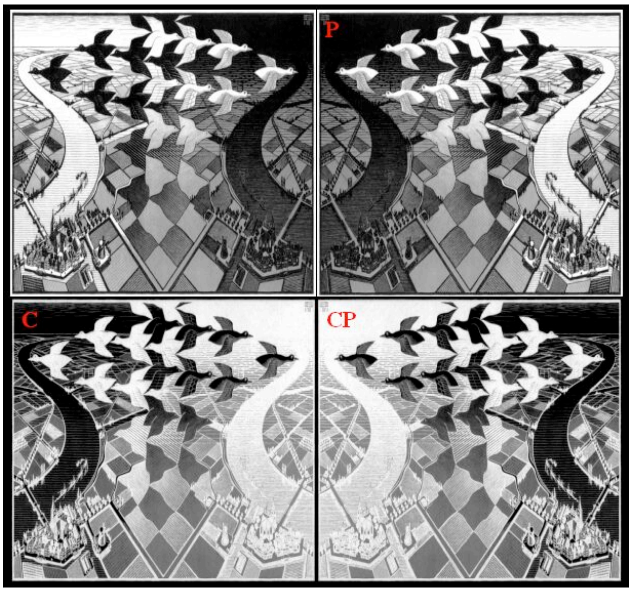
Particle Physics

Dr Victoria Martin, Spring Semester 2012
Lecture 13: Symmetries



★Symmetries of QED and QCD
★Parity, Charge Conjugation and Time Reversal
★Parity Violation in Weak Decay
★CP and CPT

From Last Tuesday: Decays of Hadrons

- Hadron decays give us insight into forces in particle physics.

- **Strong decays** are characterised by very short lifetimes, $\tau \sim 10^{-20}$ - 10^{-23} s appearing as **resonances** with a large width $\Gamma \sim$ MeV.
  - ➡ Final states are all hadronic. All quantum numbers are conserved.
- **Electromagnetic decays** are characterised by $\tau \sim 10^{-20}$ - 10^{-16} s.
  - ➡ Decays containing photons are electromagnetic.
  - ➡ All quantum numbers conserved except total isospin, I.
- **Weak decays** characterised by long lifetimes, $\tau \sim 10^{-13}$ - 10^{3} s.
  - ➡ Responsible for decay of lightest baryons with a strange, charm or bottom quark.
  - ➡ Particles can live long enough to reach the detector.
  - ➡ Final states may be leptonic, semi-leptonic or hadronic.
  - ➡ Allows access to the elements of the **CKM matrix**.
  - ➡ Isospin, I, I_3, Parity, P, Flavour quantum numbers not conserved.

- **CKM matrix** relates the mass eigenstates to the weak eigenstates. Contains a complex phase.

Cabibbo-Kobayashi-Maskawa Matrix

- **Mass eigenstates** and **weak eigenstates** of quarks are not identical.
  - ➡ Decay properties measure mass eigenstates with a definite lifetime and decay width
  - ➡ The weak force acts on the weak eigenstates.
- Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix a mixture of the down-type quarks:

$$\text{weak eigenstates} \quad \begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix} \quad \text{mass eigenstates}$$

- *e.g.* weak eigenstate of the strange quark is a mixture between down, strange and bottom mass eigenstates

$$\mathrm{s}' = V_{\mathrm{cd}}\mathrm{d} + V_{\mathrm{cs}}\mathrm{s} + V_{\mathrm{cb}}\mathrm{b}$$

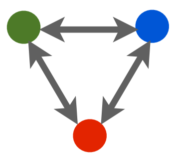
- The CKM matrix is unitary, $V_{\mathbf{CKM}}{}^{\dagger}V_{\mathbf{CKM}} = \mathbf{1}$; standard parameterisation in terms of three mixing angles ($\theta_1, \theta_2, \theta_3$) and one phase ($\delta$) is:

don't attempt to remember this!

$$\begin{pmatrix} \cos\theta_1 & \sin\theta_1\cos\theta_3 & \sin\theta_1\sin\theta_3 \\ -\sin\theta_1\cos\theta_3 & \cos\theta_1\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3 + \sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta_2\cos\theta_3 - \cos\theta_2\sin\theta_3 e^{i\delta} & -\cos\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_2\cos\theta_3 e^{i\delta} \end{pmatrix}$$

Introduction: Symmetries in Particle Physics

- We've already seen that symmetries play a central role in particle physics.
- Symmetries describe operations which leave a physical system unchanged.



