Particle Physics

Dr Victoria Martin, Spring Semester 2012

Lecture 17: Electroweak Theory

- ★Weak Isospin and Hypercharge
- ★SU(2) and U(1) symmetries
- ★Weak Isospin and Hypercharge currents
- ★W and Z bosons
- ★Z boson decays
- ★Precision measurements of Electroweak Physics

From Friday: Neutrino Summary

- Three neutrinos in the Standard Model: ν_e, ν_μ, ν_τ
- Only left-handed neutrinos and right-handed antineutrinos are observed.

- Mass eigenstates propagate through matter or a vacuum ν_1, ν_2, ν_3
- Masses are very small, < 1 eV absolute masses unknown.
- Large mixing is observed between the flavour eigenstates

- Many experiments and observations of neutrinos used to measure Δm^2 and mixing angle between the mass eigenstates.

- *CP* violation may be present in neutrinos, unobserved as yet!

Electroweak Unification

- Electroweak Theory was proposed in 1967 by Glashow, Salam & Weinberg. Unifies the electromagnetic and weak forces (Noble prize 1979)
- In 1970 ‘t Hooft and Veltman showed how to renormalise electroweak theory. (Noble prize 1999)

- At high energies ($E \gtrsim m_Z$) the electromagnetic force and the weak force are unified as a single **electroweak force**.
- At low energies ($E \lesssim m_Z$) the manifestations of the electroweak force are separate weak and electromagnetic forces.

- We will see today:
  1. The coupling constants for weak and electromagnetism are unified:
  $$e = g_W \sin\theta_W$$
     ➡Where $\sin\theta_W$ is the **weak mixing angle**
  2. Electroweak Unification predicts the existence of massive W^+ W^- and Z^0 bosons.
     ➡Relies on Higgs mechanism to “give mass” to the W and Z bosons.

Review from Lecture 7,8: Charged & Neutral Weak Current

- Neutral Current is the exchange of massive Z-bosons.
  ➡Couples to all quarks and all leptons (including neutrinos)
  ➡No allowed flavour changes!
  ➡Neutral weak current for fermion, f:

$$\frac{g_Z}{2}\bar{u}(f)\gamma^\mu(c_V^f - c_A^f\gamma^5)u(f)$$

Lepton	c^f_V	c^f_A	Quark	c^f_V	c^f_A
ν_e, ν_μ, ν_τ	½	½	u, c, t	0.19	½
e, μ, τ	-0.03	−½	d, s, b	-0.34	−½

- Charged Current is the exchange of massive W-bosons.
  ➡Couples to all quarks and leptons and changes fermion flavour:
  ➡Allowed flavour changes are: $e\leftrightarrow\nu_e$, $\mu\leftrightarrow\nu_\mu$, $\tau\leftrightarrow\nu_\tau$, d’↔u, s’↔c, b’↔t
  ➡Acts only on the left-handed components of the fermions: $V-A$ structure.

$$g_W\frac{1}{2\sqrt{2}}\bar{u}(\nu_e)\gamma^\mu(1-\gamma^5)u(e^-)$$

Weak Isospin and Hypercharge

- QED couples to electric charge; QCD couples to colour charge...
- Electroweak force couples to two "charges".
  - **Weak Isospin:** total and third component T, T_3. Depends on handedness
  - **Hypercharge,** Y In terms of electric charge Q: $Y = 2\,(Q - T_3)$
    - All right-handed fermions have $T=0$, $T_3=0$
    - All left-handed fermions have $T=½$, $T_3=±½$
    - All left-handed antifermions have $T=0$, $T_3=0$
    - All right-handed antifermions have $T=½$, $T_3(\bar{f})=-T_3(f)$

Lepton	T	T_3	Y	Quark	T	T_3	Y
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	½	+½	-1	u_L, c_L, t_L	½	+½	⅓
e_L, μ_L, τ_L	½	−½	-1	d_L, s_L, b_L	½	− ½	⅓
ν_R	0	0	0	u_R, c_R, t_R	0	0	4/3
e_R, μ_R, τ_R	0	0	-2	d_R, s_R, b_R	0	0	−⅔

