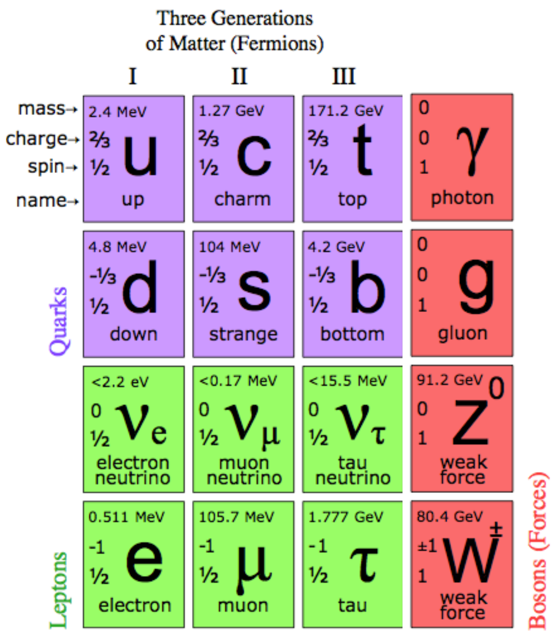
Interactions and Particles

- Standard Model describes the interactions of the known fermions.



- Three forces:
  - Electromagnetic (QED) - exchange of photons, γ, between particles with electric charge
  - Strong (QCD) - exchange of colour-charged gluons, g, between particles with colour charge
  - Weak - exchange of W^+, W^-, Z^0 bosons between particles with weak isospin and hypercharge
- Plus Higgs boson, H, exchange between massive boson and fermions

Lorentz Notation

- μ, ν on quantities are Lorentz indices and run from 0 to 3
- Quantities with repeated Lorentz indices are implicitly summed over. e.g. scalar product of a four vector:

$$p_\mu p^\mu = (p^0)^2 - (p^1)^2 - (p^2)^2 - (p^3)^2$$

- Metric tensor $g_{\mu\nu}$:

$$g_{\mu\nu} = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Scalar product of two four-vectors, implicitly uses the metric tensor:

$$a \cdot b = a_\mu b^\mu = g_{\mu\nu} a^\nu b^\mu = +1 \times (a^0 b^0) - 1 \times (a^1 b^1) - 1 \times (a^2 b^2) - 1 \times (a^3 b^3)$$

- The factors of $+1$ and -1 are due to the metric tensor.

- Objects with two different indices e.g. μ and ν, multiplied by $g_{\mu\nu}$ all the indices to be changed to be the same.

Dirac Equation

- **Dirac Equation** provides linear relationship between energy and momentum, consistent with relativity.
- Use momentum and energy operators and parameters $\gamma^0, \gamma^1, \gamma^2, \gamma^3$ in the relationship:

$$\hat{p} = -i\hbar\vec{\nabla} \qquad \hat{E} = i\hbar\frac{\partial}{\partial t} \qquad i\left(\gamma^0\frac{\partial\psi}{\partial t} + \vec{\gamma}\cdot\vec{\nabla}\right)\psi = m\psi$$

- Equation is solved using making it consistent with:

$$E^2 = \vec{p}^2c^2 + m^2c^4 = \vec{p}^2 + m^2$$

- Solutions for $\gamma^0, \gamma^1, \gamma^2, \gamma^3$ are 4 x 4 matrices with the properties:

$$(\gamma^0)^2 = 1 \qquad (\gamma^1)^2 = -1 \qquad (\gamma^2)^2 = -1 \qquad (\gamma^1)^3 = -1$$

$$\{\gamma^i, \gamma^j\} = \gamma^i\gamma^j + \gamma^j\gamma^i = 0$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

- Also use $\gamma^5 = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3$

Spinors

- In covariant form, Dirac Equation is: $(i\gamma^\mu\partial_\mu - m)\psi = 0$
- Solutions wavefunctions describing the motion of spin-½ particles (quarks and leptons). For a four momentum p^μ

$$\psi = u(p^\mu)e^{-ip\cdot x} \qquad \psi = v(p^\mu)e^{+ip\cdot x}$$

- There u and v terms are known as **spinors**. Spinors have four components and are solutions of:

$$(\gamma^\mu p_\mu - m)u = 0 \qquad (\gamma^\mu p_\mu + m)v = 0$$

- For a given momentum, there are four solutions:
  - spin-up particle
  - spin-down particle
  - spin-up anti-particle
  - spin-down anti-particle
- Four spinor solutions for p=**0** are:

$$u^1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad u^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad v^1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad v^2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Helicity and Handedness

- Helicity, $\hat{h}$ is the component of the spin along a particle's direction of flight.

$$\hat{h} = \frac{\vec{S}\cdot\vec{p}}{|\vec{S}||\vec{p}|} = \frac{2\vec{S}\cdot\vec{p}}{|\vec{p}|}$$

- For a S=½ fermion, eigenvalues of $\hat{h}$ are ±1.
  - h=+1 is "right-handed",
  - h=−1 is "left handed".

