Particle Physics

Dr Victoria Martin, Spring Semester 2013

Lecture 3: Feynman Diagrams, Decays and Scattering

- ★Feynman Diagrams continued
- ★Decays, Scattering and Fermi's Golden Rule
- ★Anti-matter?

Notation Review

- A μ sub- or super- script represents a four vector, e.g. x^μ, p^μ, p_μ
  - μ runs from 0 to 3

$$p^\mu = (p^0, p^1, p^2, p^3) = (E, p_x, p_y, p_z)$$

- This lecture also introduce other quantities with μ index, $\mu=0,1,2,3$

- The scalar product of two four vectors

$$a^\mu b_\mu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3$$

- The three dimension differential operator

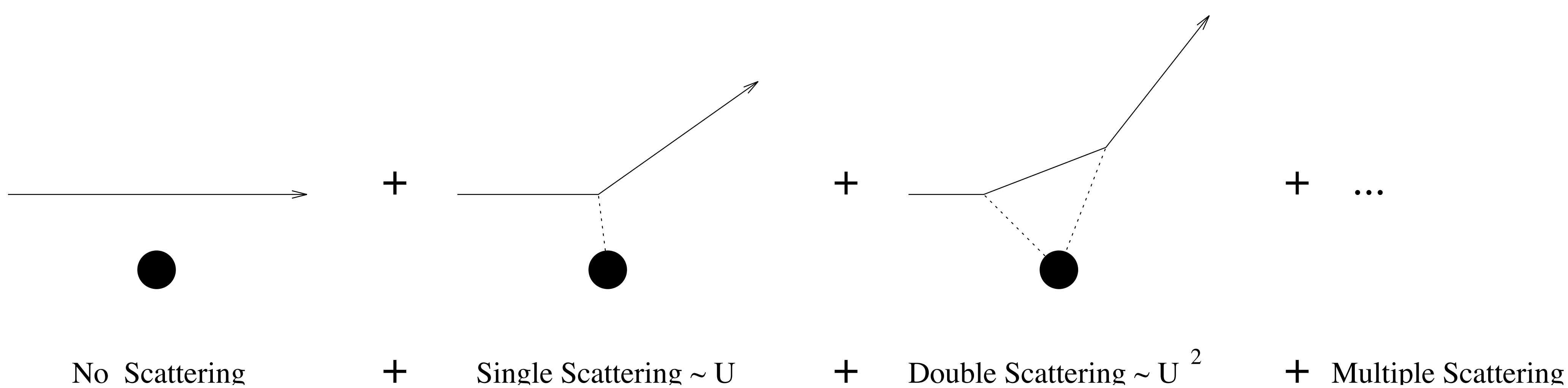
$$\vec{\bigtriangledown} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

- Four dimension differential operator

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$$

Scattering Theory

- Consider the interactions between **elementary particles.**
- Review from Quantum Physics, Lecture 12, 13: Quantum Scattering Theory & the Born Approximation
- Born Series: we can think of a scattering in terms of series of terms



