m_\gamma^2$). A virtual particle can only be an intermediate particle. You have to join two of these diagrams together to make a real process.

3. **The following processes are all due to the electromagnetic force. By considering the quark content of the baryons where necessary, try to draw the Feynman diagrams for these processes:**

   - $\boldsymbol{e^+e^- \rightarrow \tau^+\tau^-}$
   - $\boldsymbol{e^+e^- \rightarrow \Upsilon}$
   - $\boldsymbol{J/\psi \rightarrow \mu^+\mu^-}$
   - $\boldsymbol{\pi^0 \rightarrow \gamma\gamma}$ **(choose one of the possible quark combinations for the $\boldsymbol{\pi^0}$)**

4. **Show that the photon propagator is the origin of the $\boldsymbol{1/\sin^4(\frac{\theta}{2})}$ dependence of the Rutherford cross section for $\boldsymbol{e^-p \rightarrow e^-p}$ scattering.**

   The cross section, σ, is proportional to $|\mathcal{M}|^2$, thus the photon propagator adds a $1/q^4$ term to the cross section where q is the four-momentum transferred from the electron to the proton. Write the initial electron four-momentum as $p_1 = (E_1, \vec{p}_1)$

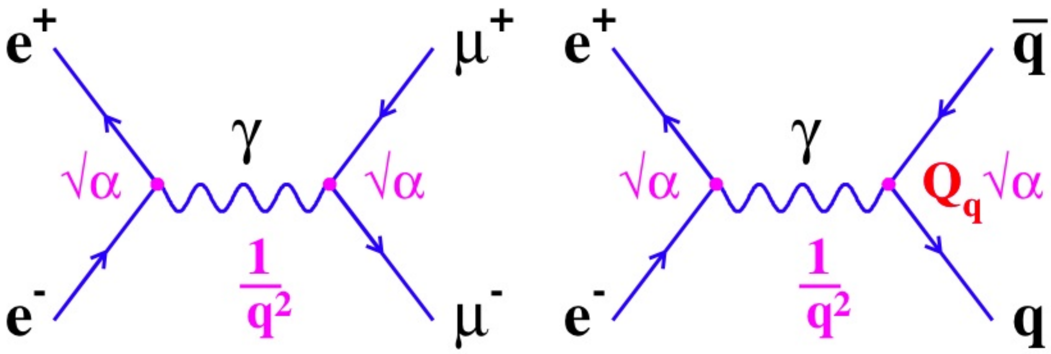
and the final electron four-momentum as $p_3 = (E_3, \vec{p_3})$ we obtain $q = p_1 - p_3$ (and using natural units, $c = 1$):

$$\begin{aligned} q^2 &= (p_1 - p_3)^2 = p_1^2 + p_3^2 - 2p_1 \cdot p_3 \\ &= 2m_e^2 - 2(E_1E_3 - |\vec{p_1}||\vec{p_3}|\cos\theta) \\ &\approx -4E_1E_3 \sin^2(\theta/2) \end{aligned}$$

In the last line we have neglected the electron mass m_e which leads to $E_1 = |\vec{p_1}|$ and $E_3 = |\vec{p_3}|$. We also used the trigonometric relation $2\sin^2 x = 1 - \cos 2x$.

$$\text{Propagator} \qquad \Longrightarrow \qquad \sigma \propto 1/\mathrm{q}^4 \propto 1/\sin^4(\frac{\theta}{2})$$

5. **Draw the lowest order Feynman diagrams for our favourite process: $e^+e^- \longrightarrow \mu^+\mu^-$. Discuss the corresponding Matrix element, $\mathcal{M}(e^+e^- \longrightarrow \mu^+\mu^-)$.**



$$\mathcal{M}(e^+e^- \to \mu^+\mu^-) \propto \frac{e^2}{\underline{\underline{q}}^2}$$

where:

$$\underline{\underline{q}} = \underline{\underline{p}}_{e^+} + \underline{\underline{p}}_{e^-}$$

A similar process can be used to create pairs of quarks, $e^+e^- \longrightarrow \mathrm{q\overline{q}}$. Discuss the corresponding Matrix element for this process $\mathcal{M}(e^+e^- \longrightarrow \mathrm{q\overline{q}})$.