- Projection operators can be use to find left-handed and right-handed components of a fermion $P_L = ½\,(1 - \gamma^5)$ $P_R = ½\,(1 + \gamma^5)$

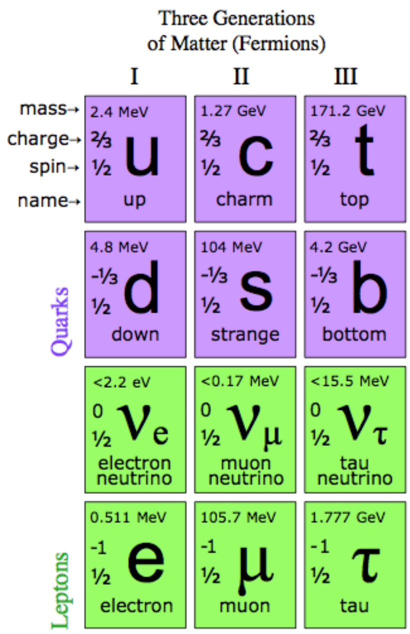
- The four spinor solutions can also be characterised as:
  - Left-handed particle
  - Right-handed particle
  - Left-handed anti-particle
  - Right-handed anti-particle

- All quarks and charged leptons (e, μ, τ) can be left-handed and right-handed.
- Neutrinos are observed to be left-handed only.
- The values for weak isospin are different for left-handed and right-handed fermions.

Standard Model Fermion Charges

- Key quantum numbers: charge (Q), isospin (I_Z), baryon number (B), lepton number (L, L_e, L_μ, L_τ), weak isospin (T_3), hypercharge ($Y = 2\,(Q - T_3)$).
- Quarks also carry colour charge: red, green and blue.
- These are intrinsic charges - cannot be removed

Q Charge	T_3 Weak Isospin (LH)	T_3 Weak Isospin (RH)	Y Hypercharge (LH)	Y Hypercharge (RH)	Three Generations of Matter (Fermions)	B Baryon Number	L Lepton Number
+2/3 e	+1/2	0	+1/2	+4/3	u, c, t	+1/3	0
−1/3 e	−1/2	0	−1/2	−2/3	d, s, b	+1/3	0
0	+1/2	0	−1	0	ν_e, ν_μ, ν_τ	0	+1
−1 e	−1/2	0	−1	−2	e, μ, τ	0	+1



Hadrons: Mesons and Baryons

- At low energy quarks are found in colour-neutral bound states called hadrons.
- **Mesons** are bosons (***S=0,1,...***) consisting of quark and anti-quark. Colour structure of wavefunction:

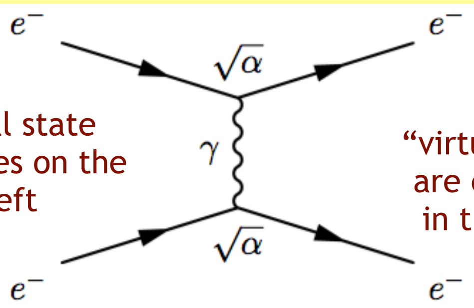
$$\chi_c = \frac{1}{\sqrt{3}}\left[\mathrm{r\bar{r}} + \mathrm{b\bar{b}} + \mathrm{g\bar{g}}\right]$$

- **Baryons** are fermions (***S=½, 3/2, ...***) consisting of three quarks. Colour structure of wavefunction:

$$\chi_c = \frac{1}{\sqrt{6}}\left[\mathrm{rgb} - \mathrm{rbg} + \mathrm{gbr} - \mathrm{grb} + \mathrm{brg} - \mathrm{bgr}\right]$$

- **Anti-baryons** have three anti-quarks
- Baryons wavefunctions must be antisymmetric under exchange of any two fermions e.g. $\psi = \chi_c\, \chi_f\, \chi_S\, \chi_L = \chi_{\text{colour}}\, \chi_{\text{flavour}}\, \chi_{\text{Spin}}\, \chi_{\text{Angular-momentum}}$

Feynman Diagrams



Final state particles on the right

Initial state particles on the left

“virtual” bosons are exchanged in the middle

Each interaction vertex has a coupling constant

Times flows from left to right

fermions | antifermions | photons, ***W, Z*** bosons | gluons | ***H*** bosons

- Feynman diagram illustrate processes to fixed order in perturbation theory.
- Lowest order Feynman diagrams (with the smallest number of bosons exchange) given reasonable estimate for weak and QED processes.
- Feynman rules applied to calculate the matrix element, $\mathcal{M}$, for a given process.

Measuring Scattering and Decays

- Standard Model can be tested by looking at particle scatterings and decays.
- Measure decay rates and scattering cross sections.