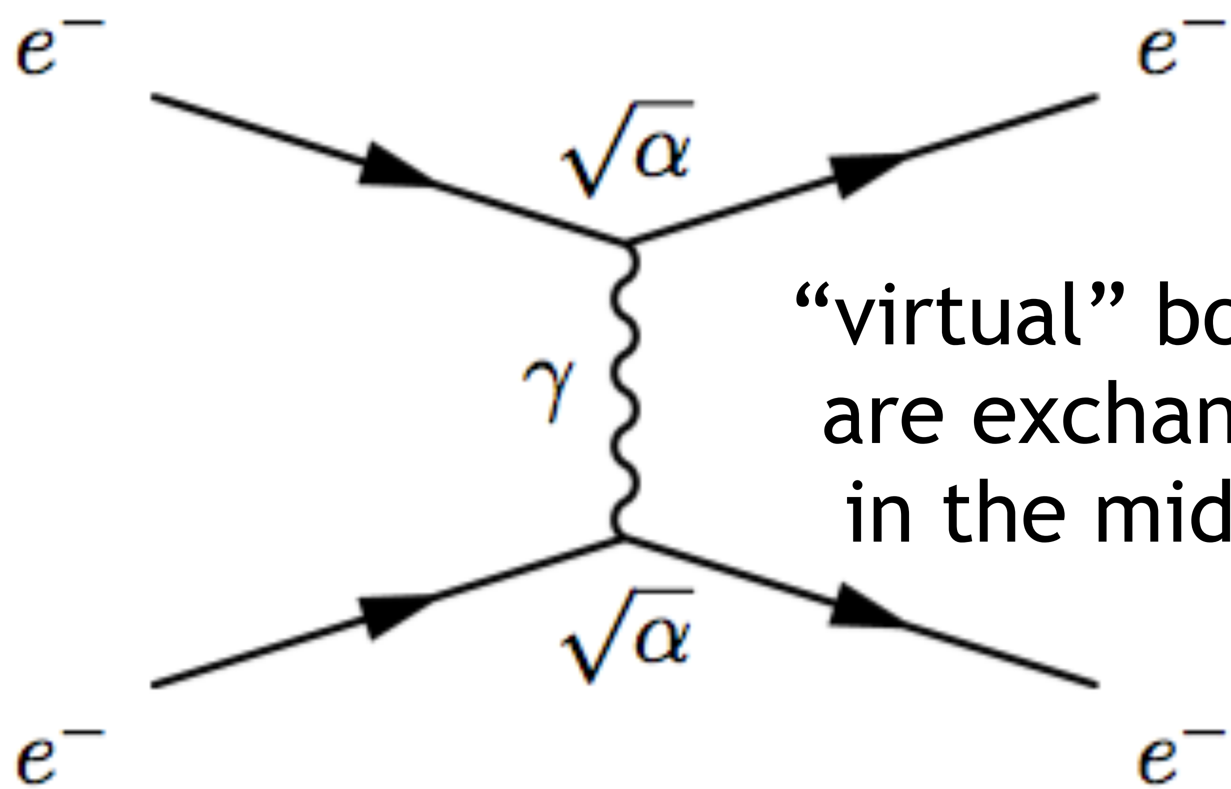
- 1 boson exchange is more probable than 2 boson exchange which is more probable than 3 boson exchange...
- The total probability for a scattering is the sum of all possible numbers of boson exchange:

$$\mathcal{M}_{\text{tot}} = \mathcal{M}_1 + \mathcal{M}_2 + \mathcal{M}_3 \ldots$$

- Feynman diagrams make use of the Born series to calculate the individual terms in the **matrix elements** series $\mathcal{M}_i$

Drawing Feynman Diagrams

Initial state particles on the left

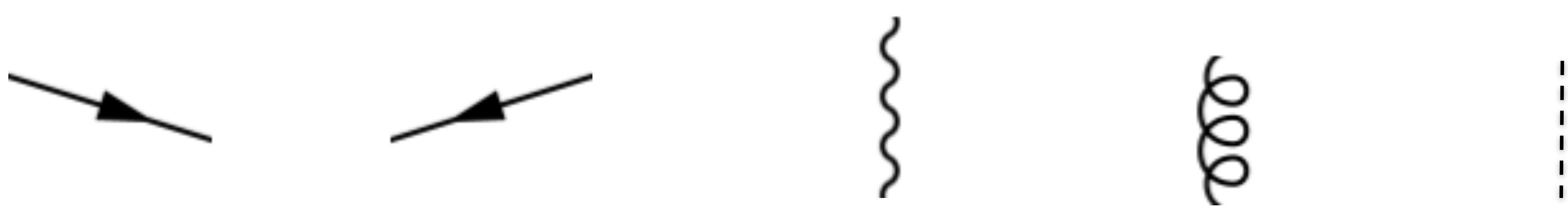


"virtual" bosons are exchanged in the middle

Final state particles on the right

Times flows from left to right

Each interaction vertex has a coupling constant



fermions | antifermions | photons, *W*, *Z* bosons | gluons | *H* bosons

The Feynman Rules

- Each part of the feynman diagram has a function associated with it. Multiply all parts together to get a term in the Born expansion

- Initial and final state particles have **wavefunctions**:
  - Spin-0 bosons are plane waves
  - Spin-½ fermions have Dirac spinors
  - Spin-1 bosons have polarization vectors ε^μ

- Vertices have dimensionless **coupling constants**:
  - Electromagnetism has $\sqrt{\alpha} = e$
  - Strong interaction has $\sqrt{\alpha_s} = g_s$
  - Weak interactions have g_L and g_R (or c_A and c_V)