This time the second vertex has a charge of $+2/3e$ or $-1/3e$. Let's write that as: Q_qe. Then we get:

$$\mathcal{M} \propto \frac{Q_qe^2}{\underline{\underline{q}}^2}$$

where: $\underline{\underline{q}}$ is the same as before.

What can you say about the ratio of the cross sections,

$$\frac{\sigma(e^+e^- \longrightarrow \mathrm{q\overline{q}})}{\sigma(e^+e^- \longrightarrow \mu^+\mu^-)}?$$

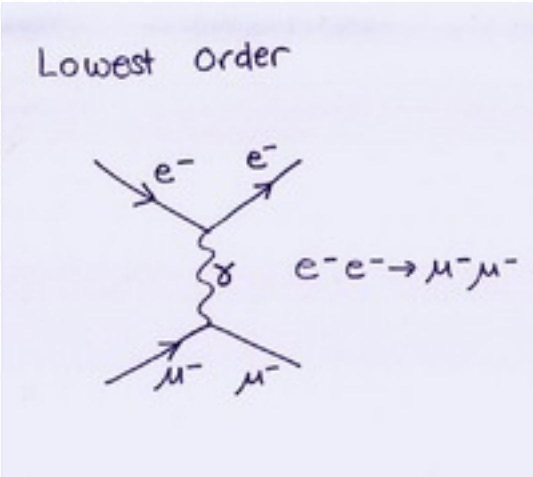
Cross sections, $\sigma \propto |\mathcal{M}|^2$.

Therefore

$$\frac{\sigma(e^+e^- \to \mathrm{q\overline{q}})}{\sigma(e^+e^- \to \mu^+\mu^-)} \propto \frac{Q_q^2e^4\underline{\underline{q}}^4}{e^4\underline{\underline{q}}^4} = Q_q^2$$

Please note: this is not the whole answer to the problem! We'll look more at this process in the coming weeks.

6. **Draw the lowest and second order Feynman diagrams for electron-muon scattering $e^- \mu^- \rightarrow e^- \mu^-$.**



Discuss the corresponding matrix element, $\mathcal{M}$, and cross section for the lowest order.

- The top vertex is proportional to the charge of the electron, e.
- The bottom vertex is proportional the charge of the muon, e
- The propagator is proportional to $1/\underline{\underline{q}}^2$; where $\underline{\underline{q}}$ is the four momentum transfer squared between the electron and the muon.

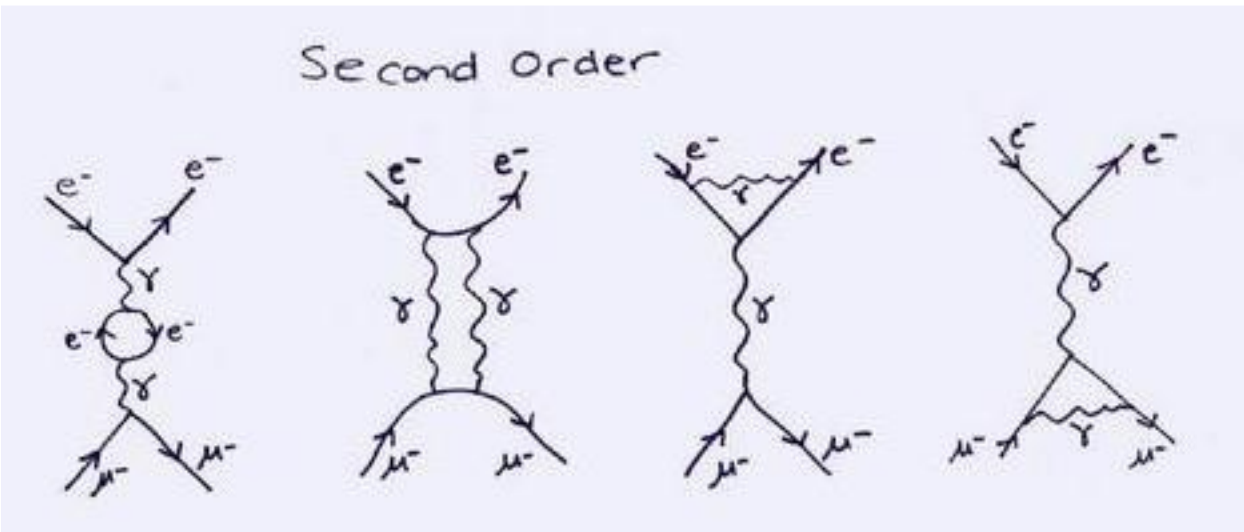
The matrix element, at lowest order, is proportional to the product of these terms:

$$\mathcal{M}_0 \propto e^2/\underline{\underline{q}}^2$$

Cross section is proportional to the square of the matrix element:

$$\sigma_0 \propto \mathcal{M}_0^2 \propto e^4/\underline{\underline{q}}^4 \propto \alpha^2/\underline{\underline{q}}^4$$

Estimate the contribution of the second order diagrams to the cross section.



The second order diagrams all have four vertices, each proportional to e. To calculate the matrix elements we would also have to worry about the propagator term, but we are only doing an estimate here. Therefore each contribution to the matrix element, $\mathcal{M}_1$, is proportional to e^4, and the contribution to the cross section is e^8 or α^4.

All second order diagrams are suppressed by a factor α in the matrix element and a factor α^2 in the cross section. We can write this as:

$$\begin{aligned} \mathcal{M} &= \mathcal{M}_0 + \mathcal{M}_1 \;=\; \mathcal{M}_0(1 + \Sigma a_i\,\alpha) \\ \sigma &\propto \mathcal{M}^2 = \mathcal{M}_0^2 + \mathcal{M}_1^2 \;=\; \sigma_0(1 + k\alpha^2) \end{aligned}$$

The amplitude coefficients a_i are some factors relative to $\mathcal{M}_0$, they can be calculated, be were are not going to do it! But we can assume for $\alpha_i \sim 1$. The cross section σ_1 will not change significantly compared to the first order estimate, σ_0, as the correction from the next orders is proportional to $\alpha^2 = 1/137^2$.

7. **Some of what we have learned about QED is applicable to the weak force. The weak force can be propagated by the $W^{\pm}$-boson with mass $m_W = 80$ GeV/c^2. For example, nuclear beta decay can be described as d $\rightarrow$ uW^-, followed by the decay of the W^- into $e^-\bar{\nu}_e$.**

   Estimate the maximum range of the weak force propagated by the W-boson.

   The masses of all the other particles involved as much smaller than the mass of the W-boson. So then the violation of energy, $\Delta E \approx m_W c^2$. So the time that we can have the violation for is: $\Delta t \approx \hbar/\Delta E = \hbar/(m_W c^2)$. The maximum speed of the W-boson is c. Therefore the maximum range of the force, $R = \hbar/(m_W c)$.

   To evaluate this, we can write $R = \hbar c/(m_W c^2) = 197$ MeV fm/80,000 MeV $\approx 2.5 \times 10^{-3}$fm $= 2.5$ am. The effective range of the weak force is only 2.5 attometres!

   What does the Yukawa potential look like for exchange of a W-boson? The coupling of the W-boson, is written as g_W.

$$V(r) = -\frac{g_W^2}{4\pi r}\exp\left(-\frac{r}{R}\right) \quad \text{with } R = \frac{\hbar}{m_W c}$$

   Note that the R is the same as the expression for the range of the force we calculated above.