Particle Physics (SH/IM)

Spring Semester 2011

Dr Victoria Martin

Lecture Notes

Introductory Material

0.1 Organisation

Teaching Weeks: 16 January - 17 February; 27 February - 6 April

Lectures	**Tuesday**	**12:10-13:00**	JCMB 5215
	Friday	**12:10-13:00**	JCMB 5215
Tutorials:	**Mondays**	**15:00-16:30**	JCMB 3211
Office Hours:	**Tuesdays**	**15:00-17:00**	JCMB 5419

Not all of these slots will be used: No lecture 10 February or 5 April.

Copies of lecture notes and problem sheets will be provided (and posted online). Slides from lectures and solutions to problem sheets will just be posted online:
`http://www2.ph.ed.ac.uk/teaching/course-notes/notes/list/75`

0.2 Synopsis

Particle physics is described by a “Standard Model” which deals with the interactions of the most fundamental constituents of matter: quarks and leptons. However, we believe that the Standard Model is not the complete story. In the first sections of this course the known fundamental particle and their interactions through electromagnetic, weak and strong interactions are discussed, with the emphasis on interpreting experimental observations in terms of underlying gauge theories. The importance of symmetries and selection rules are also emphasised. The later sections of the course provide an overview of the most interesting areas of current research, including neutrinos, matter-antimatter asymmetries, electroweak unification, Higgs bosons, and extensions of the Standard Model. The latest results from the Large Hadron Collider at CERN will be discussed.

0.3 Textbooks

- **Recommended Textbook - I will recommend reading from this textbook.** D. Griffiths - Introduction to Elementary Particles, 2nd edition, Wiley (2008)
  The most up-to-date textbook, covering topics of current interest well. At the level of this course, it includes an introduction to gauge theories.

- B.R. Martin & G. Shaw - Particle Physics, 2nd edition, Wiley (1997)
  A good introductory textbook.

- D.H. Perkins - Introduction to High Energy Physics, 4th edition, Cambridge University Press (2000)
  An update of a classic textbook.

- F. Halzen & A.D. Martin - Quarks and Leptons, Wylie (1984)
  Getting a bit dated, but still a good reference for the Standard Model. Includes some more advanced topics.

- A. Seiden - Particle Physics: A Comprehensive Introduction, Addison-Wesley (2005)
  As it says, rather comprehensive.

- I.J.R. Aitchison & A.J.G. Hey - Gauge Theories in Particle Physics, 2nd edition, Adam Hilger (1989)
  Everything you wanted to know about calculating Feynman diagrams.

- **... and the biennial particle physics bible**

  Particle Data Group (PDG), `http://pdg.lbl.gov` *Compendium of everything that is known about particle physics. Includes good reviews of some topics.*

- CERN websites: for latest news on LHC and LHC physics results
  - `http://public.web.cern.ch/public`
  - `http://atlas.ch/`
  - `http://lhcb.web.cern.ch/lhcb/`
  - `http://cms.web.cern.ch/`
  - `http://www.lhcportal.com`

0.4 Syllabus

These are topics, rather than lectures.

Fundamental: particles & interactions of the Standard Model

1. Introduction: "The mysteries of the Standard Model."
2. Forces & Feynman diagrams.
3. Kinematics & scattering.
4. Dirac equation & spinors.
5. Electromagnetic interactions: Quantum Electrodynamics (QED).
6. Weak Interactions, Weak decays & Neutrino scattering.
7. Deep inelastic scattering, The parton model & Parton density functions.
8. Strong interactions: Quantum Chromodynamics (QCD) and Gluons.
9. Quark model of hadrons. Isospin and Strangeness. Heavy quarks.

Current Topics in Particle Physics

10. Hadron production at Colliders, Fragmentation and jets.
11. Weak decays of hadrons. CKM matrix.
12. Symmetries. Parity. Charge conjugation. Time reversal. CP and CPT.
13. Mixing and CP violation in K and B decays.
14. Neutrino oscillations. MNS matrix. Neutrino masses.
15. Electroweak Theory. W and Z masses. Precision tests at LEP.
16. Spontaneous symmetry breaking. The Higgs boson.
17. Beyond the Standard Model. Supersymmetry. Grand unification.
18. Recent physics results at the LHC.

1 Mysteries of the Standard Model

See the lecture slides for the real mysteries and highlights of 2011 particle physics. Below are the examinable parts of the lecture.

1.1 Review: Quantum Mechanical Spin

(See JH Quantum Mechanics lecture 14)

The total angular momentum (J) of a quantum state is composed of the orbital angular momentum (L) and the intrinsic angular momentum or **spin** (S). Spin is an internal quantum number of the system and cannot be removed! Two quantum numbers describe the spin of the state: the total spin (operator $\hat{S}^2$) and the third component of the spin (operator $\hat{S}_z$). Total spin has eigenvalues $s(s+1)\hbar$, where $s = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$ and the third component has eigenvalues $m_s\hbar$ where m_s runs between s and $-s$ in integer steps.

All elementary (and composite) particles can be thought of as quantum states with an intrinsic spin. Particles with half-integer unit values of s are known as **fermions**; particles with integer unit values of s are known as **bosons**.

For example:

- Electrons have $s = 1/2$, we say “spin-one-half”. m_s can be either $m_s = +1/2$ (“spin-up”) or $m_s = -1/2$ (”spin-down”).
- Photon have $s = 1$, or “spin-one”; $m_s = +1, 0, -1$, referring to the three different polarisation states of the photon.

The notation used in this course is capital S is used for total spin, corresponding the s in the total spin eigenvalue $s(s+1)\hbar$.

1.2 The Elementary Fermions with Spin, $S = 1/2$

In the Standard Model all the elementary matter particles are observed to be fermions with $S = 1/2$. We classify them into two types: **leptons** and **quarks**. See tables 1 and 2.

Notes on the fundamental particles:

- Stable matter is made up of the lightest charged fermions: e^-, u and d. The u and d are bound into protons and neutrons (and eventually nuclei) by the strong force.
- The three charged leptons – e, μ, τ – are sometimes denoted as $\ell = e, \mu, \tau$.
- The three neutrinos – ν_e, ν_μ, ν_τ – are sometimes denoted as ν_i, where i runs from 1 to 3.