- Virtual particles have **propagators**, q^μ is momentum transferred by boson
  - Virtual photon propagator is $1/q^2$
  - Virtual W/Z boson propagator is $1/(q^2 - M_W^2)$; $1/(q^2 - M_Z^2)$
  - Virtual fermion propagator is $(\gamma^\mu q_\mu + m)/(q^2 - m^2)$

γ^μ (Gamma matrices) and spinors … next lecture

Plane Waves

- Plane wave can be used to describe spinless, chargeless particles:

$$\psi = e^{-ip\cdot x} \qquad\qquad p\cdot x = p^\mu x_\mu = \hbar(\vec{k}\cdot\vec{x} - \omega t)$$

- Define the **probability current** for the particle

$$j_\mu = i\left[\psi^*\,\partial_\mu\psi - \psi\,\partial_\mu\psi^*\right]$$

- Note this is a four-dimensional quantity:

$$j^\mu = (j^0, \vec{j})$$

- For a scattered particle changing momentum $\boldsymbol{p_i \longrightarrow p_f}$:

Wave for initial state

$$\psi_i = e^{-ip_i\cdot x}$$

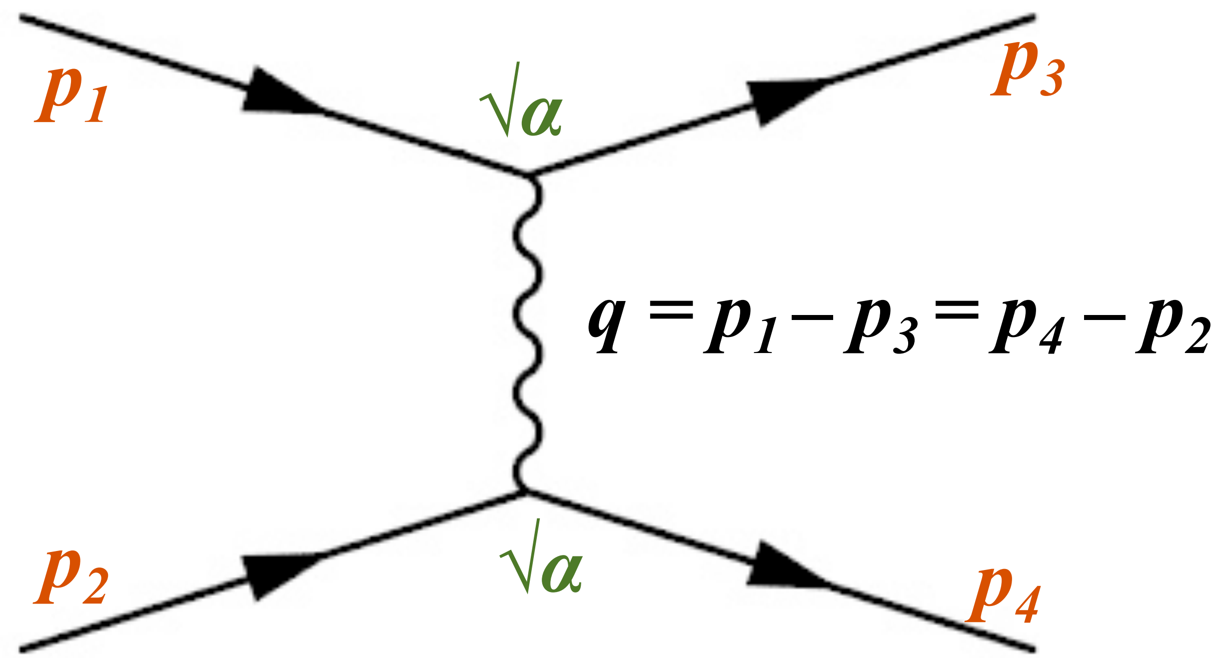
Wave for final state

$$\psi_f = e^{-ip_f\cdot x}$$

$$\begin{aligned} j_\mu(i\to f) &= i\left[\psi_f^*\,\partial_\mu\psi_i - \psi_i\,\partial_\mu\psi_f^*\right] \\ &= (p_i + p_f)e^{-i(p_i - p_f)\cdot x} \end{aligned}$$

Matrix Element for Spinless Scattering

Hypothetical interaction in which two ***spinless*** charged particles exchange one virtual photon



Vertex Couplings

$$\mathcal{M} = \frac{\alpha}{q^2}(p_1 + p_3)(p_2 + p_4)\,\delta^4(p_1 + p_2 - p_3 - p_4)$$