Weak Isospin Doublets

Lepton	T	T_3	Y	Quark	T	T_3	Y
$\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$	½	+½	-1	u_L, c_L, t_L	½	+½	⅓
e_L, μ_L, τ_L	½	−½	-1	d_L, s_L, b_L	½	− ½	⅓
ν_R	0	0	0	u_R, c_R, t_R	0	0	4/3
e_R, μ_R, τ_R	0	0	-2	d_R, s_R, b_R	0	0	−⅔

- Neutrinos and left-handed charged leptons from a "**weak isospin doublet**":

$$\chi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \qquad T = 1/2; \; \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix}$$

- Doublet consists of "charged current flavour change pair".
  - ➡They share a quantum number, total weak isospin $T=½$.
  - ➡They differ by the third component of weak isospin $T_3=±½$.
- Left-handed up-type quarks and left-handed down-type quarks also form isospin doublet

$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix}_L \quad \begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix}_L \quad \begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix}_L \qquad T = 1/2; \; \begin{matrix} T_3 = +1/2 \\ T_3 = -1/2 \end{matrix}$$

Weak Isospin and Hypercharge Currents

- Weak Isospin and Hypercharge couple to a different set of bosons.

- Weak isospin doublets χ_L couple to a set of three W-bosons: W^1, W^2, W^3, with SU(2) symmetry described by the 3 Pauli matrices:

$$
\begin{aligned}
j_\mu^{W1} &= (g_W|T_3|)\,\overline{\chi_{\mathrm{L}}}\gamma^\mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \chi_{\mathrm{L}} \\
j_\mu^{W2} &= (g_W|T_3|)\,\overline{\chi_{\mathrm{L}}}\gamma^\mu \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \chi_{\mathrm{L}} \\
j_\mu^{W3} &= (g_W|T_3|)\,\overline{\chi_{\mathrm{L}}}\gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \chi_{\mathrm{L}}
\end{aligned}
$$

  - Strength of the fermion interaction with bosons is: $g_W|T_3|$

- Particles with hypercharge couple to one B-boson: B^0 with U(1) symmetry. Use electron as an example:

$$j_\mu^Y = (\frac{1}{2}g'_W\,Y_e)\,\bar{e}\gamma^\mu e = \frac{1}{2}g'_W(Y_{e\mathrm{L}}\bar{e_{\mathrm{L}}}\gamma^\mu e_{\mathrm{L}} + Y_{e\mathrm{R}}\bar{e_{\mathrm{R}}}\gamma^\mu e_{\mathrm{R}})$$

  - Strength of the fermion interaction with bosons is: $g'_W\,Y/2$

Physical Bosons

- The physical W^+, W^-, Z^0, γ bosons are linear superpositions of the W^1, W^2, W^3 and B^0 bosons.

$$W^+ = \frac{1}{\sqrt{2}}(W^1 - iW^2) \qquad W^- = \frac{1}{\sqrt{2}}(W^1 + iW^2)$$

$$Z^0 = W^3\cos\theta_W - B^0\sin\theta_W \qquad \gamma = W^3\sin\theta_W + B^0\cos\theta_W$$

- The coupling of the W^+, W^- bosons are

$$\tfrac{1}{\sqrt{2}}(g_W|T_3|) = \tfrac{1}{2\sqrt{2}}g_W$$

  - The $(1\text{-}\gamma^5)$ term is integrated into the definition of the χ_L doublet.