- Why bother studying symmetries?
  - Noether's theorem: every symmetry corresponds to a conservation law.
  - Conservation laws can be experimentally verified
  - Symmetries can be used to unify the description of particle physics
- Some symmetries are observed to only hold under certain conditions, e.g. conservation of quark flavour for strong and electromagnetic interactions
- Breaking of symmetries can be either "dynamical" or "spontaneous" (e.g. Higgs mechanism).

Types of Symmetries

- Continuous symmetries:
  - ➡ Translation (conservation of momentum)
  - ➡ Time (conservation of energy)
  - ➡ Rotation (angular momentum, spin)
- Gauge transformations:
  - ➡ Electromagnetic field (conservation of charge)
  - ➡ SU(3) colour
  - ➡ Electroweak symmetry
- Flavour symmetries:
  - ➡ Lepton and baryon number
  - ➡ Isospin, SU(3) flavour
- Discrete symmetries:
  - ➡ Parity (***P***)
  - ➡ Charge Conjugation (***C***)
  - ➡ Time Reversal (***T***)

U(1) Symmetry of QED

Don't need to memorise this Lagrangian or terms within.

- The Lagrangian of QED can be written as:

$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma_\mu\partial^\mu - m)\psi}_{\text{freely moving fermion}} + \underbrace{e\,\bar{\psi}\gamma_\mu A^\mu\psi}_{\text{coupling of charged fermion to photon}} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{electromagnetic field}}$$

  - ➡ ψ: fermion (electron) spinor wavefunction
  - ➡ m: fermion mass
  - ➡ $A^\mu=(\phi,\underline{A})$: electromagnetic four-potential / photon field
  - ➡ $F^{\mu\nu}$: Electromagentic Tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}$$

- Demand Lagrangian is invariant phase change of ψ: phase shift by any value $\alpha(x^\mu)$ (equivalent to local U(1) symmetry)

$$\psi \to e^{i\alpha}\psi \qquad A^\mu \to A^\mu - \tfrac{i}{e}\partial^\mu\alpha$$

- Leads to conservation of electric and magnetic charge.

$$J^\mu = e(\bar{\psi}\gamma^\mu\psi) \qquad \partial_\nu J^\mu = 0$$

- No photon mass term($mA_\mu A^\mu$) in Lagrangian ⇒ photon is massless; EM has infinite range.

SU(3) Colour Symmetry of QCD

- The Lagrangian of QCD can be written as:

$$\mathcal{L} = \bar{\mathrm{q}}\,(i\gamma_\mu\partial^\mu - m)\,\mathrm{q} + g_s\,\bar{\mathrm{q}}\,(\gamma_\mu\lambda^a G^\mu_a)\,\mathrm{q} - \tfrac{1}{4}G^a_{\mu\nu}G^{a\,\mu\nu}$$

freely moving quark — coupling of quarks gluons — colour field

  - ➡ **q**: quark spinor wavefunction $\mathrm{q} = \begin{pmatrix}\psi_r \\ \psi_b \\ \psi_g\end{pmatrix}$
  - ➡ m: quark mass
  - ➡ g_s: strong coupling constant
  - ➡ λ^a: Lambda matrices (a=1,8)
  - ➡ $G^\mu{}_a$, $G^a{}_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu - g_s f^{abc} G^b{}_\mu G^c{}_\nu$: gluon field and colour field tensor

Don't need to memorise this Lagrangian or details.

