Particle Physics - Problem Sheet 1

Discussion Questions

D1 Composite models of quarks and leptons have been proposed using "preons", which are spin-1/2 fermions, and their antipreons with equal and opposite quantum numbers. Consider these two simple models:

(a) The two "rishon" model with T (charge $+1/3e$) and V (charge 0)
Harari, Phys.Lett. B86:83 (1979)

(b) The three preon model with α, δ (both $+1/3e$) and β ($-2/3e$)
Dugne et al., Europhys.Lett.57:188 (2002) hep-ph/0208135

For each model discuss how the quarks and leptons can be constructed from the preons. What things might you hope to explain with a preon model? What problems are there with preon models?

D2 You have access to a high energy linear collider which can accelerate either e^- or e^+ beams and collide any of e^-e^-, e^-e^+ or e^+e^+. Discuss how such a facility could be used to search for lepton flavour violation in electromagnetic interactions.

Standard Problems

S1 Draw the lowest order Feynman diagrams, and describe qualitatively how the angular distributions of the following high energy scattering processes differ from each other:

(a) Bhabha scattering $e^+e^- \to e^+e^-$

(b) Muon pair production $e^+e^- \to \mu^+\mu^-$

(c) Moeller scattering $e^-e^- \to e^-e^-$

(d) Electron-muon scattering $e^-\mu^- \to e^-\mu^-$

S2 The Mandelstam variables s, t, u in the scattering process $1+2 \to 3+4$ are defined in terms of the momentum 4-vectors (p^μ) as:

$$\begin{aligned} s &= (p_3^\mu + p_4^\mu)^2 \\ t &= (p_1^\mu - p_3^\mu)^2 \\ u &= (p_1^\mu - p_4^\mu)^2 \end{aligned}$$

(a) Show that $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$.

(b) Show that $\sqrt{s}$ is the total energy of the collision in the centre of mass frame.

S3 Draw the Feynman Diagram for elastic scattering of two spinless particles by a photon.

Show that the Matrix element $\mathcal{M}$ for this process can be written as:

$$\mathcal{M} = \frac{\alpha}{q^2}\,(p_1 + p_3)(p_2 + p_4)\,\delta^4(p_1 + p_2 - p_3 - p_4)$$

Where p_1, p_2 are the initial four-momenta and p_3, p_4 are the final momenta of the particles.

What is the differential cross section $d\sigma/d\Omega$ for this process?

S4 In a 2-body decay, $1 \to 2 + 3$, show that the three-momentum of the final state particles in the centre of mass frame has magnitude:

$$|\vec{p}^*| = \frac{c}{2m_1}\sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2 - 2m_1^2m_3^2 - 2m_2^2m_3^2}$$

S5 The π^+meson decays almost entirely via the two body decay process $\pi^+ \to \mu^+\nu_\mu$ with an matrix element given by

$$|\mathcal{M}|^2 = G_F^2 f_\pi^2 m_\mu^2 (m_\pi^2 - m_\mu^2)$$

where $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant, and f_π is related to the size of the pion wavefunction (the pion being a composite object).

(a) Obtain a formula for the $\pi^+ \to \mu^+\nu_\mu$ decay rate. Assuming $f_\pi \sim m_\pi$, calculate the pion lifetime in natural units and in seconds, and compare to the measured value. Note that $m_\pi = 139.6$ MeV, $m_\mu = 105.7$ MeV, $\tau_\pi = 2.6 \times 10^{-8}$ s.

(b) By replacing m_μ by m_e, show that the rate of $\pi^+ \to e^+\nu_e$ is $1.28 \times 10^{?4}$ times smaller than the corresponding decay rate to muons. Show also that, on the basis of phase space alone (i.e. neglecting the factor $|\mathcal{M}|^2$), the decay rate to electrons would be expected to be greater than the rate to muons.