- The electron current associated with the γ is:

$$
\begin{aligned}
&j_\mu^{W3}\sin\theta_W + j_\mu^Y\cos\theta_W \\
&\quad = (g_W|T_3|\sin\theta_W)\,\overline{\chi_{\mathrm{L}}}\gamma^\mu \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \chi_{\mathrm{L}} + (\tfrac{1}{2}g'_W Y_e\cos\theta_W)\,\overline{e}\gamma^\mu e \\
&\quad = -(\tfrac{1}{2}g_W\sin\theta_W)\overline{e_{\mathrm{L}}}\gamma^\mu e_{\mathrm{L}} + (\tfrac{1}{2}g'_W\cos\theta_W)(-\overline{e_{\mathrm{L}}}\gamma^\mu e_{\mathrm{L}} - 2\overline{e_{\mathrm{R}}}\gamma^\mu e_{\mathrm{R}})
\end{aligned}
$$

- Consistent with the photon coupling if $e = g'_W\cos\theta_W = g_W\sin\theta_W$

$\sin^2\theta_W$ and Z-boson couplings

- The mixing angle between g_W and g'_W is not a prediction of the model, it must be measured experimentally.

$$\sin^2\theta_W = \frac{g'^2_W}{g^2_W + g'^2_W} \approx 0.23$$

- The Z-boson the orthogonal mixture to the γ:

$$Z^0 = W^3 \cos\theta_W - B^0 \sin\theta_W$$

- predicts the couplings of the Z^0 boson in terms of T_3 and $Y = 2\,(Q - T_3)$
- e.g. for electron:

$$\begin{aligned} j^z_\mu &= \frac{g_W}{\cos\theta_W}\left[(T_3 - Q\sin^2\theta_W)(\overline{e_L}\gamma^\mu e_L) - (Q\sin^2\theta_W)(\overline{e_R}\gamma^\mu e_R)\right] \\ &= \frac{g_Z}{2}\bar{e}\gamma^\mu(c^e_V - c^e_A\gamma^5)e \end{aligned}$$

- with:

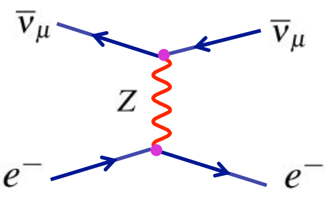
$$g_Z = \frac{g_W}{\cos\theta_W} \qquad c_V = T_3 - 2Q\sin^2\theta_W \qquad c_A = T_3$$

Summary of Electroweak Unification

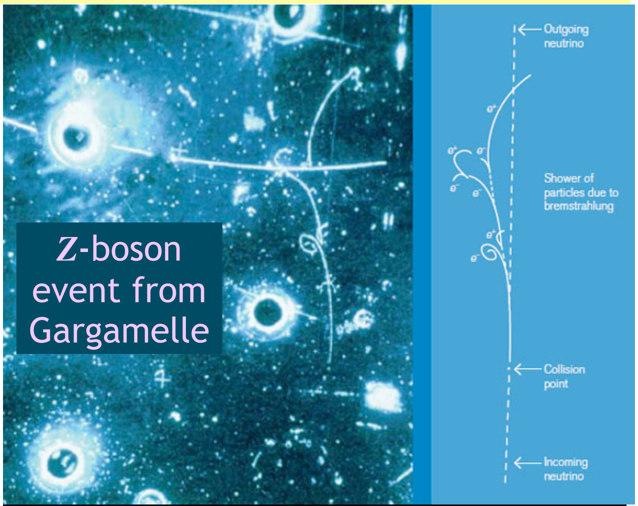
- We have recovered the behaviour of the $W^\pm$, Z and γ
  - ➡We introduced an SU(2) symmetry (3 bosons) coupling to weak isospin with a coupling constant g_W
  - ➡We introduced a U(1) symmetry (1 boson) coupling to weak hypercharge with a coupling constant g'_W
  - ➡Together predicts four bosons we identify with W^+, W^-, Z and γ
  - ➡Electroweak Theory is often called **SU(2) ⊗ U(1)** model
- All of the properties of electroweak interactions described by:
  - the intrinsic charges of the fermions
  - the SU(2) ⊗ U(1) symmetry
  - g_W and g'_W: free parameters that need to be measured
- Along with QCD, Electroweak Theory is the Standard Model.

