Particle Physics - Problem Sheet 2

Discussion Questions

D1 Weak charged currents are described by $\gamma^\mu(1-\gamma^5)$, whereas electromagnetic currents are just γ^μ.

(a) Explain why the weak charged current only couples to left-handed fermions or right-handed antifermions.

(b) Why does the weak neutral current have coefficients c_V and c_A? What is the significance of the values $c_V = c_A = 1$ for neutrinos? Why do other fermions have different values of c_V and c_A?

Standard Problems

S1 The normalisation condition for Dirac spinors is $u^\dagger u = 2E$. Show that the normalisation constant is $1/N = 1/\sqrt{E+m}$.

S2 For the Dirac spinors u^1 and u^2 show that the lower components are smaller than the upper ones by a factor of $\beta = v/c$ for a relativistic particle. What happens to the spinors for a non-relativistic particle?

S3 (i) Draw the Feynman diagram for electron-muon scattering $e^-\mu^- \to e^-\mu^-$ and write down the matrix element for this process in terms of initial and final state particle spinors.

(ii) What are the possible initial helicity configurations? In the high energy limit, what are the possible final energy helicity configurations?

For $e^-\mu^- \to e^-\mu^-$ scattering we found at lowest order masses of e and μ we found:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2\pi s}\left(\frac{s^2+u^2}{t^2}\right) \tag{1}$$

Where s, t and u are the Mandelstam variables.

(iii) Show that by integrating over the full solid angle $d\Omega = d\phi\, d(\cos\theta)$, over the limits $\phi = -\pi, +\pi$ and $\cos\theta = -1, +1$ that the total cross section in this approximation is infinite. (θ is the scattering angle between the incoming and outgoing electron.) What is the physical interpretation of this result?

You may wish to use:

$$\int \frac{dx}{(1-x)^2} = \frac{1}{1-x} \qquad \int \frac{(1+x)^2}{(1-x)^2}dx = x - \frac{4}{x-1} - 4\log(x-1)$$

S4 Consider the operators:

$$P_L = \frac{1}{2}(1 - \gamma^5) \qquad P_R = \frac{1}{2}(1 + \gamma^5)$$

(a) Show that the operators satisfy the following relations:

$$P_L^2 = P_L \qquad P_R^2 = P_R \qquad P_L + P_R = 1 \qquad P_L P_R = 0$$

Explain why this makes them projection operators in quantum mechanics.

(b) In the ultrarelativistic limit $\beta \to 1$ show that the operators acting on the Dirac spinors have the following properties:

$$P_L u^1 = 0 \qquad P_L v^1 = 0 \qquad P_L u^2 = u^2 \qquad P_L v^2 = v^2$$

$$P_R u^1 = u^1 \qquad P_R v^1 = v^1 \qquad P_R u^2 = 0 \qquad P_R v^2 = 0$$

Hence explain what it is that they project out.

S5 (i) Draw the Feynman diagram for muon decay: $\mu^- \to e^- \bar{\nu}_e \nu_\mu$. Write down the matrix element, $\mathcal{M}$ for the process, and show that the decay width for this decay must be proportional G_F^2, where G_F is the Fermi coupling constant.

(ii) Explain why this is the only possible decay of a muon.

(iii) The full calculation for the decay width gives:

$$\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Determine the value of G_F from muon decay. What is the dimensionless weak coupling constant g_W and how does it compare with e?

S6 Explain how the following measurements demonstrate lepton universality between weak couplings:

$$m_\mu = 105.65869 \pm 0.000009 \text{ MeV} \qquad \tau_\mu = 2.19703 \pm 0.00004\ \mu\text{s}$$

$$m_\tau = 1776.99 \pm 0.26 \text{ MeV} \qquad \tau_\tau = 290.6 \pm 1.0 \text{ fs}$$

$$\text{BR}(\mu \to e \bar{\nu}_e \nu_\mu) = 100\%$$

$$\text{BR}(\tau \to e \bar{\nu}_e \nu_\tau) = 17.85 \pm 0.05\% \qquad \text{BR}(\tau \to \mu \bar{\nu}_\mu \nu_\tau) = 17.36 \pm 0.05\%$$