Lepton	Symbol	Charge	Mass	Lepton Family Number		
		e	MeV/c^2	L_e	L_μ	L_τ
Electron Neutrino	ν_e	0	$< 2 \times 10^{-6}$	+1	0	0
Electron	e^-	-1	0.5110	+1	0	0
Muon Neutrino	ν_μ	0	< 0.19	0	+1	0
Muon	μ^-	-1	105.66	0	+1	0
Tau Neutrino	ν_τ	0	< 18	0	0	+1
Tau	τ^-	-1	1777	0	0	+1

Table 1: The three charged leptons and three neutrinos. Their antiparticles, the positron e^+, μ^+, τ^+, $\bar{\nu}_e$, $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$, have equal mass and spin, but opposite charge and lepton number. You do not have to remember the mass of the leptons.

Quark Flavour	Symbol	Charge e	Mass MeV/c^2	Isospin (I, I_z)	Quark flavour	Baryon Number $\mathcal{B}$
up	u	+2/3	1.7 - 3.3	$(1/2, +1/2)$	-	1/3
down	d	$-1/3$	4.1 - 5.8	$(1/2, -1/2)$	-	1/3
charm	c	+2/3	1180 - 1340	-	$C = +1$	1/3
strange	s	$-1/3$	80 - 130	-	$S = -1$	1/3
top	t	+2/3	$(172.9 \pm 1.5) \times 10^3$	-	$T = +1$	1/3
bottom	b	$-1/3$	4130 - 4370	-	$B = -1$	1/3

Table 2: The three up-type and three down-type quarks, each with three possible colours, r(ed), b(lue) or g(reen). The antiquarks ($\bar{\mathrm{u}}$, $\bar{\mathrm{d}}$, $\bar{\mathrm{c}}$, $\bar{\mathrm{s}}$, $\bar{\mathrm{t}}$, $\bar{\mathrm{b}}$) have equal mass and spin, but opposite charge, colour ($\bar{r}$, $\bar{b}$, $\bar{g}$), flavour and baryon number. You do not have to remember the mass of the quarks; but it's useful to remember the top quark is very heavy. *Non-examinable comment for those interested: The masses are defined in the* $\overline{\mathrm{MS}}$ *scheme; uncertainties are due to strong interaction effects.*

- The quarks with charge $Q = +2/3e$ – up, charm and top – are collectively known as up-type quarks. They are sometimes denoted as $\mathrm{u_i}$, where i runs from 1 to 3.

- The quarks with charge $Q = -1/3e$ – down, strange and bottom – are collectively known as down-type quarks. They are sometimes denotes as $\mathrm{d_i}$, where i runs from 1 to 3.

- Every quark carries a colour-charge: red, green or blue. It is more correct to consider there to be $3 \times 6 = 18$ fundamental quarks in the Standard Model: a red-charged up quark (u_r), a blue-charged up quark (u_b), a green-charged up quark (u_g), and similarly for the other quark flavours ($\mathrm{d}_r, \mathrm{d}_b, \mathrm{d}_g, \mathrm{s}_r, \mathrm{s}_b, \mathrm{s}_g, \mathrm{c}_r, \mathrm{c}_b, \mathrm{c}_g, \mathrm{b}_r, \mathrm{b}_b, \mathrm{b}_g, \mathrm{t}_r, \mathrm{t}_b, \mathrm{t}_g$).

- It is observed that there is almost no anitmatter in the universe (which is one of the mysteries of the Standard Model).

1.3 The Fundamental Interactions

Table 3 lists the forces of nature, and the properties of the bosons that transmit the force.

Interaction	Coupling Strength	Couples To	Symmetry Group	Gauge Bosons	Charge e	Mass GeV$/c^2$
Strong	$\alpha_s \approx 1$	colour-charge	SU(3)	Gluons (g)	0	0
Electromagnetic	$\alpha = 1/137$	electric charge	U(1)	Photon (γ)	0	0
Weak	$G_F = 1 \times 10^{-5}$	weak hyercharge	$\mathrm{SU(2)_L}$	$W^\pm$	± 1	80.4
				Z^0	0	91.2
Gravity	0.53×10^{-38}	mass		Graviton	0	0

Table 3: The fundamental interactions with their associated couplings, gauge symmetries, and exchanged bosons. For an exam, I would not expect you to remember the exact strength of the interactions, symmetry groups and exact masses of the W and Z boson.

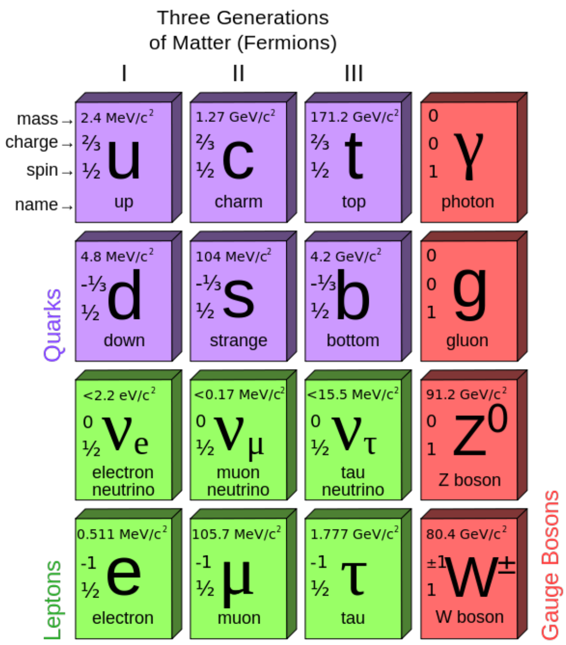

Figure 1.1: The Particles of the Standard Model

2 The Forces of the Standard Model

In this section we review the interactions between the Standard Model fermions and bosons.

2.1 Natural Units

The SI units of mass (kg), length (m) and time (s) are all defined at the "human" scale for convenience.

Unit	Particle Physics	Conversion to SI
Mass	MeV/c^2 or GeV/c^2	1 MeV = 1.6×10^{-13} J
Length	Fermi (fm)	1 fm = 10^{-15}m
Time	μs, ns, ps	10^{-6}, 10^{-9}, 10^{-12}s

Table 4: Particle physics units and their SI values

It is conventional to use $\hbar = c = 1$ in particle physics, and then substitute factors of $\hbar$ and c where required by using dimensional analysis, if an answer is required in SI or particle physics units.