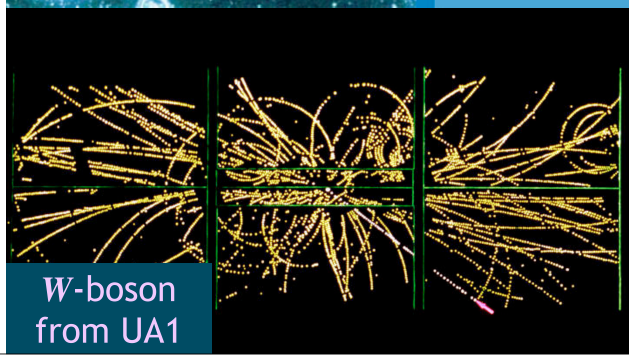
W and Z bosons

- W-boson interactions already been observed in beta decay
- Z-boson interactions were first observed 1973 at CERN Gargamelle experiment through $\overline{\nu}_\mu e^- \rightarrow \overline{\nu}_\mu e^-$

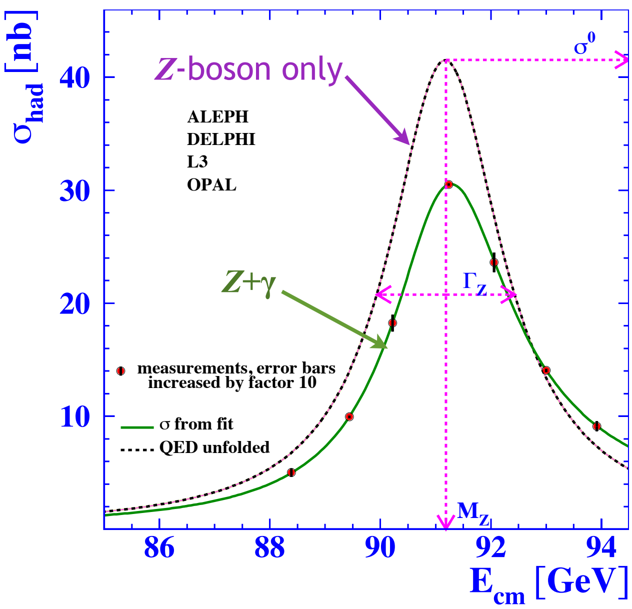


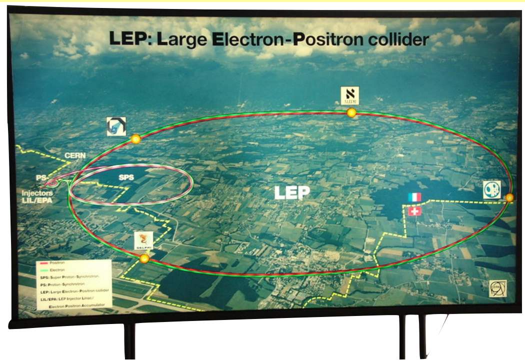
- Real (as opposed to virtual) W and Z bosons were first observed 1983 at UA1 and UA2 experiments at CERN in $p\overline{p} \rightarrow WX$, $p\overline{p} \rightarrow ZX$ (Noble prize 1984)





Z-boson measurements

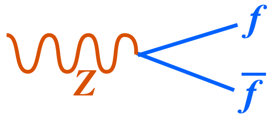




- Z^0 bosons were studied in detail by the Large Electron Positron (LEP) Collider at CERN.
- Approximately ~1M events at $\sqrt{s} \sim m_Z$: $e^+e^- \rightarrow Z^0 \rightarrow f\overline{f}$
- Measured total width of Z^0 boson: $\Gamma_Z = 2.4952(23)$ GeV

Decays of the Z-boson

➡The Z-boson can decay to any pair of fermions, except tops as $m_Z/2 < m_t$



- Decay width of Z-boson is:

$$\Gamma(Z \to f\bar{f}) = \frac{g_W^2[(c_V^f)^2 + (c_A^f)^2]}{48\pi \cos^2\theta_W} m_Z = 322[(c_V^f)^2 + (c_A^f)^2][\mathrm{MeV}]$$

$$\Gamma(Z \to e^+e^-) = \Gamma(Z \to \mu^+\mu^-) = \Gamma(Z \to \tau^+\tau^-) = 84\ \mathrm{MeV}$$