Fermi's Golden Rule

- Fermi's Golden Rule relates the transition rate from initial state i to final state f: $T_{i \to f}$
- Transition rates calculated from two quantities:
  - ➡ The **amplitude** or **matrix element** for the process, $\mathcal{M}$
  - ➡ The **available phase space** (density of final states), ρ

$$T_{i \to f} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \rho$$

- $\mathcal{M}$ contains the **dynamics** of the process. It can be calculated (to a given order in perturbation theory) from Feynman diagrams.
- Phase space ρ contains the kinematic constraints.
- Transition rate $T_{i \to f}$ is related to decay rates Γ and cross sections σ

$$\Gamma = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \rho \qquad \sigma = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \frac{\rho}{f_i}$$

with f_i incident flux

decay rate — scattering cross section

Measuring Scattering

- Fire a beam of particles at a target, or another beam.
- The effective area of the interaction is the cross section, **σ.**
- Measured in units of area, usually barn, $1\text{b}=10^{-28}\text{m}^2$
  - Incident flux: the number of particles per unit area per unit
  - Scattered flux: number of particles per unit time scattered into solid angle dΩ
- Measure the differential cross section

$$\frac{d\sigma}{d\Omega} \equiv \frac{\text{scattered flux}}{\text{initial flux}}$$

- Total cross section (Lorentz invariant)

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega$$

- exclusive cross section to given final state: *e.g.* **σ(pp→WH)** or
- inclusive cross section **σ(pp→anything)** sum of all possible exclusive cross sections
- Counting number of event observed e.g. $N(pp\to WH) = \sigma(pp\to WH) \times \int L dt$

Measuring Decays

- Measure the lifetime of a particle in its own rest frame.
- Define the decay rate, **Γ**: the probability per unit time the particle will decay:

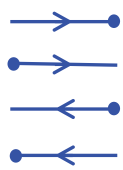
$$dN = -\Gamma N dt \qquad N(t) = N(0)e^{-\Gamma t}$$

- Mean lifetime is $\tau = 1/\Gamma$ (natural units).
  - For τ in seconds can use $\tau = \hbar/\Gamma$
- Most particles decay more than one different route: add up all decay rates to obtain the **total decay rate**: $$\Gamma_{\text{tot}} = \sum_{i=1}^{n} \Gamma_i$$
- The lifetime is the reciprocal of Γ_{tot}: $$\tau = \frac{1}{\Gamma_{\text{tot}}}$$
- The different final states of the particle are known as the **decay modes.**
- The **branching ratio** for the *i*th decay mode is: $\Gamma_i/\Gamma_{\text{tot}}$

Feynman Rules for QED

- **External Lines**

spin 1/2
- incoming particle $u(p)$
- outgoing particle $\overline{u}(p)$
- incoming antiparticle $\overline{v}(p)$
- outgoing antiparticle $v(p)$



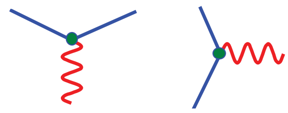
u and v are spinors

- **Internal Lines (propagators)**

spin 1 **photon** $\frac{g_{\mu\nu}}{q^2}$

- **Vertex Factors**

spin 1/2 **fermion (charge -|e|)** $e\gamma^\mu$



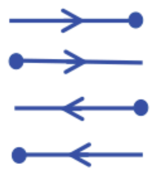
- Matrix element $\mathcal{M}$ is product of all factors

Feynman Rules for QCD

- Gluons propagate QCD, carry colour and anti-colour, described by 8 Gell-Mann matrices, λ.

- **External Lines**

spin 1/2
- incoming quark $u(p)$
- outgoing quark $\overline{u}(p)$
- incoming anti-quark $\overline{v}(p)$
- outgoing anti-quark $v(p)$



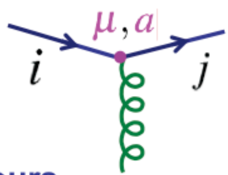
- **Internal Lines (propagators)**

spin 1 gluon $\boxed{\frac{g_{\mu\nu}}{q^2}\delta^{ab}}$

a, b = 1,2,…,8 are gluon colour indices

- **Vertex Factors**

spin 1/2 quark $\boxed{g_s \frac{1}{2}\lambda^a_{ji}\gamma^\mu}$

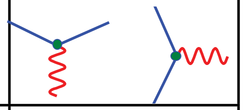


i, j = 1,2,3 are quark colours,

λ^a **a = 1,2,..8 are the Gell-Mann SU(3) matrices**

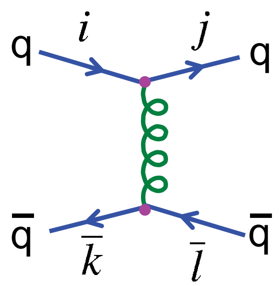
Feynman Rules for Weak

- Left-handed W interactions known as $V-A$ theory
  - ➡ γ^μ gives a vector current (V)
  - ➡ $\gamma^\mu\gamma^5$ gives an axial vector current (A)
- Photon interactions are purely vector
- Z-boson interactions contain both vector and axial-vector terms

	propagator 	interaction vertex 
W-boson	$\frac{g_{\mu\nu}}{q^2 - m_W^2}$	$\frac{1}{2\sqrt{2}} g_W \, \gamma^\mu (1-\gamma^5)$
Z-boson	$\frac{g_{\mu\nu}}{q^2 - m_Z^2}$	$\frac{1}{2} g_Z \, \gamma^\mu (c_V^f - c_A^f \gamma^5)$
photon, γ	$\frac{g_{\mu\nu}}{q^2}$	$e \, \gamma^\mu$

- Quark couplings have extra factor of $V_{u_i d_j}$
- $P_L = (1-\gamma^5)/2$ is the Left Handed projection operator, therefore W-boson interactions only act on **left-handed components** of fermions
- For low energy interactions $q << m_W$: effective propagator is $g_{\mu\nu}/m_W^2$

Feynman Rules Examples



- Follow fermion lines backwards!