To convert to particle physics units or SI units use:

$$\hbar = 6.58 \times 10^{-22}\ \text{MeV} \cdot \text{s} = 1.05 \times 10^{-34}\ \text{J} \cdot \text{s} \tag{2.1}$$

$$c = 3 \times 10^{8}\text{ms}^{-1} \qquad \hbar c = 197\ \text{MeV fm} \tag{2.2}$$

Cross sections are measured in barns (b), or more usually nb, fb or pb.

$$1\ \text{b} = 10^{-28}\ \text{m}^2 \qquad (\hbar c)^2 = 0.389\ \text{GeV}^2\ \text{mb} \tag{2.3}$$

2.2 Quantum Electrodynamics

Quantum Electrodynamics (QED) is the relativistic quantum mechanical description of the electromagnetic force. In QED the fundamental interaction is the absorption or emission of a photon from a charged fermion, as shown in top left hand of figure 2.1. This is often referred to as the fundamental vertex of QED. The term vertex here refers to more than two particles meeting at the same position in space-time.

The strength of this interaction, or equivalently the probability per unit time for the interaction to happen, is proportional to the charge of interacting fermion. For an electron the strength of the interaction would be e.

The electron charge e can be defined as a dimensionless quantity using the fine structure constant, α:

$$\alpha = \frac{e^2}{(4\pi\epsilon_0)\hbar c} = \frac{e^2}{4\pi} \approx \frac{1}{137} \tag{2.4}$$

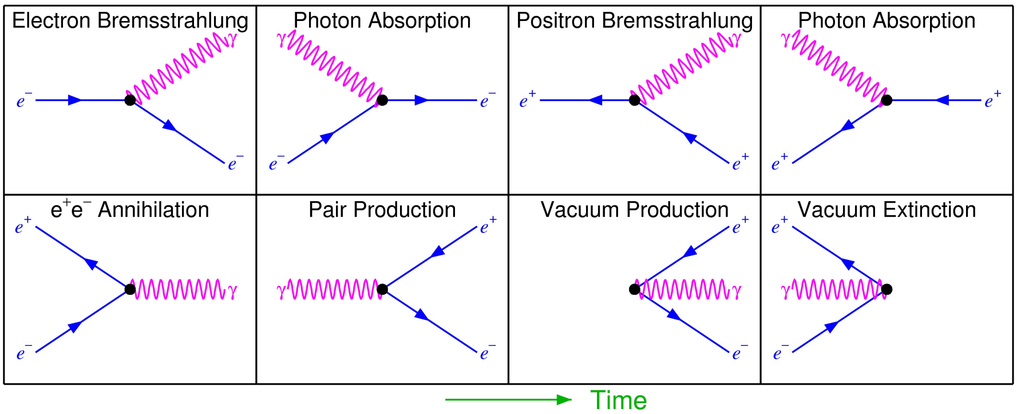

Figure 2.1: The fundamental QED interaction vertex in various time configurations. The electron is used as a example fermion, all charged fermions can interact with a photon in this way.

where the middle equality uses natural units. For an electron, we write $\sqrt{\alpha}$ at the vertex to indicate the strength of the interaction. For a quark vertex, the strength of the interaction is modified to $2/3\sqrt{\alpha}$ or $1/3\sqrt{\alpha}$ (as appropriate).

As space and time are equivalent, the fundamental vertex can occur in any orientation: you can move the lines to be in any orientation, so long as they continue to meet at one point. This is illustrated in figure 2.1.

A few notes:

- Although we have used electrons to illustrate the vertex, this can happen to any fundamental charged fermion.
- As we will see in chapter 5, if a fermion appears to be travelling back in time, it represents the equivalent anti-fermion, e.g. a positron (e^+) in the case of an electron.
- Four-momentum, electric charge, and fermion flavour are conserved at the QED vertex.

2.3 Quantum Chromodynamics

Quantum Chromodynamics (QCD) is the relativistic quantum mechanical description of the strong force. Similar to QED, the fundamental interaction is the absorption or emission of a gluon from a colour-charged fermion, as shown in figure 2.2. As quarks are the only fermions which carry colour-charge, quarks are the only fermions which interact due to the strong force. Leptons do not feel the strong force.

Gluons exchange colour-charge between the quarks. For example a red-coloured up quark may transition into a green-coloured up quark if it emits a gluon: $\mathrm{u}_r \rightarrow \mathrm{u}_g \mathrm{g}_{(r,\bar{g})}$. To conserve colour-charge the gluon must be bi-coloured, in this example the gluon is red and anti-green, $(r, \bar{g})$.

Symmetry considerations tell us there are eight distinct kinds of gluon, each carrying a different colour-charge.

The strength of the gluon interactions with any of the quarks is equal. By analogy with QED, we the strength of the coupling at the gluon-quark vertex as $\sqrt{\alpha_s}$.

Four-momentum, electric charge, colour-charge and fermion flavour are also conserved at the QCD vertex.

2.3.1 Gluon self-interactions

As gluons themselves are colour-charged they can also interact due to the strong force. As illustrated in figure 2.2 underlying symmetry considerations imply that the only two allowed vertices are three gluons meeting at a point or four gluon meeting at a point. Colour-charge will be conserved at these verticies.

Gluon self-interactions do not have an equivalent in QED as the photon does not carry electric charge.

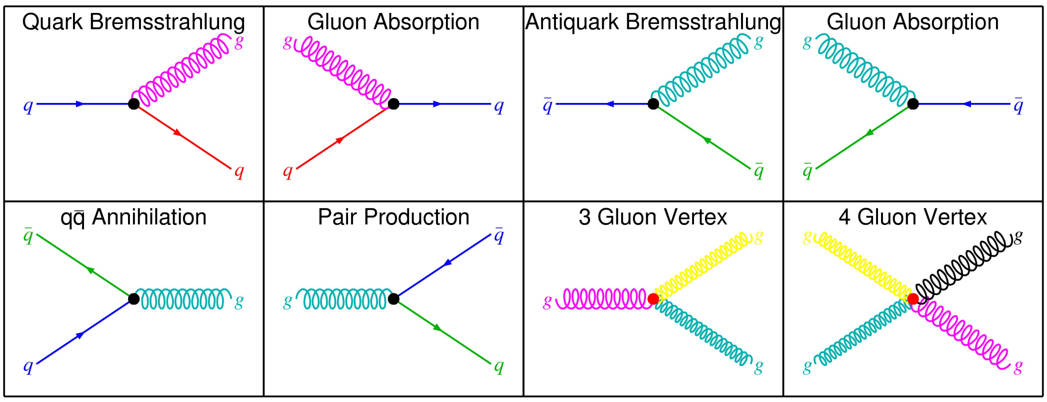

Figure 2.2: The fundamental QCD interaction vertices, in various time configuration, plus the two gluon self-interaction vertices. q represents any quark.