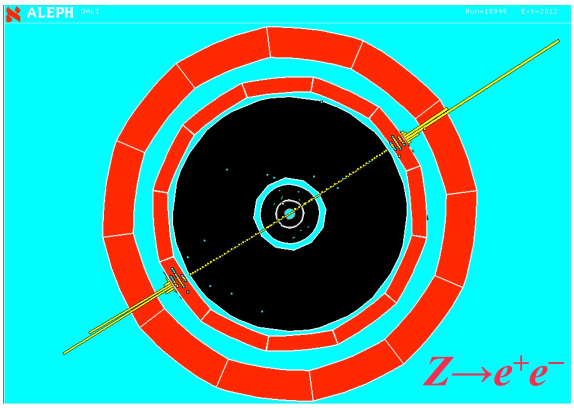
$$\Gamma(Z \to \nu_e\bar{\nu}_e) = \Gamma(Z \to \nu_\mu\bar{\nu}_\mu) = \Gamma(Z \to \nu_\tau\bar{\nu}_\tau) = 166\ \mathrm{MeV}$$

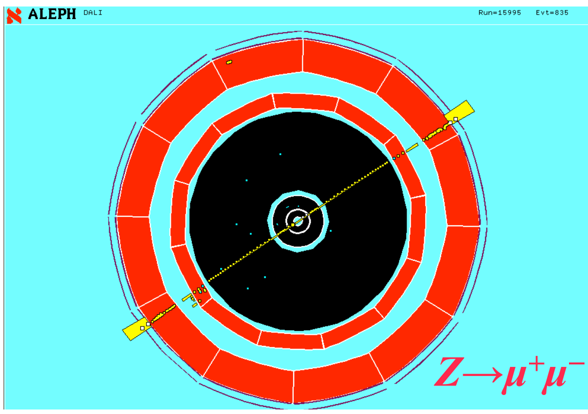
- For each colour of quark:

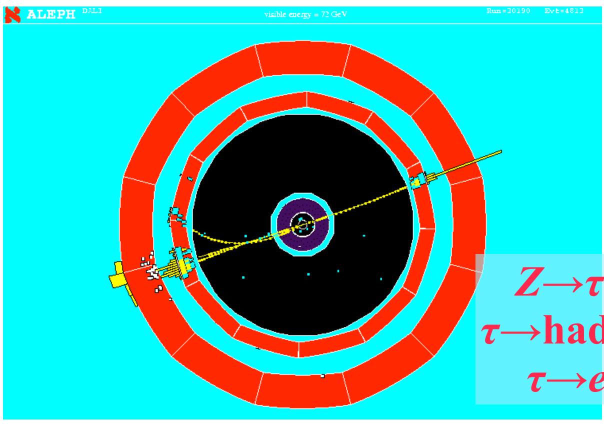
$$\Gamma(Z \to \mathrm{d\bar{d}}) = \Gamma(Z \to \mathrm{s\bar{s}}) = \Gamma(Z \to \mathrm{b\bar{b}}) = 118\ \mathrm{MeV}$$
$$\Gamma(Z \to \mathrm{u\bar{u}}) = \Gamma(Z \to \mathrm{c\bar{c}}) = 92\ \mathrm{MeV}$$

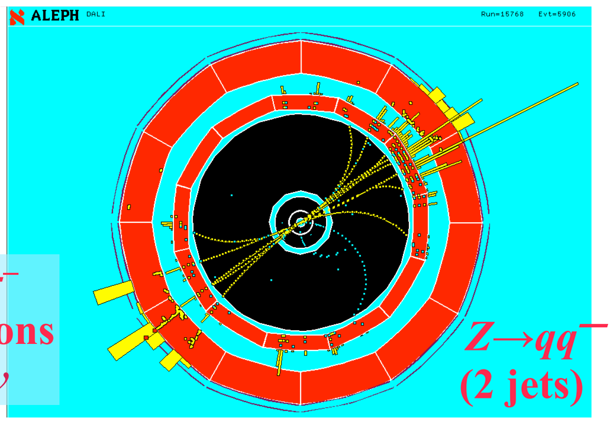
Lepton	$c^f_V = T_3 - 2Q\sin^2\theta_W$	$c^f_A = T_3$	Quark	$c^f_V = T_3 - 2Q\sin^2\theta_W$	$c^f_A = T_3$
ν_e, ν_μ, ν_τ	½	½	**u, c, t**	0.19	½
e, μ, τ	-0.03	–½	**d, s, b**	-0.34	–½