- Lagrangian is invariant under "SU(3) Gauge Transformation" (change of phase for each part of wavefunction)

$$\mathrm{q} \rightarrow (1 + i\alpha_a\lambda_a)\mathrm{q} \qquad\qquad G^\lambda_a \rightarrow G^\lambda_a - \frac{1}{g_s}\partial^\mu - f^{abc}\alpha_b G^\mu_c$$

- Leads to conservation of colour charge
- No mass term $mG^\mu G_\mu$ in Lagrangian. Gluon is massless.

Parity (P)

- Parity, P, is a spatial inversion through the origin: $P\psi(\vec{r}) = \psi(-\vec{r})$
(not a mirror reflection in a plane!)

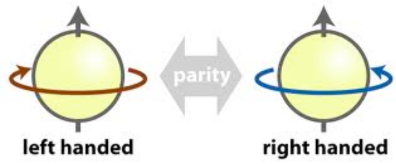
- If you act P twice on a state you get the original state back $\Rightarrow P^2 = 1$.
  - Eigenvalues of P are either $+1$ (even) or -1 (odd)
    - e.g. $\psi(x) = \sin kx$ is odd $\quad \psi(x) = \cos kx$ is even

- e.g. Hydrogen atom states described by Legendre polynomials:
$Y_L{}^m = P_L{}^m(\cos\theta)\, e^{im\phi}$

  - Parity of state described by $Y_L{}^m$ is $(-1)^L$ (depends on orbital angular momentum L)
  - S (L=0) and D (L=2) states are even, P (L=1) states are odd

Intrinsic Parity

- Intrinsic parity is property of the elementary particles, we define:
  - Fermions to be eigenstates with $P = +1$ (even)
  - Antifermions to be eignestates w. $P = -1$ (odd)



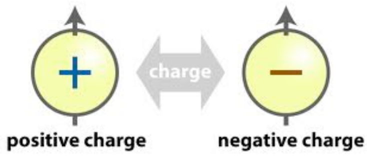
- Parity operator (P) can be represented by $\gamma^0 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix}$
  - Check this by looking at Dirac spinors (on problem sheet)

- Photons have odd parity, $P = -1$
  - Parity operation changes the direction of Electric fields.
  - Check this from transformation of electromagnetic fields (A^μ)

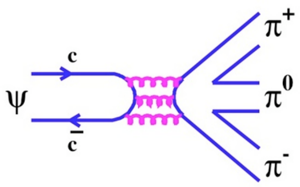
- In general:
  - Scalar ($\overline{\psi}\,\psi$) and axial-vector ($\overline{\psi}\,\gamma^5\,\gamma^\mu\,\psi$) quantities have $P = +1$ (even)
  - Pseudoscalar ($\overline{\psi}\,\gamma^5\psi$) and vector ($\overline{\psi}\,\gamma^\mu\,\psi$) quantities have $P = -1$ (odd)

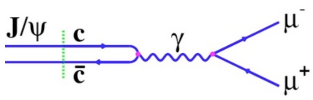
Charge Conjugation (C)

- A change from particle to antiparticle: $C\bar{f} = f$; $Cf = \bar{f}$
- $C^2 = 1$ $\Rightarrow$ eigenvalues of C are either $+1$ (even) or -1 (odd)
  - (C can be represented by $i\gamma^2$)
- Fermions and antifermions are not eigenstates of C !



- Photons have $C = -1$. C changes sign of the electric charges, and therefore of the electromagnetic field. Similarly gluons have $C = -1$.
- C and P observed to be conserved in electromagnetic and strong interactions.
- Mesons ($q\bar{q}$) have $C = (-1)^{L+S}$ and $P = (-1)^{L+1}$
- Lightest mesons pseudoscalars (↑↓) with $J^{PC} = 0^{-+}$ Second-lightest states are vectors (↑↑ or ↓↓) with J^{PC}=1^{--}

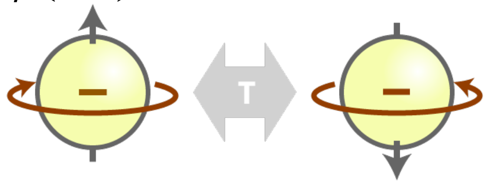




- e.g. J/ψ ($c\bar{c}$) meson has J^{PC}=1^{--}. EM and strong decays via odd number of photons or gluons to conserve C and P