2.4 The Weak Force

The weak force is sometimes refereed to as quantum flavour dynamics (QFD), in analogy with QED and QCD. The weak force describes the interactions of the massive $W^{\pm}$ and Z^0 bosons. The $W^{\pm}$ and Z^0 boson both interact with all flavours of quarks and all

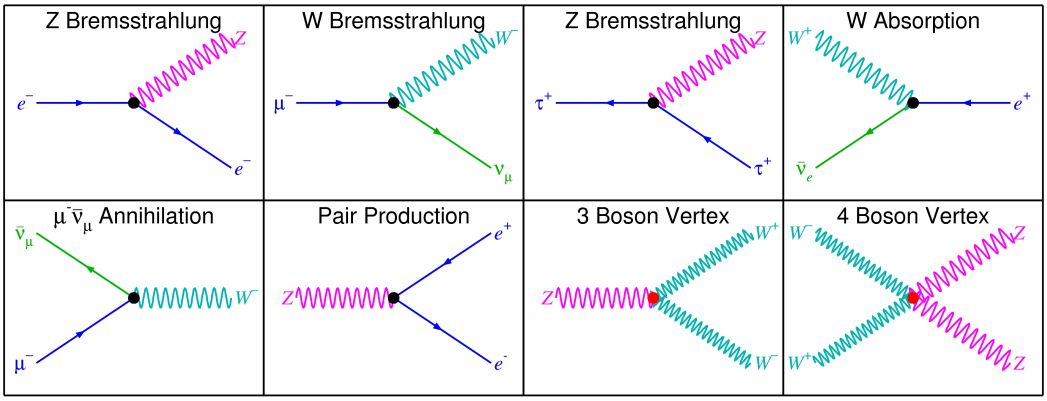

Figure 2.3: Examples of interactions of leptons with W and Z bosons in the Standard Model, plus the two vertices describing the allowed interactions between the W and Z bosons. For completeness, there is a further fundamental vertex of four W bosons meeting at a point.

flavours of leptons. Various examples of W and Z boson verticies are illustrated in figure 2.3.

Fermion interactions with the Z^0 boson do not change the flavour of the fermion. The Z^0 boson is simply emitted or absorbed by the fermion. (We will consider the strength of these interactions later.)

Fermion interactions with the $W^{\pm}$ boson, however, **must** change the flavour of the fermion. For example, to conserve electric charge! Experimental observations tell us which of these fermion flavour changes are allowed. The strength of the W-boson interactions with fermions is proportional to g_W.

2.4.1 Interactions Between Leptons and the W-boson

For leptons, we observe the following flavour changes: $e^- \leftrightarrow \nu_e$, $\mu^- \leftrightarrow \nu_\mu$, $\tau^- \leftrightarrow \nu_\tau$. We use this observation to motivate the conservation of electron number, L_e, muon number L_μ and tau number L_τ. The probability of each of these transitions at the W-boson vertex is g_W.

2.4.2 Interactions Between Quarks and the W-boson

For quark interactions with the W-boson it is observed that any electric charge conserving transitions are allowed. i.e. any up-type quark ($\mathrm{u_i}$, $Q = +\frac{2}{3}e$) may transition to any down-type quark ($\mathrm{d_j}$, $Q = -\frac{1}{3}e$): $\mathrm{u}_i \leftrightarrow \mathrm{d}_j$. There are nine allowed quark transitions:

$\mathrm{u}_i \leftrightarrow \mathrm{d}_j$:

$$\begin{array}{ccc} \mathrm{u} \leftrightarrow \mathrm{d} & u \leftrightarrow s & u \leftrightarrow b \\ c \leftrightarrow d & c \leftrightarrow s & c \leftrightarrow b \\ t \leftrightarrow d & t \leftrightarrow s & t \leftrightarrow b \end{array} \tag{2.5}$$

We use this observation to motivate the preservation of baryon number, $\mathcal{B}$.

Experimentally we observe these nine allowed transitions is not equally likely. We write this as a modified coupling at the W-boson vertex: e.g. $V_{\mathrm{ud}} g_W$. The observed values of the V constants are:

$$\begin{pmatrix} V_{\mathrm{ud}} = 0.974 & V_{\mathrm{us}} = 0.227 & V_{\mathrm{ub}} = 0.004 \\ V_{\mathrm{cd}} = 0.230 & V_{\mathrm{cs}} = 0.972 & V_{\mathrm{cb}} = 0.042 \\ V_{\mathrm{td}} = 0.008 & V_{\mathrm{ts}} = 0.041 & V_{\mathrm{tb}} = 0.999 \end{pmatrix} \tag{2.6}$$

This is the CKM matrix. More about this later.

There are no observed flavour changing quarks into leptons or vice versa.

2.4.3 Weak boson interactions

For completeness we should note that the W and Z bosons can self-interact just like the gluons. The interactions vertices between W and Z are shown in figure 2.3. There is further vertex with four W-bosons meeting at a point.

The $W^{\pm}$ bosons have an electric charge, which means they also feel the electromagnetic force. From underlying symmetry considerations, we find there are two possible vertices: $WW\gamma\gamma$ and $WW\gamma$.

3 Measurements in Particle Physics

In this section we discuss the measurements that can be made in experimental particle physics: decays and scattering. The terms scattering and collisions are used interchangeably.

We start with a review of relativistic kinematics, particularly as applied to scattering processes and then introduce the measurements that can be made of decays and scattering.

3.1 Relativistic Kinematics (review)

A revision of material from Dynamics & Relativity.

In this course, four-vectors are denoted as $a^\mu = (a^0, a^1, a^2, a^3) = (a^0, \vec{a})$. For example, the four-vector for spacetime: is $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z) = (ct, \vec{x})$. Four-momentum is $p^\mu = (E/c, \vec{p})$.

The scalar (or inner) product of two four-vectors are Lorentz-invariant quantities and give the same answer independent of the frame.