Photon Propagator

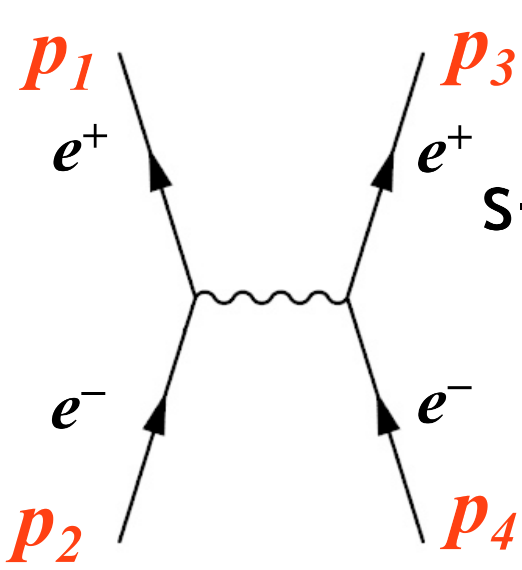
Plane Waves

Four momentum conservation

(in terms of Mandelstam variables) $\mathcal{M} = \dfrac{\alpha\,(s-u)}{t}$

Mandelstam Variables

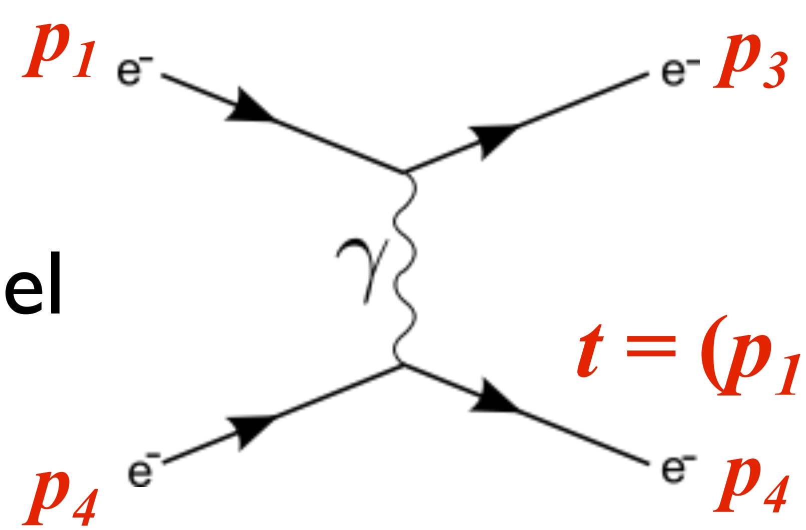
Introduce the Lorentz invariant scattering variables: s, t and u



s-channel

$$s = (p_1 + p_2)^2$$

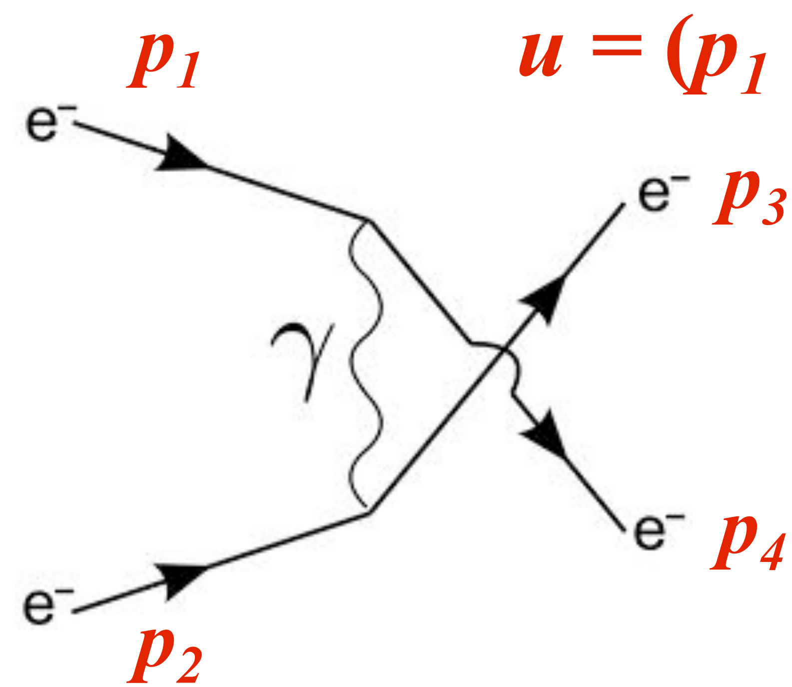
t-channel



$$t = (p_1 - p_3)^2$$

u-channel

$$u = (p_1 - p_4)^2$$



For highly relativistic elastic scattering
$p \sim E$, $m \ll E$:

$$s = 4\,p^{*2}$$
$$t = -2\,p^{*2}\,(1 - \cos\theta^*)$$
$$u = -2\,p^{*2}\,(1 + \cos\theta^*)$$

with $p^* = p_1 = p_2$ is the CM momentum of the particles, and θ^* is the CM scattering angle