Squaring the Matrix Element

- Squaring spinors, ***u*** and ***v*** to get terms only dependent on momentum is beyond the scope of the course. You may have to write down $\mathcal{M}$ using Feynman rules, but you won't have to square it.

- Usually $\mathcal{M}^2$ is for a particular spin configuration.

- To calculate an unpolarised cross-section need to **average over initial state spins** and **sum over possible spins configurations.**

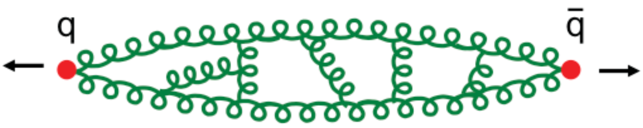
- e.g. for a electron-positron scattering.
  - Sum over all possible spin combinations.
    - Unpolarised electrons are ½ spin up, ½ spin down.
    - Unpolarised positrons are ½ spin up, ½ spin down.
  - Therefore average is by ¼.

QCD Potential

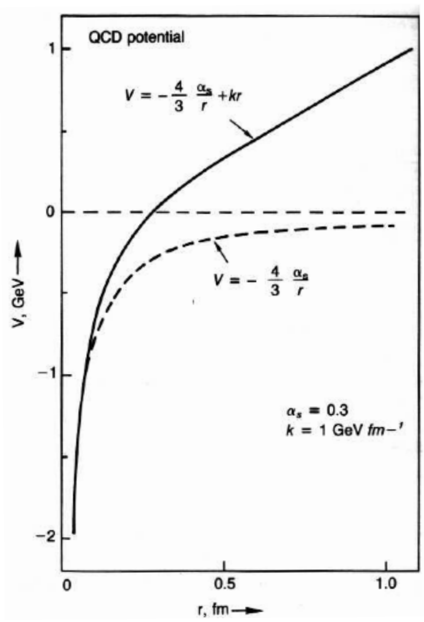
- At short distances gluons display a potential of

$$V_{q\bar{q}} = -\frac{4}{3}\frac{\alpha_S}{r}$$

- (review where the 4/3 comes from!)
- Gluons also carry colour charge and can therefore self-interact.
  - To separate an quark anti-quark pair to long distances $V_{\rm QCD}(r) \sim \lambda r$



- A gluon **flux tube** of interacting gluons is formed. Energy **~1 GeV/fm.**
- Gluon-gluon interactions are responsible for holding quarks in mesons and baryons.



$$V_{\rm QCD}(r) = -\frac{4}{3}\frac{\alpha_S}{r} + \lambda r$$

Jets

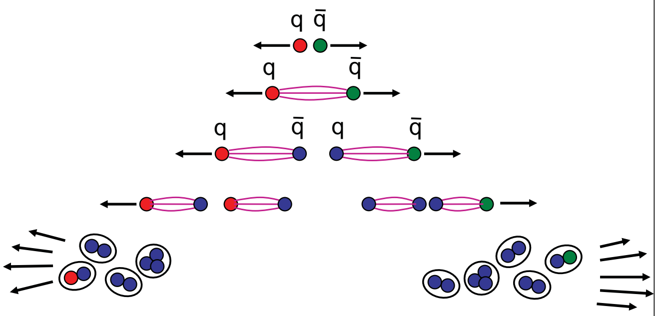
- Consider a quark and anti-quark produced in electron positron annihilation

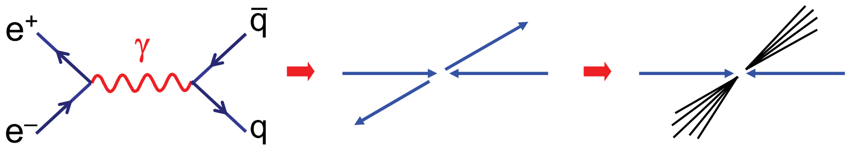
(i) Initially Quarks separate at high velocity

(ii) Colour flux tube forms between quarks

(iii) Energy stored in the flux tube sufficient to produce $q\bar{q}$ pairs

(iv) Process continues until quarks pair up into jets of colourless hadrons

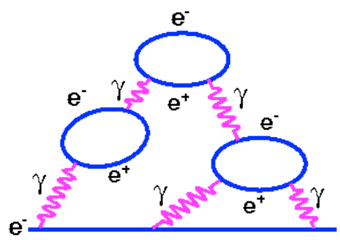


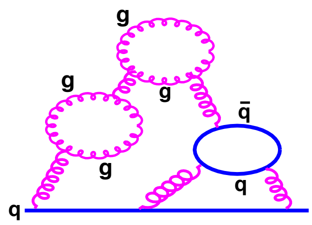


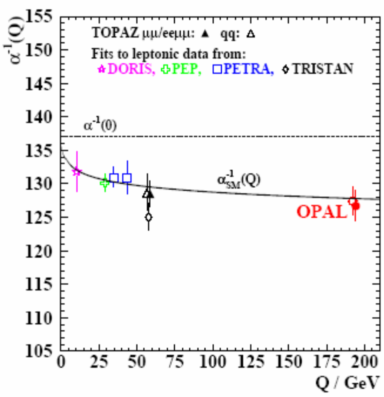
- This process is called **hadronisation**. It is not (yet) calculable.
- The main consequence is that at collider experiments quarks and gluons observed as **jets** of particles