The simplified notation for four-vector products is:

$$a \cdot b = a_\mu b^\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 = a^0 b^0 - \vec{a} \cdot \vec{b} \tag{3.1}$$

Note minus sign on the 1st, 2nd and 3rd components. This might be familiar to some of you as:

$$a \cdot b = a_\mu b^\mu = g_{\mu\nu} a^\nu b^\mu \tag{3.2}$$

where $g_{\mu\nu}$ is the metric tensor:

$$g_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{3.3}$$

3.1.1 Invariant Mass

One important quantity is the 'Lorentz invariant mass' of a particle. It is defined as the scalar product of the four-momentum of the particle with itself:

$$p \cdot p = p^2 = \frac{E^2}{c^2} - |\vec{p}|^2 = m^2 c^2 \tag{3.4}$$

3.1.2 Lorentz Transformations

The Lorentz transformations from a frame S to a frame S', moving with constant velocity $(v, 0, 0)$ respect to frame S are given by:

$$\begin{aligned} x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ ct' &= \gamma(ct - \beta x) \end{aligned} \tag{3.5}$$

Where β and γ are defined as:

$$\beta = \frac{v}{c} = \frac{|\vec{p}|c}{E} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{E}{mc^2} \tag{3.6}$$

In natural units the dimensionless quantity $\beta = v$ i.e. speed is always measured relative to the speed of light and $\gamma = E/m$.

- In the non-relativistic (classical) limit:

$$|\vec{p}| \ll mc \qquad E = mc^2 + \frac{1}{2}mv^2 \qquad \beta \to 0 \qquad \gamma \to 1 \tag{3.7}$$

- Highly relativistic limit (massless or very high energy particles):

$$mc^2 \ll E \qquad E = |\vec{p}|c \qquad \beta \to 1 \qquad \gamma \to \infty \tag{3.8}$$

3.1.3 Collision Kinematics

Notes on notation:

- CM refers to the centre of mass, or centre of momentum, frame.
- A star on a quantity means it is evaluated in the CM frame, e.g. $\vec{p}^*, \theta^*, E^*$.
- An arrow above a momentum indicates that it is three momentum, $\vec{p}$.
- No arrow, or a μ sub- or superscript, indicates that it is a four-momentum, p, p^μ.

Consider a collision $1+2 \to 3+4$ with particle four-momenta p_1, p_2, p_3, p_4 as illustrated in figure 4.2.

The Lab frame is defined by:

$$\vec{p}_1 = p_{\text{beam}} \qquad \vec{p}_2 = 0 \tag{3.9}$$

The CM frame is defined by:

$$\vec{p}_1 = -\vec{p}_2 = \vec{p}_i^{\,*} \qquad \vec{p}_3 = -\vec{p}_4 = \vec{p}_f^{\,*} \tag{3.10}$$

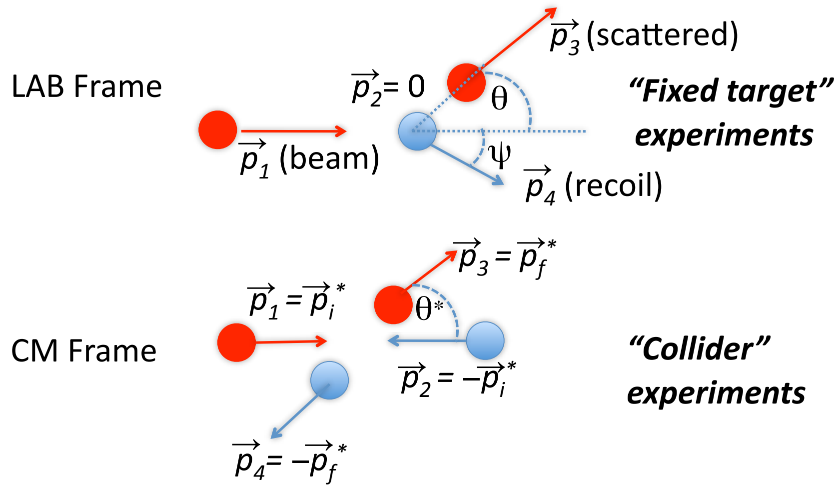

Figure 3.1: A particle collision $1 + 2 \to 3 + 4$ as seen in the Lab and Centre of Mass frames.

- For an elastic collision (with no loss of kinetic energy) in the CM frame:

$$m_1 = m_3 \qquad m_2 = m_4 \qquad \vec{p}_i = \vec{p}_f = \vec{p}^* \tag{3.11}$$

- The Lorentz transformation from Lab to CM frame:

$$\beta == \frac{p^*}{E_2^*} = \frac{p_{\text{beam}}}{(E_{\text{beam}} + m_2)} \tag{3.12}$$

- The four momentum transfer in the collision is:

$$q = p_1 - p_3 = p_4 - p_2. \tag{3.13}$$

  $q^2 = (p_1 - p_3)^2 = (p_4 - p_2)^2$ is, like all squares of four vectors, a Lorentz invariant quantity. This is an important quantity in many calculations.

3.1.4 Mandelstam variables

A particularly useful set of Lorentz invariant quantities for describing collisions are the Mandelstam variables, s, t, u. The total CM energy squared is:

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2 = 4p^{*2} \tag{3.14}$$

The usual four momentum transfer squared is:

$$t = (p_1 - p_3)^2 = (p_4 - p_2)^2 = 2p^{*2}(1 - \cos\theta^*) \tag{3.15}$$

and there is one further four momentum transfer squared:

$$u = (p_1 - p_4)^2 = (p_3 - p_2)^2 = 2p^{*2}(1 + \cos\theta^*) \tag{3.16}$$

where θ^* is the CM scattering angle, and we have taken the highly relativistic limit $p^* = E^*$ for all particles (and $c = 1$).