Measuring Interactions

- To test the Standard Model (or any other model) of particle physics, relate Feynman diagrams with measurable quantities.

- Two main measurable processes in particle physics:
  - ★ particle decay e.g. $A \to c\, d$
    - ➡ measure decay width, $\Gamma\,(A \to c\, d)$
  - ★ scattering e.g. $a\, b \to c\, d$
    - ➡ measure cross section, $\sigma\,(a\, b \to c\, d)$

- Related to $\mathcal{M}$ (calculated from Feynman diagrams) through **Fermi's Golden Rule:**

$$\Gamma \sim \frac{2\pi}{\hbar}|\mathcal{M}|^2\rho \qquad\qquad \sigma \sim \frac{2\pi}{\hbar}|\mathcal{M}|^2\rho$$

- ρ is the phase space - a purely kinematic quantity

(see tutorial sheet and/or Griffiths Appendix B)

Measuring Decays

- Measure the lifetime of a particle in its own rest frame.
- Define the decay rate, Γ: the probability per unit time the particle will decay:

$$dN = -\Gamma N dt \qquad N(t) = N(0)e^{-\Gamma t}$$

- Mean lifetime is $\tau = 1 / \Gamma$ (natural units).
  - For τ in seconds can use $\tau = \hbar / \Gamma$

- Most particles decay more than one different route: add up all decay rates to obtain the **total decay rate**:

$$\Gamma_{\text{tot}} = \sum_{i=1}^{n} \Gamma_i$$

- The lifetime is the reciprocal of Γ_{tot}: $\tau = \frac{1}{\Gamma_{\text{tot}}}$
- The different final states of the particle are known as the **decay modes**.
- The **branching ratio** for the *i*th decay mode is: $\Gamma_i / \Gamma_{\text{tot}}$

Example: Decays of the K^0_S meson

- Collated by the particle data group: http://pdglive.lbl.gov

K^0_S $\qquad I(J^P) = \frac{1}{2}(0^-)$

Mean life $\tau = (0.8953 \pm 0.0005) \times 10^{-10}$ s

K^0_S DECAY MODES		Fraction (Γ_i/Γ)	Scale factor/ Confidence level	p (MeV/c)
Hadronic modes				
$\pi^0\pi^0$		(30.69 ± 0.05) %		209
$\pi^+\pi^-$		(69.20 ± 0.05) %		206
$\pi^+\pi^-\pi^0$		$(3.5 \ {}^{+1.1}_{-0.9}) \times 10^{-7}$		133
Modes with photons or $\ell\overline{\ell}$ pairs				
$\pi^+\pi^-\gamma$	[f,m]	$(1.79\pm0.05) \times 10^{-3}$		206
$\pi^+\pi^-e^+e^-$		$(4.79\pm0.15) \times 10^{-5}$		206
$\pi^0\gamma\gamma$	[m]	$(4.9 \pm1.8) \times 10^{-8}$		231
$\gamma\gamma$		$(2.63\pm0.17) \times 10^{-6}$	S=3.0	249
Semileptonic modes				
$\pi^\pm e^\mp \nu_e$	[n]	$(7.04\pm0.08) \times 10^{-4}$		229

- Which decay modes happen and which not provide information on symmetries and quantum numbers

Key Points

- Interactions in particle physics are caused by the exchange of bosons (photon, gluon, W, Z).
- Use perturbation theory to describe the interactions in terms of numbers of bosons.
  - ➡ 1 boson exchange is most probable
  - ➡ 2 boson exchange is next most probable
  - ➡ 3 " " " " " "
  - ➡ 4 …
- Use Feynman diagrams to illustrate these terms in the perturbation series
- Use Feynman rules to calculate a value for each Feynman diagram, the matrix element, $\mathcal{M}$
- The matrix element is used to calculate cross sections and decay widths to compare to experimental results.