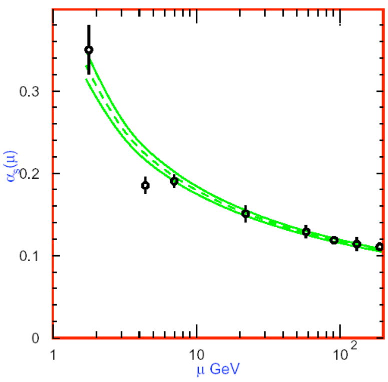
Running Coupling Constants

- The effective mass and charge of the fermions change depending on the momentum transferred by the boson probing the interaction.
- Small effect for QED - coupling strengthens at higher momentum transfer, Q.
- Different for QCD due to gluon self-couplings. Coupling strengthens strongly at low Q. When $\alpha_S > \sim 1$, perturbation theory no longer useful



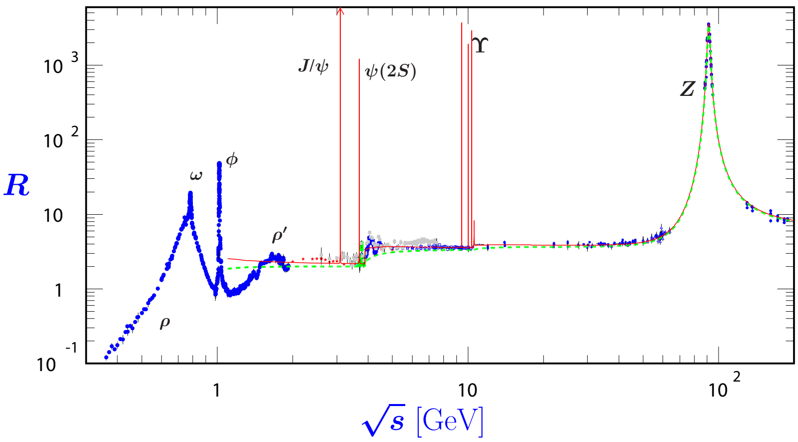






Measurement of R

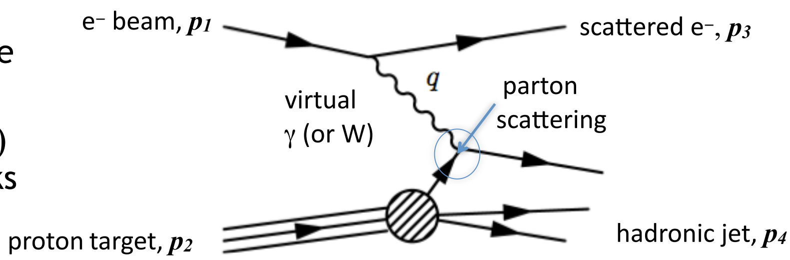
$$\mathbf{R} = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_q \frac{e_q^2}{e^2}$$



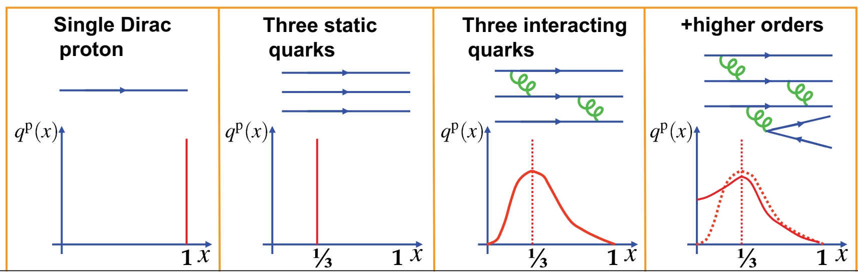
- N_C=3, one of the key pieces of evidence for three quark colours.
- At quark thresholds, $\sqrt{s} \sim 2m_q$ "resonances" occur as bound states of $\mathbf{q\bar{q}}$ more easily produced.
- Steps at ~**4** and ~**10 GeV** due to charm and bottom quark threshold
- At $\sqrt{s} \sim$ **100 GeV**, Z-boson exchange takes over.

Electron-Proton Scattering

- Electromagnetic $e^-p \to e^-p$ scattering is used to probe the structure of the proton.
- Deep inelastic scattering (DIS) occurs when the proton breaks up and reveals partonic structure of the proton.