Citation: K. Nakamura *et al.* (Particle Data Group), JP G **37**, 075021 (2010) and 2011 partial update for the 2012 edition (URL: http://pdg.lbl.gov)

K_S^0 DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
		Hadronic modes		
$\pi^0\pi^0$		(30.69 ± 0.05) %		209
$\pi^+\pi^-$		(69.20 ± 0.05) %		206
$\pi^+\pi^-\pi^0$		$(3.5\ ^{+1.1}_{-0.9}) \times 10^{-7}$		133
		Modes with photons or $\ell\overline{\ell}$ pairs		
$\pi^+\pi^-\gamma$	[*f,m*]	$(1.79\pm0.05) \times 10^{-3}$		206
$\pi^+\pi^-e^+e^-$		$(4.79\pm0.15) \times 10^{-5}$		206
$\pi^0\gamma\gamma$	[*m*]	$(4.9\ \pm1.8\ ) \times 10^{-8}$		231
$\gamma\gamma$		$(2.63\pm0.17) \times 10^{-6}$	S=3.0	249
		Semileptonic modes		
$\pi^\pm e^\mp \nu_e$	[*n*]	$(7.04\pm0.08) \times 10^{-4}$		229

Figure 3.2: The measured decay modes and branching ratios of the K_S^0 meson. The main decay modes as $K_S^0 \rightarrow \pi^+\pi^-$ with a branching ratio of 30.69%, and $K_S^0 \rightarrow \pi^0\pi^0$ with a branching ratio of 69.20%

3.2 Particle Decays

The following properties of particle decay can be measured experimentally:

- The **decay rate** of a particle in its own rest frame Γ is defined as the probability per unit time the particle will decay: $dN = -\Gamma N dt \Rightarrow N(t) = N(0)e^{-\Gamma t}$.
- Most particles decay more than one different route. Add up all decay rates to obtain the total decay rate: $\Gamma_{\text{tot}} = \sum_{i=1}^{n} \Gamma_i$.
- The **lifetime** $\tau = \hbar/\Gamma_{\text{tot}}$, is measured in units of time. In natural units: $\tau = 1/\Gamma_{\text{tot}}$.
- The final states of the particle decays are the **decay modes**.
- How often the particle decays into a given decay mode, is known as the **branching ratio** (or branching fraction), $\Gamma_i/\Gamma_{\text{tot}}$. It is often is often measured in %.
- The sum of all branching ratios of a given particle will sum to 1.

An example of some of these meausrements are given in figure 3.2.

3.3 Collisions

For particle collisions we measure the **cross section** of the process. The cross section is a measure of how often a process happens per unit of incident flux. It has dimensions of area, and is measured often measured in **barn, b**. $1\text{ b} \equiv 10^{-28}\text{ m}^2$.

At the LHC typical cross sections rate from 0.1 b for the inclusive cross section to 0.1 pb for Higgs boson production.

A few notes:

- An **elastic collision** (or elastic scattering) occurs when the initial and final state particles are the same. Kinetic energy and three-momentum is conserved.
- An **inelastic collision** (or inelastic scattering) occurs when the initial and final state particles are different. Kinetic energy and three-momentum will not be conserved. (However four-momentum will be conserved!)
- The **exclusive cross section** is one with a given final state. e.g. at the LHC we are interested in measuring how often Higgs bosons are created along with W-bosons, so we would like to measure $\sigma(pp \to WH)$. Exclusive cross sections are easier to calculate.
- The **inclusive cross section** is sum of all possible exclusive cross sections for a given initial state. e.g. $\sigma(pp \to \text{anything})$. Inclusive cross sections are often easier to measure, as experimentalists do not have to identify which kind of particles were produced in the collision.
- Cross sections are measured experimentally by simply counting the number of times that a process occurs:

  $$N(pp \to WH) = \sigma(pp \to WH) \times \int \mathcal{L} dt \tag{3.17}$$

  where $\mathcal{L}$ is the luminosity of the colliding beams, and $\int \mathcal{L} dt$ is the integrated luminosity for the period of time the experiment has run for.
- Luminosity, also known as instantaneous luminosity, can be (roughly) defined as:

  $$\mathcal{L} = \frac{N_a \times N_b \times \text{collision frequency}}{\text{Overlap Area}} \tag{3.18}$$

  Where N_a and N_b are the numbers of particle in the overlap area in colliding bunches a and b. Luminosity is measured in dimensions of inverse area per time unit. At the LHC luminosity is often quoted in units 10^{32} cm^{-2}s^{-1}, equivalent to 1 inverse barn per second. Integrated luminosity is measured in units of b^{-1}. At the LHC is it often quoted in units of inverse femptobarns, fb^{-1}.

4 Feynman Diagrams and Fermi's Golden Rule

This section describes how we can calculate the observable quantities of cross section, σ, and decay width, Γ, using Feynman diagrams and Fermi's Golden Rule.

4.1 Fermi's Golden Rule

Fermi's Golden Rule tells us the transition probability for an initial state i to a final state f, $T_{i\to f}$:

$$T_{i\to f} = \frac{2\pi}{\hbar}|\mathcal{M}|^2\rho \tag{4.1}$$

Where:

- $\mathcal{M}$ is known as the amplitude of the **matrix element** of the process. It contains the dynamics of the process. It can be calculated (to a given order in perturbation theory) from Feynman diagrams.
- ρ is the available **phase space** or density of final states. It contains the kinematic constraints.

Transition rate $T_{i\to f}$ is related to decay rates Γ and and cross section σ:

$$\Gamma = \frac{2\pi}{\hbar}|\mathcal{M}|^2\rho \qquad \sigma = \frac{2\pi}{\hbar}|\mathcal{M}|^2\frac{\rho}{f_i} \tag{4.2}$$

where f_i is the incident flux of the colliding particles.

4.2 Feynman Diagrams

A Feynman diagram is a *pictorial* representation of an interaction in which fermions couple to bosons. The sum of *all possible* diagrams is used to calculate the matrix amplitude $\mathcal{M}$ for a particle physics interaction. Note that it is often sufficient to consider only the *lowest order* diagrams with the fewest vertices.

We saw the fundamental electromagnetic interaction vertex in figure 2.1. All other electromagnetic processes are built up from such basic diagrams. Figure 4.1 shows the lowest order Feynman diagrams for electron-electron scattering $e^-e^- \to e^-e^-$, and electron-positron annihilation into muon pairs $e^+e^- \to \mu^+\mu^-$.

4.2.1 Rules for Feynman Diagrams

- Initial state particles enter from the left.
- Final state particles exit to the right.