Time Reversal (T)

- A reversal of the arrow of time: $T\,\psi(t) = \psi(-t)$

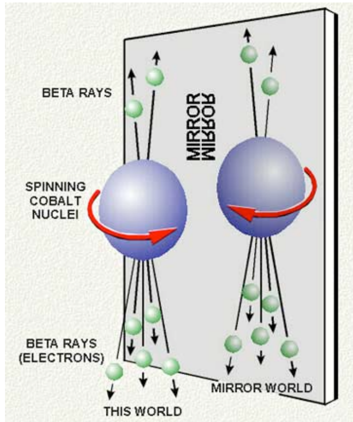


- $T^2 = 1$ so eigenvalues of T are either $+1$ (even) or -1 (odd)
- T corresponds to interchanging the initial and final states in a Feynman diagram.
- T reverses the sign of time-dependent variables including momentum and angular momentum (spin)
- It is evident that the Universe is not T symmetric!
  - Big Bang, Hubble expansion, increase in Entropy …

Discrete Symmetries of Physical Quantities

Quantity	Notation	P	C	T
Position	$\vec{r}$	-1	$+1$	$+1$
Momentum (Vector)	$\vec{p}$	-1	$+1$	-1
Spin (Axial Vector)	$\vec{\sigma} = \vec{r} \times \vec{p}$	$+1$	$+1$	-1
Helicity	$\vec{\sigma} \cdot \vec{p}$	-1	$+1$	$+1$
Electric Field	$\vec{E}$	-1	-1	$+1$
Magnetic Field	$\vec{B}$	$+1$	-1	-1
Magnetic Dipole Moment	$\vec{\sigma} \cdot \vec{B}$	$+1$	-1	$+1$
Electric Dipole Moment	$\vec{\sigma} \cdot \vec{E}$	-1	-1	-1
Transverse Polarization	$\vec{\sigma} \cdot (\vec{p_1} \times \vec{p_2})$	$+1$	$+1$	-1

Parity Violation in Weak Decays



- First observed by Chien-Shiung Wu in 1957 through β-decay of polarised ^{60}Co nuclei: $^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^- + \overline{\nu}_e$
  - Recall: under parity momentum changes sign but not nuclear spin
- Electrons were observed to be emitted to opposite to nuclear spin direction
- Particular direction is space is preferred!

- P is found to be violated maximally in weak decays
- Still a good symmetry in strong and QED.

- Recall the vertex term for the weak force is $g_w\, \gamma^\mu(1-\gamma^5)/\sqrt{8}$
- Fermion currents are proportional to $\overline{\psi}(e)\,(\gamma^\mu - \gamma^\mu\gamma^5)\,\psi(\nu_\mu)$ "vector – axial vector"
- Under parity, $P\,(\overline{\psi}\,(\gamma^\mu - \gamma^\mu\gamma^5)\,\psi) = \overline{\psi}\,(\gamma^\mu + \gamma^\mu\gamma^5)\,\psi$ (mixture of eigenstates)

- Compare to QED and QCD vertices: $P\,(\,\overline{\psi}\,\gamma^\mu\,\psi\,) = \overline{\psi}\,\gamma^\mu\,\psi$

  (pure eigenstates - no P violation)

Summary

- Symmetries play a key role in describing interactions in particle physics.
- QED and QCD obey Gauge symmetries in the Lagrangian corresponding to symmetry groups. These lead to conservation of electric and colour charge.
- Three important discreet symmetries: Charge Conjugation (C), Parity (P) and Time reversal (T).
  - C: changes the sign of the charge
  - P: spatial inversion, reserves helicity. Fermions have P=+1, antifermions P=−1
  - T: changes the initial and final states
  - Gluons and photons have C =−1, P=−1

- C and P are conserved in QED and QCD, maximally violated in weak
- Only LH neutrinos and RH anti-neutrinos are found in nature.

- CPT is thought to be absolutely conserved (otherwise RQFT doesn't work!)