Particle Physics - Problem Sheet 4

Discussion Questions

D1 In a hadron collider, the existence of neutrinos in the final is inferred by "large missing transverse momentum" or "large missing transverse energy", $\not{E}_T$, defined as:

$$\not{E}_x = -\sum_i p_x^i \qquad \not{E}_y = -\sum_i p_y^i \qquad \not{E}_T = \sqrt{(\not{E}_x)^2 + (\not{E}_y)^2}$$

Where p_x^i and p_y^i are the reconstructed momentum of all the observed particles in the x and y direction, and z is the direction of the initial protons (anti-protons). Typically a value of $\not{E}_T > 25$ GeV is used to identify neutrinos in the event.

(a) Why might a values of $\not{E}_T > 25$ GeV indicate the presence of neutrinos in the detector?

(b) Why are isn't $\sum p_z^i$ included in the definition of $\not{E}_T$?

(c) (*) What other particles could be indentified by looking for large values of missing transverse energy?

D2 The weak eigenstates of the K^0 mesons are written as:

$$\mathrm{K_L} = \frac{1}{\sqrt{N}}[(1+\epsilon)\mathrm{K}^0 - (1-\epsilon)\overline{\mathrm{K}}^0] \qquad \mathrm{K_S} = \frac{1}{\sqrt{N}}[(1+\epsilon)\mathrm{K}^0 + (1-\epsilon)\overline{\mathrm{K}}^0]$$

if CP and T are violated by ϵ and CPT is conserved.

Explain why the weak eigenstates of the K^0 mesons are written as:

$$\mathrm{K_L} = \frac{1}{\sqrt{N}}[(1-\delta)\mathrm{K}^0 - (1+\delta)\overline{\mathrm{K}}^0] \qquad \mathrm{K_S} = \frac{1}{\sqrt{N}}[(1+\delta)\mathrm{K}^0 + (1-\delta)\overline{\mathrm{K}}^0]$$

if CP and CPT are violated by δ and T is conserved.

How would you try and constrain δ experimentally?

D3 In the SNO experiment, solar neutrinos are detected in heavy water (D_2O) via three reactions:

- neutral current scattering on electrons $\nu + e^- \rightarrow \nu + e^-$ (ES)
- charged current scattering on deuterium $\nu + d \rightarrow p + p + e^-$ (CC)
- neutral current scattering on deuterium $\nu + d \rightarrow n + p + \nu$ (NC)

(a) Discuss which flavours of neutrinos each reaction is sensitive to.

(b) Discuss whether you would expect to observe any of the following:
  - A day/night variation of the neutrino flux from the sun.
  - A winter/summer variation of the neutrino flux from the sun.
  - A variation in neutrino flux over the 11 year solar sunspot cycle.

Standard Questions

S1 The parity operator can be described by γ^0:

$$\gamma^0 = \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\mathbf{I} \end{pmatrix}$$

(a) Show that a transformation $\psi(t, \vec{x}) \to \psi'(t', \vec{x}\,') = \gamma^0\psi$ is also a solution to the Dirac equation, if $\vec{x} \to \vec{x}\,' = -\vec{x}, t \to t' = +t$.

(b) Show that the parity of a antiparticle spinor (v_1 or v_2) at rest is negative.

S2 (a) Draw a Feynman diagram illustrating $\mathrm{K}^0 \leftrightarrow \overline{\mathrm{K}}^0$ mixing.

(b) In the loop, the dominant contribution is from charm and top quarks. What combinations of CKM matrix elements are used to describe $\mathrm{K}^0 \leftrightarrow \overline{\mathrm{K}}^0$ mixing? Using the Wolfenstein parameterisation show that at least one of these combinations has a complex phase.

S3 By considering the semileptonic decays of the K^0 and $\overline{\mathrm{K}}^0$ derive the following expression for the semileptonic charge asymmetry:

$$\delta_\ell = \frac{\Gamma(\mathrm{K^0_L} \to \pi^-\ell^+\nu_\ell) - \Gamma(\mathrm{K^0_L} \to \pi^+\ell^-\bar{\nu}_\ell)}{\Gamma(\mathrm{K^0_L} \to \pi^-\ell^+\nu_\ell) + \Gamma(\mathrm{K^0_L} \to \pi^+\ell^-\bar{\nu}_\ell)} = \frac{2\,Re(\epsilon)}{1+|\epsilon|^2} \approx 2\,Re(\epsilon)$$

Starting with an initially pure $\overline{\mathrm{K}}^0$ beam, what is the K^0 and $\overline{\mathrm{K}}^0$ content at a later time $\tau_S < t < \tau_L$?

S4 Show the the "unitarity triangle" relation $V_{\mathrm{ud}}V^*_{\mathrm{ub}} + V_{\mathrm{td}}V^*_{\mathrm{tb}} + V_{\mathrm{cd}}V^*_{\mathrm{cb}} = 0$ is a consequence of the unitarity of the CKM matrix.

S5 Show that the probability of $\nu_\mu \to \nu_\tau$ disappearance is:

$$P(\nu_\mu \to \nu_\tau) = \sin^2 2\theta_{23} \sin^2\left(\frac{1.27\Delta m^2_{23} L}{E}\right)$$

with L in metres, Δm in eV^2 and E in MeV. Determine this probability for the K2K experiment, using $\Delta m^2_{23} = 2.4 \times 10^{-3}\mathrm{eV}^2$, $E = 1\mathrm{GeV}$ and $L = 250\mathrm{km}$.

Hint: ignore the possibility of mixing $\nu_\mu \to \nu_e$ (as θ_{13} is small) and treat this as two neutrino mixing.