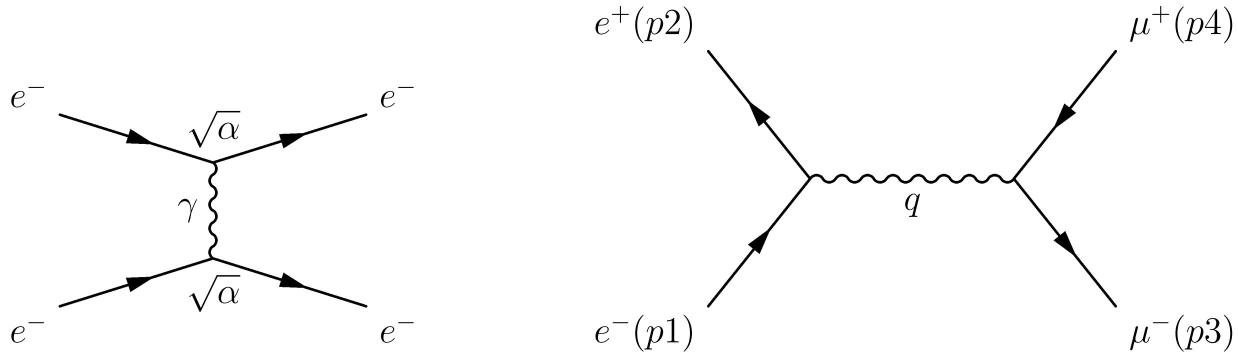

Figure 4.1: Feynman diagrams for $e^-e^- \to e^-e^-$ and $e^+e^- \to \mu^+\mu^-$.

- Initial and final state particles have wavefunctions associated with them. Wavefunctions contain information about four momenta and spin states. We use the current associated with that wavefunction.
  - Spin-0 bosons have **plane wavefunctions**.
  - Spin-$\frac{1}{2}$ fermions have wavefunctions known as **spinors**.
  - Spin-1 bosons have wavefunctions are known as **polarization vectors**.
- Four momentum is conserved at each vertex.
- Fermions are solid lines labelled with arrows pointing to the right. Antifermions have arrows pointing to the left.
- Lepton and baryon number are conserved by having the same number of arrows going into a vertex as come out of it.
- Photons are represented by wavy lines, gluons by springs, and heavy bosons by dashed or wavy lines.
- An electromagnetic vertex has a dimensionless coupling strength $\sqrt{\alpha}$.
- A strong interaction vertex has coupling strength $\sqrt{\alpha_s} = g_s$.
- Weak interactions have vector (c_V) and axial-vector (c_A) couplings, or alternatively left and right-handed couplings (g_L and g_R). The weak charged couplings via $W^\pm$ are purely left-handed or $V - A$.
- A line connecting two vertices represents a *virtual particle* which cannot be observed.
- Virtual photons have *propagators* proportional to $1/q^2$, where q is the four-momentum transfer between the vertices.
- Heavy bosons with mass m have propagators $1/(q^2 - m^2)$, where m is the mass of the boson.

- There can be virtual fermions with propagators $(\gamma^\mu q_\mu + m)/(q^2 - m^2)$, where m is the fermion mass and γ^μ are 4×4 matrices (see Lecture 5).
- Virtual particles are said to be "off their mass shell", because $q^2 \neq m^2$.

To evaluate the matrix element $\mathcal{M}$ write down

1. the current associated with each of the fermion lines,
2. the propagators associated with each of the bosons,
3. the coupling strengths at each of the interaction vertices.

$\mathcal{M}$ is the product of these terms. Additionally you may have to calculate the four-momentum transfer q and impose four momentum conservation to evaluate the matrix element.

4.3 Particle Wavefunctions

The free particle wavefunction for a colourless, chargeless spin-0 particle is a plane wave:

$$\psi = Ne^{-ip\cdot x} \qquad p \cdot x = p^\mu x_\mu = \hbar(\vec{k} \cdot \vec{x} - \omega t). \tag{4.3}$$

The probability density is time-varying:

$$\rho = i\left(\psi^* \frac{\partial \psi}{\partial t} - \frac{\partial \psi^*}{\partial t}\psi\right) \tag{4.4}$$

but can be integrated over a box of volume V to determine the normalisation of the wavefunction N:

$$\int_V \rho \, d^3x = 2E \qquad N = \frac{1}{\sqrt{V}} \tag{4.5}$$

It is conventional to choose $V = N = 1$. When calculating a physical observable such as a cross section it can be shown that the result does not depend on the choice of V.

The probability density can be generalised to a four-vector current:

$$j_\mu = i\left(\psi^* \partial_\mu \psi - \partial_\mu \psi^* \psi\right) \tag{4.6}$$

To calculate Feynman diagrams, we use the four-vector current j_μ.

For a spin-0 particle which changes four momentum from p_1 to p_3 (e.g. due to an interaction with a boson), the four-vector current can be written as:

$$j_\mu = (p_1 + p_3)e^{-i(p_3 - p_1)\cdot x} \tag{4.7}$$

4.4 Cross Sections

The **cross section**, σ, is a measurement of the effective area of the target particle in a scattering experiment.

In your Quantum Mechanics course (lectures 11, 12, 13), the cross section was defined in terms of the flux of the incident and scattered particles. The incident flux is the number of particles per unit area per unit time. The scattered flux is number of particles per unit time scattered into solid angle $d\Omega$.

$$\frac{d\sigma}{d\Omega} \equiv \frac{\text{scattered flux}}{\text{incident flux}} \tag{4.8}$$

This can be integrated over the full solid angle to give the Lorentz invariant cross section:

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega \tag{4.9}$$

4.5 Phase Space for Decay

See Griffiths section 6.2.1 for the full gory detail.

For a decay $1 \to 2 + 3 + ... + n$, the decay rate (equation 4.2) can be written as:

$$\Gamma = \frac{S}{2\hbar m_1} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_1 - p_2 - p_3 \cdots - p_n) \times \prod_{j=2}^{n} 2\pi\delta(p_j^2 - m_j^2c^2)\theta(p_j^0)\frac{d^4p_j}{(2\pi)^4} \tag{4.10}$$

Please to not attempt to memorise this! I just want to motivate the compontents as follows:

- S: statistical factor of $1/s!$ to account for s identical particles in final state.
- $\delta(p_j^2 - m_j^2c^2)$: to enforce that final state particles are real: $p_j^2 = m_j^2c^2$.
- $\theta(p_j^0)$: to enforce that final state particles have positive energy
- $\delta^4(p_1 - p_2 - p_3 - p_n)$ ensures four momentum conservation
- d^4p_j: integrate over all outgoing momenta
- 2π: every δ introduces a factor of 2π; every derivative gets $1/2\pi$.