- Parton model states that photon scatters elastically from partons within the proton: three valance quarks (**u, u, d**); sea quarks in pairs ($\mathbf{\bar{u}u, \bar{d}d, \bar{s}s, \bar{c}c, \ldots}$) and gluons, **g**
- Key parameter x, fraction of proton momentum carried by parton: $0 < x < 1$
- Parton distribution functions, $\mathbf{q}(x)$ for each quark flavour type. Represent represent the probability to find a parton in the proton with energy between x and $x+dx$



Cabibbo-Kobayashi-Maskawa Matrix

- **Mass eigenstates** and **weak eigenstates** of quarks are not identical.
  - ➡ Decay properties measure mass eigenstates with a definite lifetime and decay width
  - ➡ The weak force acts on the weak eigenstates.
- Weak eigenstates are admixture of mass eigenstates, conventionally described using CKM matrix a mixture of the down-type quarks:

weak eigenstates $$\begin{pmatrix} \mathrm{d}' \\ \mathrm{s}' \\ \mathrm{b}' \end{pmatrix} = \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} \mathrm{d} \\ \mathrm{s} \\ \mathrm{b} \end{pmatrix}$$ mass eigenstates

- The CKM matrix is unitary, $V_{\mathbf{CKM}}{}^{\dagger} V_{\mathbf{CKM}} = \mathbf{1}$ implies nine "unitarity relations"

$$\begin{pmatrix} V^*_{\mathrm{ud}} & V^*_{\mathrm{cd}} & V^*_{\mathrm{td}} \\ V^*_{\mathrm{us}} & V^*_{\mathrm{cs}} & V^*_{\mathrm{ts}} \\ V^*_{\mathrm{ub}} & V^*_{\mathrm{cb}} & V^*_{\mathrm{tb}} \end{pmatrix} \begin{pmatrix} V_{\mathrm{ud}} & V_{\mathrm{us}} & V_{\mathrm{ub}} \\ V_{\mathrm{cd}} & V_{\mathrm{cs}} & V_{\mathrm{cb}} \\ V_{\mathrm{td}} & V_{\mathrm{ts}} & V_{\mathrm{tb}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

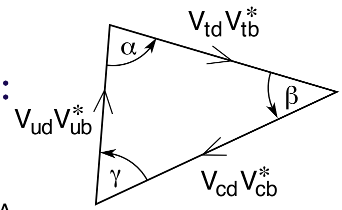
- The most frequently discussed is (1st row × 3rd column):

$$V_{\mathrm{ud}} V^*_{\mathrm{ub}} + V_{\mathrm{td}} V^*_{\mathrm{tb}} + V_{\mathrm{cd}} V^*_{\mathrm{cb}} = 0$$

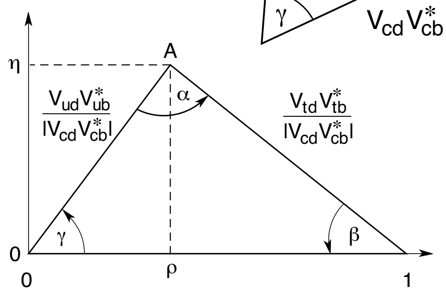
The Unitarity Triangle

$$V_{\mathrm{ud}} V^*_{\mathrm{ub}} + V_{\mathrm{td}} V^*_{\mathrm{tb}} + V_{\mathrm{cd}} V^*_{\mathrm{cb}} = 0$$

- Forms a triangle in the complex plane:



- Dividing through by $V_{cd}V_{cb}{}^*$:

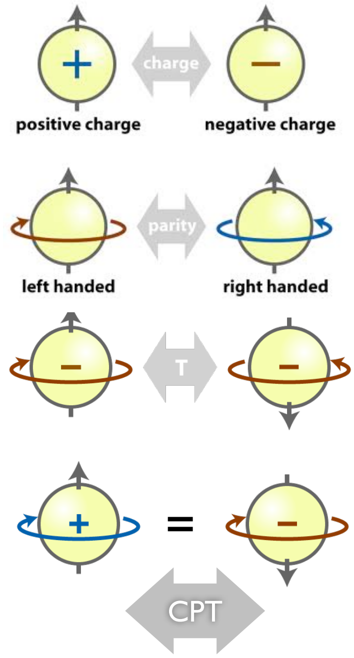


- This **unitarity triangle** is often use to present measurements of ***CP*** violation in **B**-meson decay.
- Lengths and angles of the triangle are: $\left|\frac{V_{\mathrm{ud}} V^*_{\mathrm{ub}}}{V_{\mathrm{cd}} V^*_{\mathrm{cb}}}\right| \quad \left|\frac{V_{\mathrm{td}} V^*_{\mathrm{tb}}}{V_{\mathrm{cd}} V^*_{\mathrm{cb}}}\right|$

$$\alpha \equiv \arg\left(-\frac{V_{\mathrm{td}} V^*_{\mathrm{tb}}}{V_{\mathrm{ud}} V^*_{\mathrm{ub}}}\right) \qquad \beta \equiv \arg\left(-\frac{V_{\mathrm{cd}} V^*_{\mathrm{cb}}}{V_{\mathrm{td}} V^*_{\mathrm{tb}}}\right) \qquad \gamma \equiv \arg\left(-\frac{V_{\mathrm{ud}} V^*_{\mathrm{ub}}}{V_{\mathrm{cd}} V^*_{\mathrm{cb}}}\right)$$