Z boson decays from ALEPH









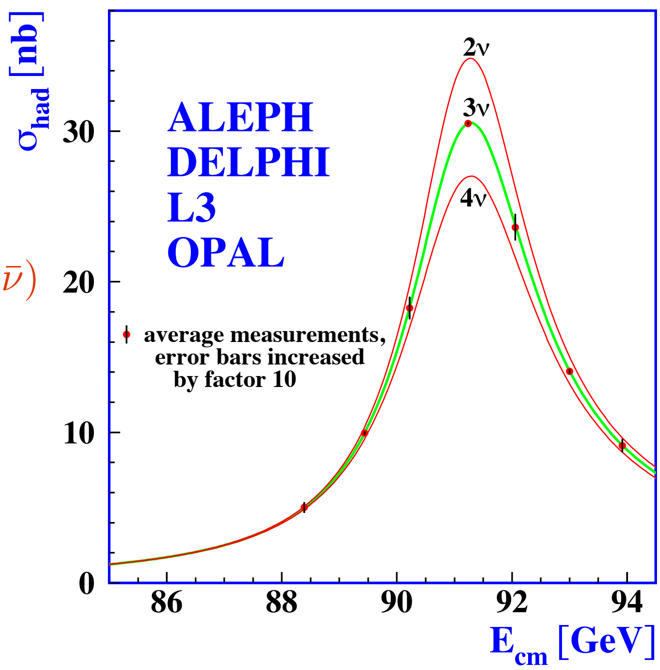
Number of Neutrinos

- Recall:
  - ➡ total width is sum of all possible partial widths
  - ➡ Z^0 boson couples to all fermions: quarks ($\mathbf{q}$), charged leptons (ℓ) neutrinos (ν)

$$\Gamma(Z) = \Gamma(\mathrm{q}\bar{\mathrm{q}}) + \Gamma(\ell^+\ell^-) + \Gamma(\nu^+\bar{\nu})$$

  - ➡ The measured total width of the Z-boson is evidence for exactly three neutrinos with $m_\nu < m_Z/2$

- Evidence there are only three generations of quarks and leptons in the Standard Model!?



Electroweak Free Parameters

- Three free parameters in the Electroweak model:
  - The gauge coupling to W-field: g_W
  - The gauge coupling to B-field: g'_W
  - The vacuum expectation value : v (from the Higgs mechanism, next lecture)

- Can be extracted from three measurements. Choose the three most precise:
  - The electric charge, e
    - measured by the electric dipole moment
  - The Fermi Constant, G_F (precision: 0.9x10^{-5})
    - measured by the muon lifetime
  - The mass of the Z boson, M_Z (precision: 2.3x10^{-5})

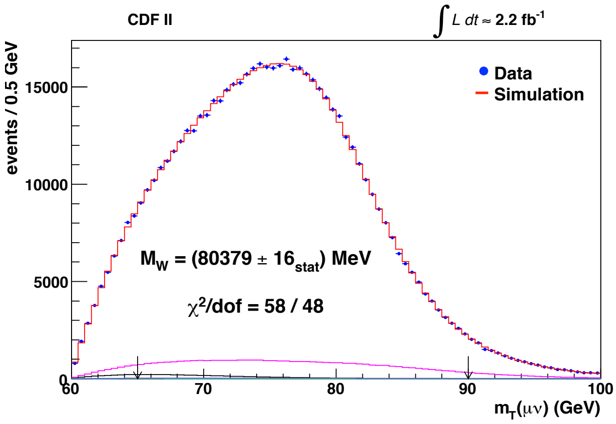
$$e = \frac{g_W\, g'_W}{\sqrt{g_W^2 + g_W'^2}} \qquad M_Z = \frac{1}{2} v \sqrt{g_W^2 + g_W'^2} \qquad G_F = \frac{1}{\sqrt{2}\, v^2}$$