In the case of a two body decay $1 \to 2 + 3$, the phase space integral can be solved without knowing the details of $|\mathcal{M}|^2$ giving:

$$\Gamma = \frac{S}{32\pi^2\hbar m_1} \int |\mathcal{M}|^2 \frac{\delta^4(p_1 - p_2 - p_3)}{\sqrt{\vec{p}_2^{\,2} + m_2^2c^2}\sqrt{\vec{p}_3^{\,2} + m_3^2c^2}} d^3\vec{p}_2\, d^3\vec{p}_3 \tag{4.11}$$

After further algebra and integration by parts (see Griffiths section 6.2.1.1) this becomes:

$$\Gamma = \frac{S|\vec{p}^*|}{8\pi\hbar m_1^2 c}|\mathcal{M}|^2 \tag{4.12}$$

where $|\vec{p}^*|$ is the outgoing momentum in the rest frame of particle 1:

$$|\vec{p}^*| = \frac{c}{2m_1}\sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2 - 2m_1^2m_3^2 - 2m_2^2m_3^2} \tag{4.13}$$

4.6 Scattering Phase Space

To apply Fermi's Golden rule to scattering (equation 4.2) we need the incident particle flux.

4.6.1 Incident Flux

The incident flux, f_i of a collision depends on the density of initial states times the relative velocity. The number of states for particle of energy of E is $2E$ (equation 4.5). For highly relativistic particles in the CM frame f_i is:

$$f_i = (2E_1)(2E_2)(v_1 + v_2) = 4p_i\sqrt{s} \tag{4.14}$$

4.6.2 Phase Space

For a scattering $1+2 \rightarrow 3+4+\ldots+n$, Fermi's Golden rule for scattering (equation 4.2) can be written as:

$$\sigma = \frac{S\,\hbar^2}{f_i}\int |\mathcal{M}|^2(2\pi)^4\delta^4(p_1+p_2-p_3\cdots-p_n) \times \prod_{j=3}^{n} 2\pi\delta(p_j^2 - m_j^2c^2)\theta(p_j^0)\frac{d^4p_j}{(2\pi)^4} \tag{4.15}$$

Please do not attempt to memorise this either! Compared to equation 4.10, we have changed the δ function to $\delta^4(p_1{+}p_2{-}p_3\ldots{-}p_n)$ to ensure four-momentum conservation.

Solving for $1+2 \rightarrow 3+4$ scattering (see Griffiths 6.2.2.1):

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S|\mathcal{M}|^2}{(E_1+E_2)^2}\frac{|\vec{p}_f^{\,*}|}{|\vec{p}_i^{\,*}|} \tag{4.16}$$

where Ω is in the centre of momentum frame. $|\vec{p}_f^{\,*}|$ and $|\vec{p}_i^{\,*}|$ are the magnitude of the initial and final momentum in the CM frame, see figure 4.2.

Substituting natural units, and using the Mandelstram variables s (equation 3.14) gives:

$$\sigma = \frac{S}{64\pi^2 s}\frac{|\vec{p}_f^{\,*}|}{|\vec{p}_i^{\,*}|}\int |\mathcal{M}|^2 d\Omega \tag{4.17}$$

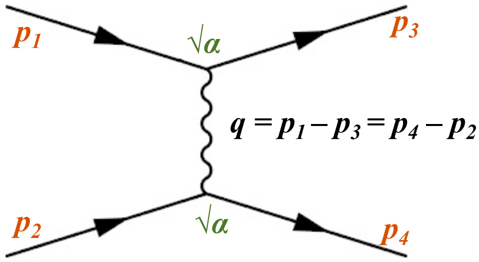

Figure 4.2: Feynman diagram for electromagnetic $1+2 \to 3+4$.

4.7 Spinless Scattering

Let's put together matrix element calculation and the phase space. We only know how to do this so far for spin-0 particles.

Note that there are no charged spinless elementary particles! It could however represent two mesons scattering such as π^+K^+ or $\pi^+\pi^+$ elastic scattering.

We consider electromagnetic scattering between two charged spinless particles $1+2 \to 3+4$ as shown in figure **??**. The matrix element includes two electromagnetic currents, a coupling constant and a virtual photon propagator:

$$\mathcal{M} = j^{13}_{\mu} \frac{\alpha}{q^2} j^{24}_{\mu} \, \delta^4(p_1 + p_2 - p_3 - p_4) \tag{4.18}$$

The plane wavefunction currents are:

$$j^{13}_{\mu} = (p_1 + p_3) e^{i(p_3 - p_1)\cdot x} \qquad j^{24}_{\mu} = (p_2 + p_4) e^{i(p_4 - p_2)\cdot x} \tag{4.19}$$

Therefore:

$$\mathcal{M} = \frac{\alpha}{q^2}(p_1 + p_3)(p_2 + p_4) \tag{4.20}$$

The four momentum squared transferred by the photon is $q^2 = (p_3 - p_1)^2 = (p_4 - p_2)^2 = t$, where t is the Mandelstam variable (equation 3.15).

As a function of the Mandelstam variables we can write:

$$\mathcal{M} = \alpha \frac{(s-u)}{t} \tag{4.21}$$

Substituting the square of the matrix element into equation 4.17, gives:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{64\pi^2 s} \frac{(s-u)^2}{t^2} \tag{4.22}$$

Note that the $1/t^2$ factor gives the characteristic steep dependence of the cross section on the scattering angle θ. The differential cross section goes to infinity as $\theta \to 0$, corresponding to zero four-momentum transfer $t \to 0$, i.e. the limit of no scattering.

The "total" scattering cross section is usually defined as the integral over a limited angular range above a specified θ_{min}.

If we assume the spinless particles are identical, there are two lowest order diagrams which result in the same final state $1+2 \to 3+4$ and $1+2 \to 4+3$. The matrix element is the sum of these:

$$\mathcal{M} = e^2 \left[\frac{(s-u)}{t} + \frac{(s-t)}{u} \right] \qquad \frac{d\sigma}{d\Omega} = \frac{\alpha^2}{64\pi^2 s} \left[\frac{(s-u)}{t} + \frac{(s-t)}{u} \right]^2 \tag{4.23}$$

In this case the matrix element squared contains terms proportional to $1/t^2$, $1/u^2$ and an interference term $1/tu$. The cross section goes to infinity at $\theta \to 0$ and $\theta \to \pi$.