- Triangle has a finite area only if relative complex phase between CKM elements

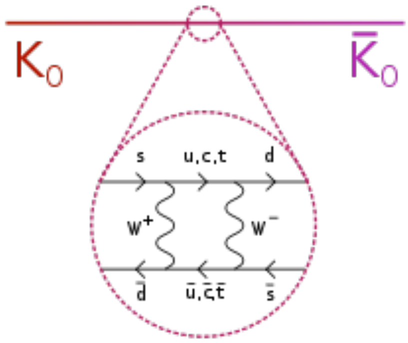
C, P and T



- Three important discreet symmetries: Charge Conjugation (C), Parity (P) and Time reversal (T).
  - C: changes sign of charge (particle ↔ anti-particle)
  - P: spatial inversion, reserves helicity. Fermions have P=+1, antifermions P=–1
  - T: changes the initial and final states
  - Gluons and photons have C =–1, P=–1
- C and P are conserved in QED and QCD, maximally violated in weak
- CPT turns a forward-going particle with LH helicity into backward-going antiparticle with RH helicity.
- CPT Theorem states all interaction invariant under combined operation of CPT

Neutral Meson Mixing

- Neutral mesons can transform into their antiparticles by exchanging two W-bosons (2nd order weak interaction)
  - e.g. neutral kaon mixing $\mathbf{K^0} \leftrightarrow \mathbf{\overline{K}^0}$ $\quad \mathbf{\bar{s}\,d} \leftrightarrow \mathbf{s\,\bar{d}}$
  - Three useful eigenbases:
  - Flavour eigenstates $\mathbf{K^0}, \mathbf{\overline{K}^0}$
  - CP eigenstates



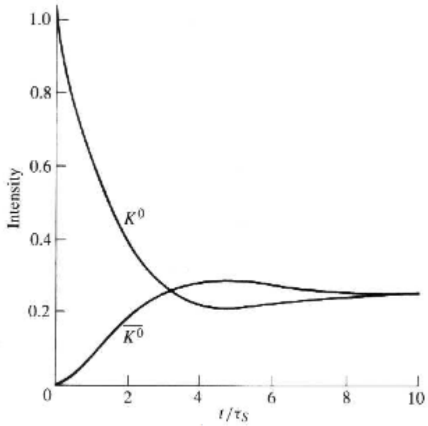
$$|\mathrm{K}_1\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{K}^0\rangle - |\overline{\mathrm{K}}^0\rangle\right) \qquad CP = +1$$

$$|\mathrm{K}_2\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{K}^0\rangle + |\overline{\mathrm{K}}^0\rangle\right) \qquad CP = -1$$

  - Decay eigenstate with measurable mass and lifetime. If these are not equal to CP eigenstates indicates the amplitudes $\mathbf{K^0} \to \mathbf{\overline{K}^0}$ and $\mathbf{\overline{K}^0} \to \mathbf{K^0}$ not equal.

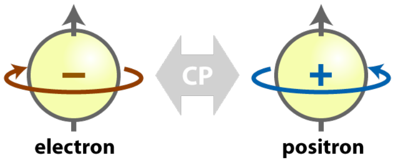
$$|\mathrm{K_S}\rangle = \frac{1}{N}\left((1-\epsilon)|\mathrm{K}^0\rangle - (1+\epsilon)|\overline{\mathrm{K}}^0\rangle\right)$$

$$|\mathrm{K_L}\rangle = \frac{1}{N}\left((1+\epsilon)|\mathrm{K}^0\rangle + (1-\epsilon)|\overline{\mathrm{K}}^0\rangle\right)$$



CP Violation

- ***CP*** turns a particle into its antiparticle with opposite helicity: it is a symmetry between matter and anti-matter



- ***CP*** violation occurs when particles related by ***CP*** symmetry do not interact in the same way.
- ***CP*** violation is only observed in weak force interactions.

- Most often measured by looking a neutral mesons decays (e.g. neutral kaons, neutral B-mesons)

- In the Standard Model ***CP*** violation is accommodated by a complex phase in the CKM matrix. This ensures the **unitarity triangles** have a finite area

Neutrinos

- In the Standard Model only left-handed neutrinos and right-handed antineutrinos are described.
  - Neutrino experiments have observed neutrino oscillations e.g. $\nu_e \longrightarrow \nu_\mu$ implying neutrinos have mass.

- Mass eigenstates of the neutrinos are not identical to the flavour eigenstates.
- Flavour eigenstates are ν_e, ν_μ, ν_τ - interact with the W and Z boson.
- Mass eigenstates are ν_1, ν_2, ν_3 - propagate through matter / vacuum.
- Eigenstates related by PMNS matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- Measurements of neutrino oscillations used to find mixing angles and Δm^2 between mass eigenstates.