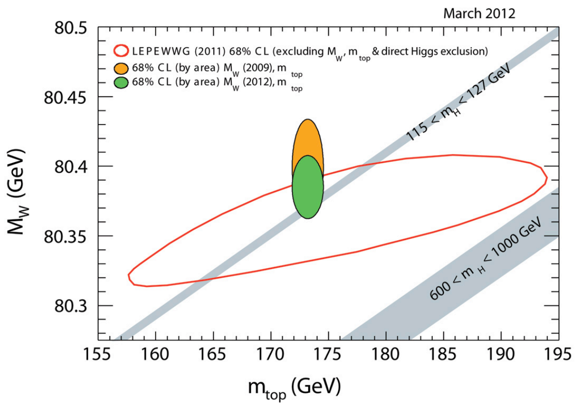
$$\sin^2\theta_W = \frac{g_W'^2}{g_W^2 + g_W'^2} \approx 0.23 \qquad M_W = \frac{v\, g_W}{2}$$

W-boson mass

- New results! from CDF experiment at Tevatron Collider near Chicago.
- Look for decays $W \to \mu\nu$ and $W \to e\nu$

Fit simulated data with different values of m_W. The simulated data that fits best gives actual mass of m_W





$$m_T = \sqrt{E_T(\ell)E_T(\nu)(1-\cos\Delta\phi(\ell,\nu))}$$

Averaging all measurements:

$m_W = 80.395 \pm 0.015$ GeV

Electroweak Precision Data

Very high Q^2 physics at LEP, SLC, and the Tevatron:
More than 1000 measurements with (correlated) uncertainties
Combined to 17 precision electroweak observables

Z boson physics (LEP-1,SLD):
5 Z lineshape and leptonic forward-backward asymmetries
2 Polarised leptonic asymmetries P_τ, $A_{LR(FB)}$
1 Inclusive hadronic charge asymmetry
6 Heavy quark flavour results (Z decays to b and c quarks)

W boson & top quark physics – ongoing at Tevatron's Run-II:
2 W boson mass and width (LEP-2, Tevatron)
1 Top quark mass (Tevatron)

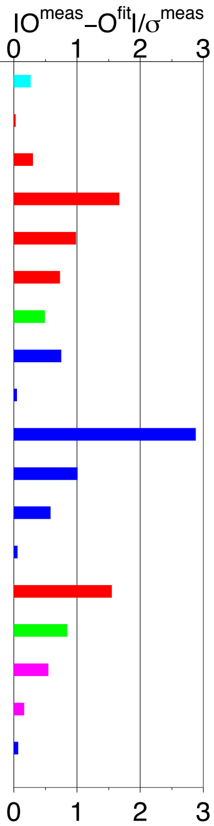
17

Fit the 17 measured values for consistency with single values of g_W, g'_W, v, m_H, m_{top}, m_b ...

Precision Tests of Electroweak Theory

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^{0} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26

March 2012



- From `lepewwg.web.cern.ch`
- Measured value and prediction from fit parameters
- Bars show derivation between measurement and prediction, scaled to error (σ).
- If the measurement is different from the prediction by ±1σ, graph shows 1.
- Three measurements deviate by ≥1σ
- For 17 measurements we expect ~5 to deviate by ≥1σ.
- This fit is also sensitive to the mass of the Higgs boson!

Summary

- Electroweak force couples to the weak isospin and weak hypercharge quantum numbers of the fermions.
  - ➡Fermions with weak isospin couple to three SU(2) bosons with g_W
  - ➡Fermions with weak hypercharge a couple to one U(1) boson with g'_W
- Combined symmetry gives four bosons, with specific properties: W^+, W^-, Z and γ
- The W and Z bosons are massive. $m_W \sim 80$ GeV, $m_Z \sim 91$ GeV
- Many precision measurements made of the properties of W and Z bosons properties and decays.
- All consistent with the Electroweak model described by three parameters: g_W, g'_W, v
- **Friday: the Higgs mechanism!**