Two Neutrino Mixing

- Let's start with the case of two neutrino mixing. Write the mixing matrix in terms of a mixing angle θ_{12} (to reflect the unitarity of the matrix):

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

- The time evolution of the two mass eigenstates is:

$$\begin{aligned} \nu_1(t) = \nu_1(0)e^{-iE_1t} &= \quad [\nu_e(0)\cos\theta_{12} + \nu_\mu(0)\sin\theta_{12}] \quad e^{-iE_1t} \\ \nu_2(t) = \nu_2(0)e^{-iE_2t} &= \quad [-\nu_e(0)\sin\theta_{12} + \nu_\mu(0)\cos\theta_{12}] \quad e^{-iE_2t} \end{aligned}$$

- For a initial state of pure ν_e, $\nu_e(0)=1$, time evolution:

$$\begin{aligned} \nu_\mu(t) &= (\cos\theta_{12}\sin\theta_{12})(e^{-iE_1t} - e^{-iE_2t}) \\ \nu_e(t) &= (1 - \cos\theta_{12}\sin\theta_{12})(e^{-iE_1t} - e^{-iE_2t}) \end{aligned}$$

- Probability for an ν_e to turn into ν_μ: $P(\nu_e \to \nu_\mu) = |\nu_\mu(t)|^2$

$$\begin{aligned} P(\nu_e \to \nu_\mu) &= (\cos\theta_{12}\sin\theta_{12})^2 (e^{iE_1t} - e^{iE_2t})(e^{-iE_1t} - e^{-iE_2t}) \\ &= \left[\sin(2\theta_{12})\sin\left(\frac{E_2 - E_1}{2}t\right)\right]^2 = \left[\sin(2\theta_{12})\sin\left(\frac{\Delta m_{12}^2}{4E}t\right)\right]^2 \end{aligned}$$

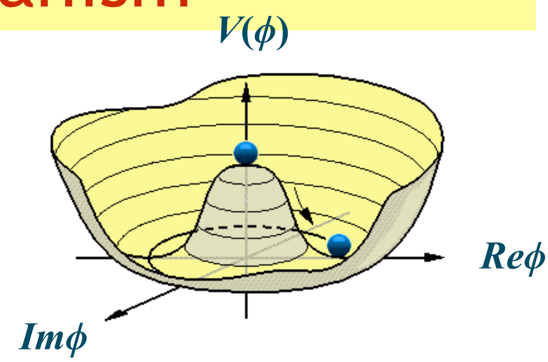
Summary of Electroweak Unification

- We have recovered the behaviour of the $W^\pm$, Z and γ
  - ➡We introduced an SU(2) symmetry (3 bosons) coupling to weak isospin with a coupling constant g_W
  - ➡We introduced a U(1) symmetry (1 boson) coupling to weak hypercharge with a coupling constant g'_W
  - ➡Together predicts four bosons we identify with W^+, W^-, Z and γ
  - ➡Electroweak Theory is often called **SU(2) ⊗ U(1)** model

- All of the properties of electroweak interactions described by:
  - the intrinsic charges of the fermions
  - the SU(2) ⊗ U(1) symmetry
  - g_W and g'_W: free parameters that need to be measured

- Along with QCD, Electroweak Theory is the Standard Model.

Higgs Mechanism



$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$$

- ϕ is complex function:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

- $V(\phi)$ is symmetric: to maximise the symmetry of the system choose $\phi=0$.
- A circle of values minimise the potential at $\phi=\phi_0 \equiv -v$ with $|\phi_0| = \frac{\mu}{\sqrt{2\lambda}}$
- The choice of a particular value of ϕ_0 spontaneously breaks the symmetry.
- When the symmetry is spontaneously broken three of these degrees of freedom are fixed and used to give mass to W^+, W^-, Z^0.

Introducing the Higgs Boson

- Consider a fluctuation of the Higgs field about its minimum:

$$\phi(x) = \phi_0 + h(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

- Substitute $\phi(x) = \frac{1}{\sqrt{2}}(v + h(x))$ into $V(\phi)$ and expand to second order in $h(x)$:

$$V(\phi) = -\mu^2 \left(\frac{v+h(x)}{\sqrt{2}}\right)^2 + \lambda \left(\frac{v+h(x)}{\sqrt{2}}\right)^4 = \ldots = V(\phi_0) + \underbrace{\lambda v^2}_{= \frac{1}{2} m_H^2} h^2 + \mathcal{O}(h(x)^3)$$

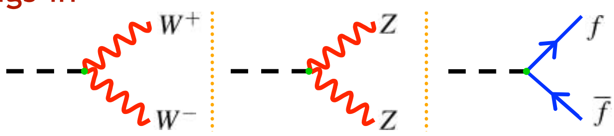
- In quantum field theory a term quadratic in the field describes a particle's mass.
- This fluctuation around the minimum of the potential describes a spin-0 particle with a mass $m = \sqrt{2\lambda} v$
- **The Higgs boson!**

Higgs Searches

- Look for decays of the Higgs boson at the LHC

The strength of Higgs boson couplings in order:

1. ***W***-boson
2. ***Z***-boson
3. fermions: from heaviest to lightest



Higgs boson also couples to photon pairs through ***W***-boson and top-quark loops